8.4 Part 2
Monday, February 21, 2022 2:04 PM

Partial fraction

① cover Method

Exp (12)

$$\int \frac{2x+1}{x^2-7x+12}\,dx$$

$$\int \frac{2x+1}{(x-4)(x-3)}\,dx$$

linear factor "distinct"

$$\int \left(\frac{A}{x-4}+\frac{B}{x-3}\right)dx$$

$$A = \frac{2\boxed{4}+1}{\boxed{4}-3} = \frac{9}{1} = 9$$

$$B = \frac{2\boxed{3}+1}{\boxed{3}-4} = \frac{7}{-1} = -7$$

$$\int \left(\frac{9}{x-4}-\frac{7}{x-3}\right)dx$$

$$9\ln|x-4| - 7\ln|x-3| + c$$

Q. $\int \frac{f(x)}{g(x)}\,dx$ f and g are poly.

- If $\deg f \geq \deg g$ we use long Division

$$g(x)\overline{\big)\,f(x)}$$

- $$\int \frac{7}{(x-1)(x+1)^2}\,dx = \int \left(\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\right)$$

$(x+1)(x+1)$

$$\frac{1}{(x+1)(x+1)}$$

$$\int \frac{x}{x^3\,(x^2+1)^2} = \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2} \right)$$

$x \cdot x \cdot x$ $\qquad$ $(x^2+1)^2$

$$= \boxed{\int \frac{Ax^2+Bx+C}{x^3}}$$

$$\int \frac{x}{(x^2+1)\,(x^3+2)}\,dx = \int \left(\frac{Ax+B}{x^2+1} + \frac{Cx^2+Dx+E}{x^3+2} \right) dx$$

irred; $\qquad$ irred;

$$\int \frac{x^2}{(x-1)^3\,(x^2+1)^2}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} +$$

$$+ \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

Exp (18)

$$\int_{-1}^{0} \frac{x^3}{x^2-2x+1}\,dx$$

long Division

$$\begin{array}{r|l} & x+2 \\ \hline x^2-2x+1 & x^3 \\ & -x^3+2x^2 \pm x \\ \hline \end{array}$$

$$= \int_{-1}^{0} \left[(x+2) + \frac{3x-2}{x^2-2x+1} \right] dx$$

$$= \int_{-1}^{0} \left[(x+2) + \frac{3x-2}{x^2-2x+1} \right] dx$$

$$\begin{array}{r} -x^3 + 2x^2 \pm x \\ \hline 2x^2 - x \\ -2x^2 + 4x \pm 2 \\ \hline 3x - 2 \end{array}$$

$$= \int_{-1}^{0} (x+2)\,dx + \int_{-1}^{0} \frac{3x-2}{(x-1)^2}\,dx$$

$$= \frac{x^2}{2} + 2x \Big|_{-1}^{0} + \int_{-1}^{0} \left(\frac{A}{x-1} + \frac{B}{(x-1)^2} \right) dx$$

$$\frac{3x-2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$= \frac{A(x-1)}{(x-1)^2} + \frac{B}{(x-1)^2}$$

$$= \frac{A(x-1)+B}{(x-1)^2}$$

$$\boxed{3x-2 = A(x-1) + B}$$

• الاشتقاق

$$3 = A$$

• مقارنة معاملات

$$\boxed{3 = A}, \quad -2 = -A + B$$

$$= -3 + B$$

$$\boxed{1 = B}$$

• التعويض في

$$x = 1 \Rightarrow 3-2 = A(0) + B$$

$$\underline{1 = B}$$

$$x = 0 \Rightarrow 0-2 = A(0-1) + B$$

$$-2 = -A + B$$

$$-2 = -A + 1$$

$$\boxed{3 = A}$$

$$= (0+0) - \left(\frac{1}{2} - 2\right) + \int \left(\frac{3}{x-1} + \frac{1}{(x-1)^2} \right) dx$$

$$= \frac{3}{2} + 3\ln|x-1| \Big|_{-1}^{0} + \int_{-1}^{0} \frac{dx}{(x-1)^2}$$

$$= \frac{3}{2} + 3\cancel{\ln 1} - 3\ln 2 + \int_{-2}^{-1} \frac{du}{u^2}$$

$$u = x - 1$$

$$du = dx$$

$$x = -1 \Rightarrow u = -2$$

$$x = 0 \Rightarrow u = -1$$

$$= \frac{3}{2} - 3\ln 2 + \int_{-2}^{-1} u^{-2}\,du$$

$$= \frac{3}{2} - 3\ln 2 + \frac{u^{-1}}{-1} \Big|_{-2}^{-1}$$

$$= \frac{3}{2} - 3\ln - \frac{1}{u} \Big|_{-2}^{-1}$$

$$= \dots = 2 - 3\ln 2$$

$$(33)\quad \int \frac{y^2+2y+1}{(y^2+1)^2}\,dy = \int \left(\frac{Ay+B}{y^2+1} + \frac{Cy+D}{(y^2+1)^2} \right) dy$$

$$\frac{y^2+2y+1}{(y^2+1)^2} = \frac{Ay+B}{y^2+1} + \frac{Cy+D}{(y^2+1)^2}$$

$$= \frac{(Ay+B)(y^2+1)}{(y^2+1)^2} + \frac{Cy+D}{(y^2+1)^2}$$

$$= \frac{(Ay+B)(y^2+1) + Cy+D}{(y^2+1)^2}$$

$$y^2+2y+1 = (Ay+B)(y^2+1) + Cy+D$$

مقارنة معاملات

$$\boxed{1 = B+D}$$

$$y=0 \Rightarrow \boxed{1 = B+D} \quad (1)$$

$$\boxed{D=0}$$

$$2y+2 = (Ay+B)(2y) + (y^2+1)A + C$$

$$y=0 \quad 2 = 0 + A + C$$

$$\boxed{2 = A+C}$$

$\boxed{2 = A + C}$

$C = 2$

$2y + 2 = 2Ay^2 + 2By + Ay^2 + A + C$

$\boxed{2 = A + C}$

$$\boxed{2 = 4Ay + 2B + 2Ay}$$

$y = 0 \Rightarrow 2 = 0 + 2B + 0 \Rightarrow B = 1$

$0 = 4A + 0 + 2A$

$0 = 6A \Rightarrow A = 0$

$$\int \frac{y^2+2y+1}{(y^2+1)^2} dy = \int \frac{1}{y^2+1} dy + \int \frac{2y}{(y^2+1)^2} dy$$

$u = y^2 + 1$

$du = 2y\,dy$

$$= \tan^{-1} y + \int \frac{du}{u^2}$$

$$= \tan^{-1} y + \int u^{-2}\, du$$

$$= \tan^{-1} y + \frac{u^{-1}}{-1} + C$$

$$\tan^{-1} y - \frac{1}{} + C$$

$$= \tan^{-1} y - \frac{1}{\boxed{y^2+1}} + C$$

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(30) $$\int \frac{x^2+x}{x^4-3x^2-4}\,dx = \int \frac{x^2+x}{(x^2-4)(x^2+1)}\,dx$$

$$= \int \frac{x^2+x}{(x-2)(x+2)(x^2+1)}\,dx$$

$$= \int \left(\frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1} \right) dx$$

$$\frac{x^2+x}{(x-2)(x+2)(x^2+1)} = \frac{\overset{3/10}{A}}{(x-2)} + \frac{\overset{-1/10}{B}}{x+2} + \frac{\overset{-1/5}{C}x+\overset{1/5}{D}}{x^2+1}$$

$$= \frac{A(x+2)(x^2+1)}{(x-2)(x+2)(x^2+1)} + \frac{B(x-2)(x^2+1)}{(x+2)(x-2)(x^2+1)} + \frac{(Cx+D)(x-2)(x+2)}{(x^2+1)(x-2)(x+2)}$$

$$x^2+x = A(x+2)(x^2+1) + \underbrace{B(x-2)(x^2+1)}_{0} + \underbrace{(Cx+D)(x-2)(x+2)}_{0}$$

$$x = 2 \Rightarrow 4 + 2 = A(4)(5)$$

$$6 = 20A$$

$$A = 3$$

$6 = 20A$

$A = \frac{3}{10}$

$x=-2 \Rightarrow 4-2 = 0 + B(-4)(5) + 0$

$2 = -20B$

$B = \frac{-1}{10}$

$x=0 \Rightarrow 0+0 = 2A - 2B - 4D$

$0 = 2\frac{3}{10} - 2\left(\frac{-1}{10}\right) - 4D \Rightarrow 4D = \frac{8}{10} \Rightarrow D = \frac{1}{5}$

$x=1 \Rightarrow 1+1 = A(3)(2) + B(-1)(2) + \left(C + \frac{1}{5}\right)(-3)$

$\uparrow \frac{3}{10}$ $\qquad \uparrow -\frac{1}{10}$

$C = \frac{-1}{5}$

$$\int \frac{x^2+x}{x^4-3x^2-4}\,dx = \int \frac{\frac{3}{10}}{x-2}\,dx - \int \frac{\frac{1}{10}}{x+2}\,dx + \int \frac{-\frac{1}{5}x + \frac{1}{5}}{x^2+1}\,dx$$

$$= \frac{3}{10}\ln|x-2| - \frac{1}{10}\ln|x+2| + -\frac{1}{5}\frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{dx}{x^2+1}$$

$$= \frac{3}{10}\ln|x-2| - \frac{1}{10}\ln|x+2| - \frac{1}{5}\frac{1}{2}\ln(x^2+1) + \frac{1}{5}\tan^{-1}x + c$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

....

34) $\displaystyle\int \frac{x^4}{x^2-1}\,dx$

اضربه

المقسوم عليه

$$x^4 \div (x^2-1) = x^2 + 1$$

$$-x^4 + x^2 \quad +$$

$$x^2$$

$$\int \left(x^2+1 + \frac{1}{x^2-1}\right)dx$$

$$\int \left(x^2+1 + \frac{1}{x^2-1} \right) dx$$

$$\begin{array}{l} x^2 \\ \underline{-x^2+1} \quad + \\ \quad 1 \end{array}$$

ابامنا

$$\int (x^2+1)\,dx + \int \frac{1}{(x-1)(x+1)}\,dx$$

$$\frac{x^3}{3} + x + \int \left(\frac{A}{x-1} + \frac{B}{x+1} \right) dx$$

$+1 \qquad -1$

$$A = \frac{1}{\boxed{1}+1} = \frac{1}{2}$$

$$B = \frac{1}{\boxed{-1}-1} = \frac{-1}{2}$$

$$\frac{x^3}{3} + x + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$