

Key

Name :

Student Number.....

Section 10/10 .

Question #1: Circle the correct answer 4.5 pt.

1) The solution of the equation $\log_3 2 + 2 \log_3 x = \log_3(7x - 3)$

a) $x = 3$

b) $x = \frac{1}{2}, x = 3$

c) $x = \frac{1}{3}$

d) $x = \frac{1}{2}$

2) Let $f(x) = x^2, x \leq 0$. Find the derivative of $f^{-1}(x)$ at $x=4$

a) $\frac{1}{2}$

b) $\frac{1}{8}$

c) $\frac{1}{4}$

d) $\frac{1}{-4}$

3) $\int \sec(5x) dx,$

a) $\frac{1}{5} \ln |\tan(5x)| + c$

b) $\frac{1}{5} \ln |\sec(5x) + \tan(5x)| + c$

c) $5 \ln |\sec^2(5x)| + c$

d) $5 \ln |\sec(5x) + \tan(5x)| + c$

Question #2: Find the derivative of the inverse function of $f(x) = x + \sin x$ at $x = \pi$ 3 Pt.

$$f(x) = x + \sin x$$

$$f'(x) = 1 + \cos x$$

$$\left. \frac{df^{-1}}{dx} \right|_{x=\pi} = \frac{1}{\left. \frac{df}{dx} \right|_{f^{-1}(\pi)}} = \frac{1}{\left. \frac{df}{dx} \right|_{x=\pi}} = \frac{1}{1 + \cos \pi} = \text{Undefined}$$

$$f^{-1}(\pi) = ??$$

$$\pi = x + \sin x$$

$$\boxed{x = \pi}$$

Question #3: Selected values of a strictly increasing function $g(x)$ and $g'(x)$ are shown on the table below. Find 25 Pt.

x	-3	-1	1	4
$g(x)$	5	1	0	-3
$g'(x)$	-4	$-\frac{1}{5}$	$-\frac{1}{6}$	-2

1) $(g^{-1})'(1)$

2) $(g^{-1})'(-3)$

$$\left. \frac{dg^{-1}}{dx} \right|_{x=1} = \frac{1}{\left. \frac{dg}{dx} \right|_{x=-1}} = \frac{1}{g'(-1)} = \frac{1}{-\frac{1}{5}} = -5$$

$$\left. \frac{dg^{-1}}{dx} \right|_{x=-3} = \frac{1}{\left. \frac{dg}{dx} \right|_{x=4}} = \frac{1}{g'(4)} = \frac{1}{-2}$$

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Question #1: Circle the correct answer 4.5 pt.

1) The solution of the equation $\ln(x^3 - 2x^2 - x) - \ln x = \ln 2$

- a) $x = 3$
- b) $x = 3, x = -1, x = 0$
- c) $x = -1$
- d) $x = 3, x = -1$

2) Let $f(x) = x^3 + 2x - 3$. Find the value of a if $f^{-1}(a) = 2$

- a) 9
- b) 1
- c) 2
- d) Does not exist

3) If $y = x^{\ln x}$, then $y'(e) =$

- a) 0
- b) 1
- c) 2
- d) e

Question #2: Given the following function

$$f(x) = 1 - \sqrt{x}$$

→ Is f 1-1?? Yes

2 Pt a) Find $f^{-1}(x)$

$$y = 1 - \sqrt{x} \rightarrow \sqrt{x} = 1 - y$$

$$x = (1 - y)^2 \rightarrow y = (1 - x)^2$$

$$f^{-1}(x) = (1 - x)^2$$

Since $f'(x) = \frac{-1}{2\sqrt{x}} < 0$

→ f is dec on $\text{Dom}(f)$

2 Pt. b) Find $\frac{df^{-1}}{dx}$ at $x = -1$

$$\frac{df^{-1}}{dx} = 2(1-x)(-1)$$

$$\left(\frac{df^{-1}}{dx}\right)\Big|_{x=-1} = -2(2) = \boxed{-4}$$

OR

$$\left(\frac{df^{-1}}{dx}\right)\Big|_{x=-1} = \frac{1}{\left(\frac{df}{dx}\right)\Big|_{x=4}}$$

$$= \frac{1}{\frac{-1}{4}} = \boxed{-4}$$

$f(x) = 1 - \sqrt{x}$
 $f'(x) = \frac{-1}{2\sqrt{x}}$

1.5 Pt c) $f^{-1}(2)$

$f^{-1}(2)$ Undefined

Since $2 \notin \text{Dom}(f^{-1})$

$$\text{Dom}(f^{-1}) = (-\infty, 1]$$

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Question #1: Circle the correct answer 4.5 Pt

1) If $y = \ln\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

- a) 10
- b) 1
- c) 0
- d) $\frac{3\pi}{4}$

2) The solution of the equation

$$\ln(x) + \ln(x - 2) = \ln(x + 10)$$

is

- a) $\{-2, 5\}$
- b) 5
- c) $\{-5, 2\}$

3) $\tan^{-1}(\tan\frac{3\pi}{4})$

- a) $-\frac{3\pi}{4}$
- b) $\frac{3\pi}{4}$
- c) $\frac{\pi}{4}$
- d) Does not exist



Question #2: Given the following function

$$f(x) = 1 - \sqrt{x} \rightarrow \text{Is } f \text{ 1-1?? Yes}$$

1.5 Pt a) Find $f^{-1}(x)$

$$\text{since } f'(x) = \frac{-1}{2\sqrt{x}} < 0$$

$\rightarrow f(x)$ dec on $\text{Dom}(f)$

$$y = 1 - \sqrt{x}$$

$$\sqrt{x} = 1 - y$$

$$x = (1 - y)^2$$

$$y = (1 - x)^2 \rightarrow$$

$$f^{-1}(x) = (1 - x)^2$$

2 Pt b) Find $\frac{df^{-1}}{dx}$ at $x = -1$

$$\frac{df^{-1}}{dx} = 2(1 - x) \cdot (-1) = -2(1 - x)$$

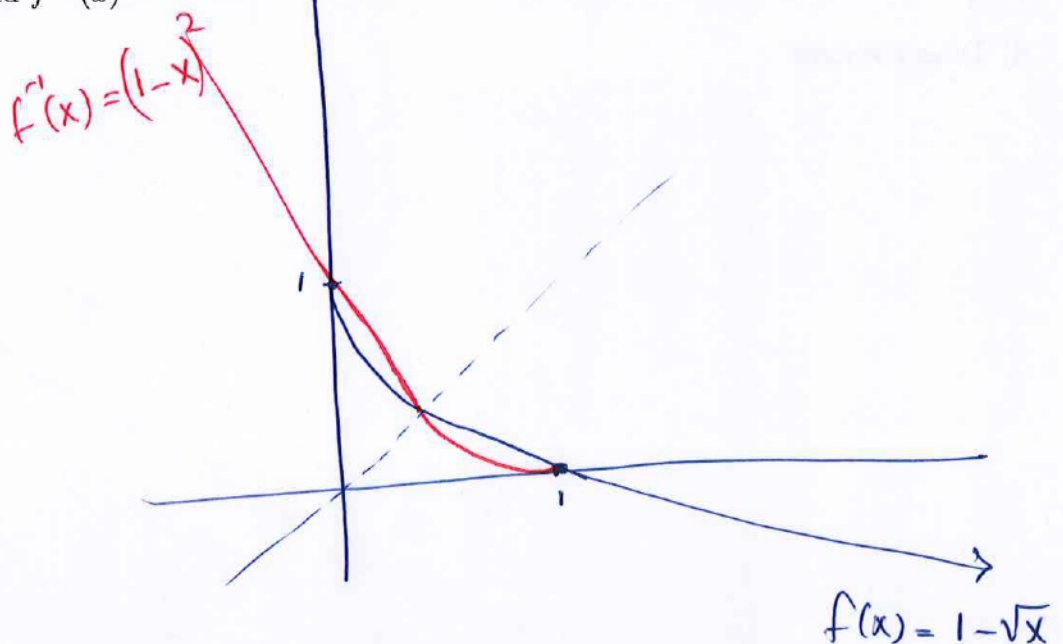
$$\left. \frac{df^{-1}}{dx} \right|_{x=-1} = -2(2) = \boxed{-4}$$

OR

$$\left. \left(\frac{df^{-1}}{dx} \right) \right|_{x=-1} = \frac{1}{\left. \left(\frac{df}{dx} \right) \right|_{x=4}} = \frac{1}{-\frac{1}{4}} = \boxed{-4}$$

$$f(x) = 1 - \sqrt{x}$$
$$f'(x) = \frac{-1}{2\sqrt{x}}$$

2 Pt c) Sketch $f(x)$ and $f^{-1}(x)$



BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH1411 - QUIZ 2

Section 21

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Section 10/10

Question #1: Circle the correct answer

PT

1) The solution of the equation $\ln x - \ln(x - 1) = 1$

a) $x = 0$

b) $x = 1, x = 0$

c) $x = \frac{-e}{e-1}$

d) $x = \frac{e}{e-1}$

2) Let $f(x) = (x - 4)^3 - 1$. Find the solution of $f^{-1}(x) = 6$

a) 9

b) 7

c) 2

d) Does not exist

3) If $\int \frac{\sec^2(2x)}{\tan(2x)}$,

a) $\frac{1}{2} \ln |\tan(2x)| + c$

b) $\frac{1}{2} \ln |\sec(2x) + \tan(2x)| + c$

c) $\frac{1}{2} \ln |\sin(2x)| + c$

Question #2: Let $f(x) = \frac{5}{x} - 2$ \longrightarrow Is f 1-1 ?? Yes

2 Pt. a) Determine $f^{-1}(x)$

$$y = \frac{5}{x} - 2$$

$$y + 2 = \frac{5}{x}$$

$$x = \frac{5}{y+2}$$

$$y = \frac{5}{x+2} \longrightarrow \boxed{f^{-1}(x) = \frac{5}{x+2}}$$

Since $f'(x) = -\frac{5}{x^2} < 0$

$f(x)$ dec. on $\mathbb{R} \setminus \{0\}$

2 Pt. b) Find and simplify $(f^{-1} \circ f)(x)$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{5}{x} - 2\right)$$

$$= \frac{5}{\left(\frac{5}{x} + 2\right) - 2} = \frac{5}{\frac{5}{x}} = 5 \cdot \frac{x}{5} = x$$

$$(f^{-1} \circ f)(x) = x, \quad \forall x \in \text{Dom}(f)$$