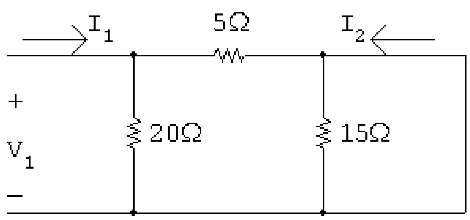
18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have



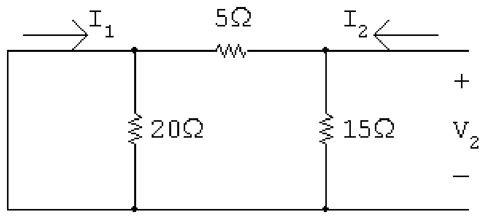
$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \qquad \frac{I_1}{V_1} = y_{11} = 0.25\,\text{S}; \qquad I_2 = \left(\frac{-20}{25}\right) I_1 = -0.8 I_1$$

When $V_2 = 0$, we have $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$

Therefore $I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $y_{21} = -0.8y_{11} = -0.2\,\text{S}$

With port 1 short-circuited, we have



$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \qquad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15}\right)\,\text{S}$$

$$I_1 = \left(\frac{-15}{20}\right) I_2 = -0.75 I_2 = -0.75 y_{22} V_2$$

$$\text{Therefore} \quad y_{12} = (-0.75)\frac{4}{15} = -0.2\,\text{S}$$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

AP 18.3

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{5\times 10^{-6}}{50\times 10^{-3}} = 0.1\,\text{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{200\times 10^{-3}}{50\times 10^{-3}} = 4$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{2\times 10^{-6}}{0.5\times 10^{-6}} = 4$$

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{10\times 10^{-3}}{0.5\times 10^{-6}} = 20\,\text{k}\Omega$$

AP 18.4 First calculate the b-parameters:

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{15}{10} = 1.5\,\Omega; \qquad b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{30}{10} = 3\,\mathrm{S}$$

$$b_{12} = \frac{-V_2}{I_1}\bigg|_{V_1=0} = \frac{-10}{-5} = 2\,\Omega; \qquad b_{22} = \frac{-I_2}{I_1}\bigg|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the z-parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15}\,\Omega; \qquad z_{12} = \frac{1}{b_{21}} = \frac{1}{3}\,\Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6\,\Omega; \qquad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2}\,\Omega$$

AP 18.5

$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$

Therefore, $\quad z_{11} = z_{22} = 95/5 = 19\,\Omega$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right) z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17\,\Omega$.

AP 18.6 **[a]**

$$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$$

$$= \frac{-50 \times 10^{-3}}{(5 \times 10^{-4})(5 \times 10^3) + 10 + (10^{-6})(100)(5 \times 10^3) + (-3 \times 10^{-2})(100)}$$

$$= \frac{-50 \times 10^{-3}}{10} = -5\,\mathrm{mA}$$

$$P_L = \frac{1}{2}(5 \times 10^{-3})^2(5 \times 10^3) = 62.5\,\mathrm{mW}$$

[b]

$$Z_{\mathrm{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{10 + (-3 \times 10^{-2})(100)}{5 \times 10^{-4} + (10^{-6})(100)}$$

$$= \frac{7}{6 \times 10^{-4}} = \frac{70}{6}\,\mathrm{k}\Omega$$

[c] $V_{\text{Th}} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{50 \times 10^{-3}}{6 \times 10^{-4}} = \dfrac{500}{6}\,\text{V}$

Therefore $V_2 = \dfrac{250}{6}\,\text{V}; \qquad P_{\max} = \dfrac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4\,\text{mW}$

AP 18.7 **[a]** For the given bridged-tee circuit, we have

$$a'_{11} = a'_{22} = 1.25, \qquad a'_{21} = \frac{1}{20}\,\text{S}, \qquad a'_{12} = 11.25\,\Omega$$

The a-parameters of the cascaded networks are

$$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$$

$$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125\,\Omega$$

$$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125\,\text{S}$$

$$a_{22} = a_{11} = 2.125, \qquad R_{\text{Th}} = (45.125/3.125) = 14.44\,\Omega$$

[b] $V_t = \dfrac{100}{3.125} = 32\,\text{V}; \qquad$ therefore $\quad V_2 = 16\,\text{V}$

[c] $P = \dfrac{16^2}{14.44} = 17.73\,\text{W}$

Problems

P 18.1 $h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

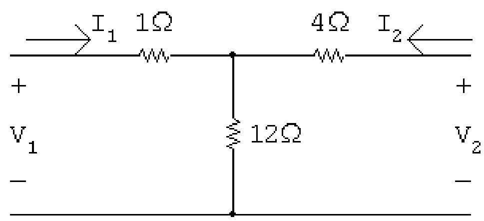
$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

P 18.2



$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = 1 + 12 = 13\,\Omega$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = 12\,\Omega$$

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = 4 + 12 = 16\,\Omega$$

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = 12\,\Omega$$

P 18.3 $\Delta z = (13)(16) - (12)(12) = 64$

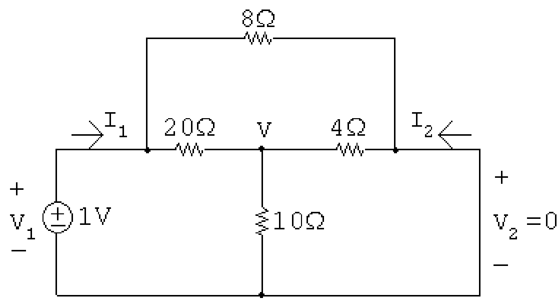
$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{16}{64} = 0.25\,\text{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-12}{64} = -0.1875\,\text{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-12}{64} = -0.1875\,\text{S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{13}{64} = 0.203125\,\text{S}$$

P 18.4 $y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$; $\qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

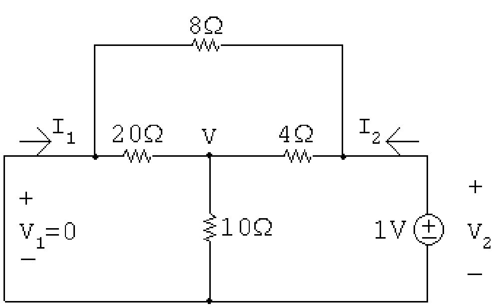


$$\frac{V-1}{20} + \frac{V}{10} + \frac{V}{4} = 0; \quad \text{so} \quad V = 0.125\,\text{V}$$

$$\therefore \quad I_1 = \frac{1-0.125}{20} + \frac{1-0}{8} = 168.75\,\text{mA}; \qquad I_2 = \frac{0-0.125}{4} + \frac{0-1}{8} = -156.25\,\text{mA}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 168.75\,\text{mS}; \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -156.25\,\text{mS}$$

$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$; $\qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$



$$\frac{V}{20} + \frac{V}{10} + \frac{V-1}{4} = 0; \quad \text{so} \quad V = 0.625\,\text{V}$$

$$\therefore \quad I_1 = \frac{0 - 0.625}{20} + \frac{0 - 1}{8} = -156.25\,\text{mA};$$

$$I_2 = \frac{1 - 0.625}{4} + \frac{1 - 0}{8} = 218.75\,\text{mA}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -156.25\,\text{mS}; \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 218.75\,\text{mS}$$

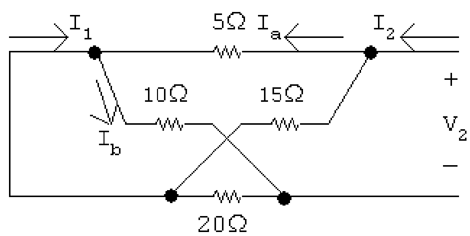
Summary:

$y_{11} = 168.75\,\text{mS} \quad y_{12} = -156.25\,\text{mS} \quad y_{21} = -156.25\,\text{mS} \quad y_{22} = 218.75\,\text{mS}$

P 18.5 $V_2 = b_{11}V_1 - b_{12}I_1$

$I_2 = b_{21}V_1 - b_{22}I_1$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$5\|15 = (15/4)\,\Omega; \qquad 10\|20 = (20/3)\,\Omega$

$$I_2 = \frac{V_2}{(15/4) + (20/3)} = \frac{12V_2}{125}; \qquad I_1 = I_\text{b} - I_\text{a}$$

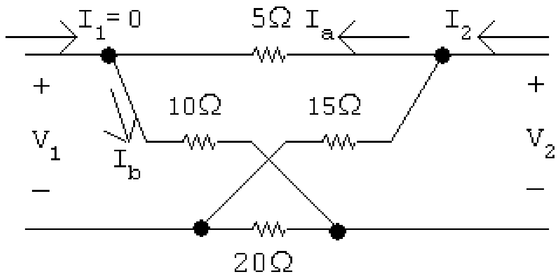
$$I_\text{a} = \frac{15}{20}I_2; \qquad I_\text{b} = \frac{20}{30}I_2$$

$$I_1 = \left(\frac{20}{30} - \frac{15}{20}\right) I_2 = \frac{-5}{60}I_2 = \frac{-1}{12}I_2$$

$$b_{22} = \frac{-I_2}{I_1} = 12$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2}\left(\frac{I_2}{I_1}\right) = \frac{125}{12}(12) = 125\,\Omega$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



$$V_1 = V_a - V_b; \quad V_a = \frac{10}{15}V_2; \quad V_b = \frac{20}{35}V_2$$

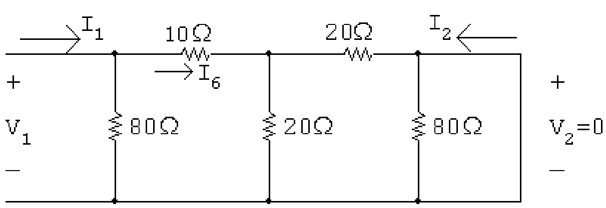
$$V_1 = \frac{10}{15}V_2 - \frac{20}{35}V_2 = \frac{2}{21}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{21}{2} = 10.5$$

$$V_2 = (10 + 5)\|(20 + 15)I_2 = 10.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{1}{10.5}\right)(10.5) = 1\,\mathrm{S}$$

P 18.6 $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

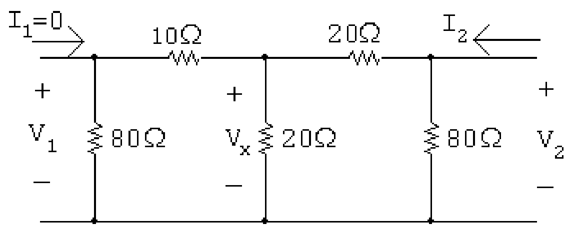


$$\frac{V_1}{I_1} = 80\|[10 + 20\|20] = 80\|20 = 16\,\Omega \qquad \therefore \quad h_{11} = 16\,\Omega$$

$$I_6 = \frac{80}{80 + 20}I_1 = 0.8I_1$$

$$I_2 = \frac{-20}{20 + 20}I_6 = -0.5I_6 = -0.5(0.8)I_1 = -0.4I_1 \qquad \therefore \quad h_{21} = -0.4$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$\frac{V_2}{I_2} = 80\|[20 + 20\|90] = 25\,\Omega \qquad \therefore \quad h_{22} = \frac{1}{25} = 40 \text{ mS}$$

$$V_x = \frac{20\|90}{20 + 20\|90} V_2$$

$$V_1 = \frac{80}{80 + 10} V_x = \frac{80(20\|90)}{90(20 + 20\|90)} V_2 = 0.4 V_2$$

$$\therefore \qquad h_{12} = 0.4$$

Summary:

$h_{11} = 16\,\Omega; \quad h_{12} = 0.4; \quad h_{21} = -0.4; \quad h_{22} = 40$ mS

P 18.7 $$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = R_1\|R_2 = 4 \quad \therefore \quad \frac{R_1R_2}{R_1 + R_2} = 4$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{-R_2}{R_1 + R_2} = -0.8$$

$$\therefore \qquad R_2 = 0.8R_1 + 0.8R_2 \quad \text{so} \quad R_1 = \frac{R_2}{4}$$

Substituting,

$$\frac{(R_2/4)R_2}{(R_2/4) + R_2} = 4 \quad \text{so} \quad R_2 = 20\,\Omega \quad \text{and} \quad R_1 = 5\,\Omega$$

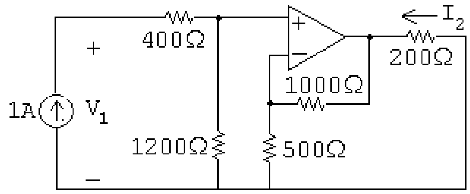
$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{1}{R_3\|(R_1 + R_2)} = \frac{1}{R_3\|25} = 0.14$$

$$\therefore \qquad R_3 = 10$$

Summary:

$R_1 = 5\,\Omega; \qquad R_2 = 20\,\Omega; \qquad R_3 = 10\,\Omega$

P 18.8 For $V_2 = 0$:



$$V_1 = (400 + 1200)I_1$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{1600}{1} = 1600\,\Omega$$

$$V_p = 1200(1\,\text{A}) = 1200\,\text{V} = V_n$$

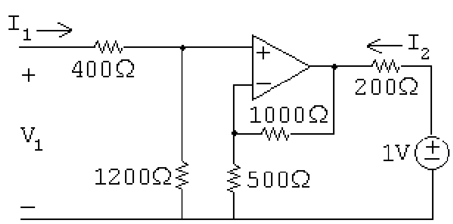
At V_n,

$$\frac{1200}{500} + \frac{1200 - V_o}{1000} = 0 \quad \text{so} \quad V_o = 3600\,\text{V}$$

$$\therefore \qquad I_2 = -\frac{3600}{200} = -18\,\text{A}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{I_1=0} = \frac{-18}{1} = -18$$

For $I_1 = 0$:



$$V_1 = 0$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{0}{1} = 0$$

At V_n,

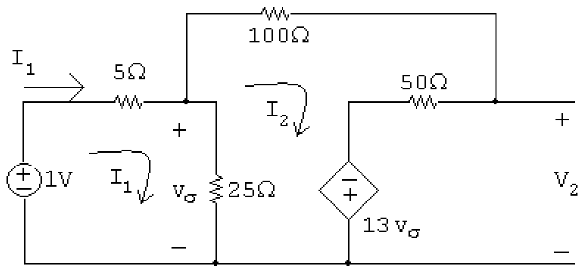
$$\frac{V_n}{500} + \frac{V_n - V_o}{100} = 0$$

But $V_n = V_p = 0$ so $V_o = 0$; therefore,

$$I_2 = \frac{1\,\text{V}}{200\,\Omega} = 5\,\text{mS}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{5\,\text{m}}{1} = 5\,\text{mS}$$

P 18.9 For $I_2 = 0$:



$$30I_1 - 25I_2 = 1$$

$$-25I_1 + 175I_2 - 13(25)(I_1 - I_2) = 0 \quad \text{so} \quad -350I_1 + 500I_2 = 0$$

Solving,

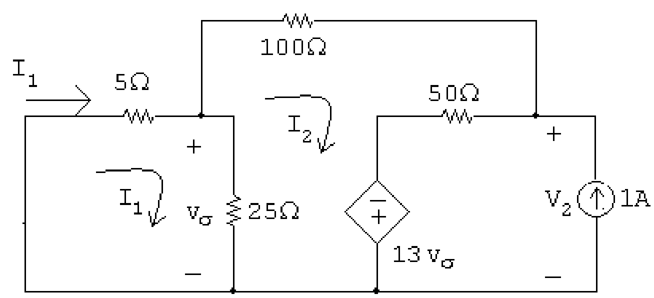
$$I_1 = 80\,\text{mA}; \qquad I_2 = 56\,\text{mA}$$

$$V_2 = 50I_2 - 13(25)(I_1 - I_2) = -5\,\text{V}$$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{80\,\text{m}}{1} = 80\,\text{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{-5}{1} = -5$$

For $V_1 = 0$:



$$30I_1 - 25I_2 = 0$$

$$-25I_1 + 175I_2 + 50 - 13(25)(I_1 - I_2) = 0 \quad \text{so} \quad -350I_1 + 500I_2 = -50$$

Solving,

$$I_1 = -200\,\text{mA}; \qquad I_2 = -240\,\text{mA}$$

$$V_2 = 50(I_2 + 1) - 13(25)(I_1 - I_2) = 25\,\text{V}$$

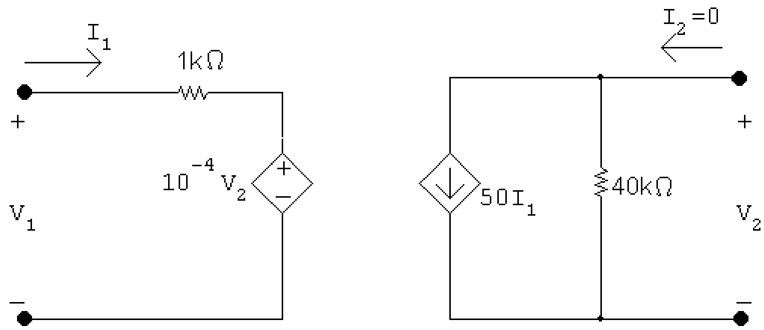
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{-200\,\text{m}}{1} = -0.2$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{25}{1} = 25\,\Omega$$

P 18.10 $V_1 = a_{11}V_2 - a_{12}I_2$

$$I_1 = a_{21}V_2 - a_{22}I_2$$

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}; \qquad a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

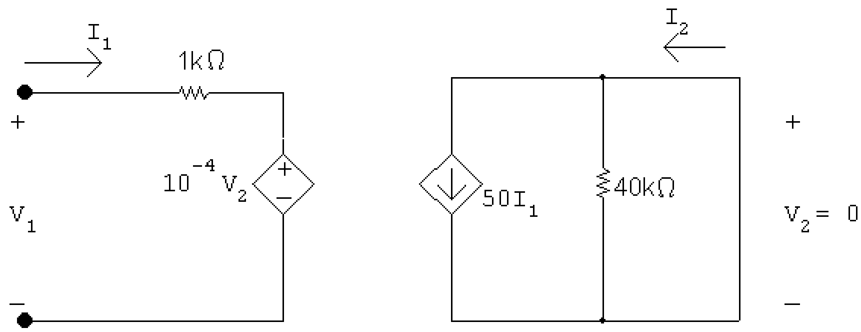


$$V_1 = 10^3 I_1 + 10^{-4}V_2 = 10^3(-0.5\times10^{-6})V_2 + 10^{-4}V_2$$

$$\therefore \quad a_{11} = -5\times10^{-4} + 10^{-4} = -4\times10^{-4}$$

$$V_2 = -(50I_1)(40\times10^3); \qquad \therefore \quad a_{21} = -\frac{1}{2\times10^6} = -0.5\,\mu\text{S}$$

$$a_{12} = \left.\frac{-V_1}{I_2}\right|_{V_2=0}; \qquad a_{22} = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$



$$I_2 = 50I_1; \qquad \therefore \quad a_{22} = -\frac{I_1}{I_2} = -\frac{1}{50}$$

$$V_1 = 1000I_1; \qquad \therefore \quad a_{12} = -\frac{V_1}{I_2} = -\frac{V_1}{I_1}\frac{I_1}{I_2} = -(1000)(1/50) = -20\,\Omega$$

Summary

$$a_{11} = -4\times 10^{-4}; \quad a_{12} = -20\,\Omega; \quad a_{21} = -0.5\,\mu\text{S}; \quad a_{22} = -0.02$$

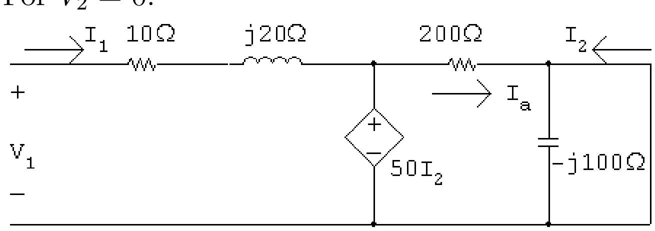
P 18.11 $g_{11} = \frac{a_{21}}{a_{11}} = \frac{-0.5\times 10^{-6}}{-4\times 10^{-4}} = 1.25\,\text{mS}$

$$g_{12} = \frac{-\Delta a}{a_{11}} = \frac{-(-4\times 10^{-4})(-1/50) - (-0.5\times 10^{-6})(-20)}{-4\times 10^{-4}} = -0.005$$

$$g_{21} = \frac{1}{a_{11}} = \frac{1}{-4\times 10^{-4}} = -2500$$

$$g_{22} = \frac{a_{12}}{a_{11}} = \frac{(-20)}{-400\times 10^{-6}} = 5\times 10^{4}\,\Omega$$

P 18.12 For $V_2 = 0$:

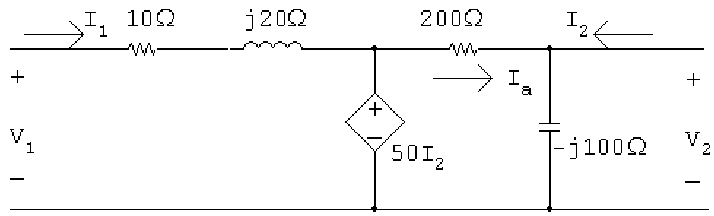


$$I_a = \frac{50I_2}{200} = \frac{1}{4}I_2 = -I_2; \quad \therefore \qquad I_2 = 0$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = 0$$

$$V_1 = (10 + j20)I_1 \quad \therefore \quad h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = 10 + j20\,\Omega$$

For $I_1 = 0$:



$$V_1 = 50I_2; \qquad I_2 = \frac{V_2}{-j100} + \frac{V_2 - 50I_2}{200}$$

$$200I_2 = j2V_2 + V_2 - 50i_2$$

$$250I_2 = V_2(1 + j2)$$

$$50I_2 = V_2\left(\frac{1 + j2}{5}\right) = (0.2 + j0.4)V_2$$

$$\therefore \quad V_1 = (0.2 + j0.4)V_2$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = 0.2 + j0.4$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{1 + j2}{250} = 4 + j8 \text{ mS}$$

Summary:

$$h_{11} = 10 + j20\,\Omega; \quad h_{12} = 0.2 + j0.4; \quad h_{21} = 0; \quad h_{22} = 4 + j8\,\text{mS}$$

P 18.13 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0} = \frac{0.25 \times 10^{-6}}{20 \times 10^{-3}} = 12.5 \times 10^{-6} = 12.5\,\mu\text{S}$$

$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0} = \frac{-5}{20} \times 10^3 = -250$$

$$0 = -250(10) + g_{22}(50 \times 10^{-6})$$

$$g_{22} = \frac{2500}{50 \times 10^{-6}} = 50\,\text{M}\Omega$$

$$200 \times 10^{-6} = 12.5 \times 10^{-6}(10) + g_{12}(50 \times 10^{-6})$$

$$(200 - 125)10^{-6} = g_{12}(50 \times 10^{-6})$$

$$g_{12} = \frac{75}{50} = 1.5$$

P 18.14 **[a]** $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{50 \times 10^{-6}}{10} = 5\,\mu\text{S}$$

$$0 = y_{21}(20 \times 10^{-3}) + y_{22}(-5)$$

$$\therefore \qquad y_{22} = \frac{1}{5}y_{21}(20 \times 10^{-3}) = 20\,\text{nS}$$

$$200 \times 10^{-6} = y_{11}(10) \quad \text{so} \quad y_{11} = 20\,\mu\text{S}$$

$$0.25 \times 10^{-6} = 20 \times 10^{-6}(20 \times 10^{-3}) + y_{12}(-5)$$

$$y_{12} = \frac{0.25 \times 10^{-6} - 0.4 \times 10^{-6}}{-5} = 30\,\text{nS}$$

Summary:

$y_{11} = 20\,\mu\text{S}; \quad y_{12} = 30\,\text{nS}; \quad y_{21} = 5\,\mu\text{S}; \quad y_{22} = 20\,\text{nS}$

[b] $y_{11} = \frac{\Delta g}{g_{22}}; \quad y_{12} = \frac{g_{12}}{g_{22}}; \quad y_{21} = \frac{-g_{21}}{g_{22}}; \quad y_{22} = \frac{1}{g_{22}}$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21} = (12.5 \times 10^{-6})(50 \times 10^6) - 1.5(-250)$$

$$= 625 + 375 = 1000$$

$$y_{11} = \frac{1000}{50 \times 10^6} = 20\,\mu\text{S}; \qquad y_{21} = \frac{250}{5 \times 10^6} = 5\,\mu\text{S}$$

$$y_{12} = \frac{1.5}{50 \times 10^6} = 30\,\text{nS}; \qquad y_{22} = \frac{1}{5 \times 10^6} = 20\,\text{nS}$$

These values are the same as those in part (a).

P 18.15 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2$

Solve the first equation for I_1 and the second equation for V_2:

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}V_2; \qquad V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}}I_1$$

Work with the I_1 equation, substituting in the expression for V_2:

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}\left[\frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}}I_1\right]$$

$$I_1\left[1-\frac{h_{12}h_{21}}{h_{11}h_{22}}\right]=\frac{V_1}{h_{11}}-\frac{h_{12}}{h_{11}h_{22}}I_2$$

Thus,

$$g_{11}=\frac{1/h_{11}}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{h_{22}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{h_{22}}{\Delta h}$$

$$g_{12}=\frac{-h_{12}/(h_{11}h_{22})}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{-h_{12}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{-h_{12}}{\Delta h}$$

Now work with the V_2 equation, substituting in the expression for I_1:

$$V_2=\frac{I_2}{h_{22}}-\frac{h_{21}}{h_{22}}\left[\frac{V_1}{h_{11}}-\frac{h_{12}}{h_{11}}V_2\right]$$

$$V_2\left[1-\frac{h_{12}h_{21}}{h_{11}h_{22}}\right]=\frac{I_2}{h_{22}}-\frac{h_{21}}{h_{11}h_{22}}V_1$$

Thus,

$$g_{21}=\frac{-h_{21}/(h_{11}h_{22})}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{-h_{21}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{-h_{21}}{\Delta h}$$

$$g_{22}=\frac{1/h_{22}}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{h_{11}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{h_{11}}{\Delta h}$$

P 18.16 $V_1=a_{11}V_2-a_{12}I_2;\qquad I_1=a_{21}V_2-a_{22}I_2$

$$I_2=\frac{a_{21}}{a_{22}}V_2-\frac{1}{a_{22}}I_1$$

$$\therefore\qquad h_{21}=\frac{-1}{a_{22}}\quad\text{and}\quad h_{22}=\frac{a_{21}}{a_{22}}$$

$$V_1=a_{11}V_2-a_{12}\left(\frac{a_{21}}{a_{22}}V_2-\frac{1}{a_{22}}I_1\right)=\left(a_{11}-\frac{a_{12}a_{21}}{a_{22}}\right)V_2+\frac{a_{12}}{a_{22}}I_1$$

$$\therefore\qquad h_{11}=\frac{a_{12}}{a_{22}}\quad\text{and}\quad h_{12}=a_{11}-\frac{a_{12}a_{21}}{a_{22}}=\frac{\Delta a}{a_{22}}$$

P 18.17 $I_1 = g_{11}V_1 + g_{12}I_2;\qquad V_2 = g_{21}V_1 + g_{22}I_2$

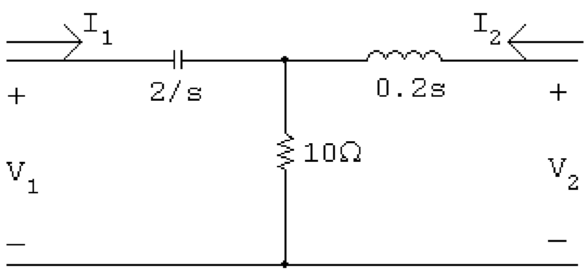
$$I_2 = \frac{1}{g_{22}}V_2 - \frac{g_{21}}{g_{22}}V_1$$

$$\therefore \qquad y_{21} = \frac{-g_{21}}{g_{22}} \quad \text{and} \quad y_{22} = \frac{1}{g_{22}}$$

$$I_1 = g_{11}V_1 + g_{12}\left(\frac{1}{g_{22}}V_2 - \frac{g_{21}}{g_{22}}V_1\right) = \left(g_{11} - \frac{g_{12}g_{21}}{g_{22}}\right)V_1 + \frac{g_{12}}{g_{22}}V_2$$

$$\therefore \qquad y_{11} = g_{11} - \frac{g_{12}g_{21}}{g_{22}} = \frac{\Delta g}{g_{22}} \quad \text{and} \quad y_{12} = \frac{g_{12}}{g_{22}}$$

P 18.18



For $I_2 = 0$:

$$V_1 = \left(\frac{2}{s} + 10\right)I_1 \qquad \text{so} \qquad z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{10s+2}{s}\,\Omega$$

$$V_2 = 10I_1 \qquad \text{so} \qquad z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = 10\,\Omega$$

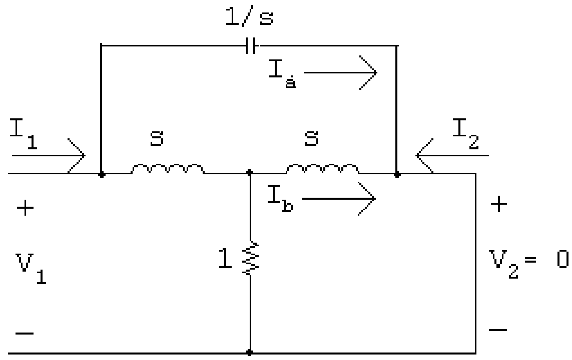
For $I_1 = 0$:

$$V_1 = 10I_2 \qquad \text{so} \qquad z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = 10\,\Omega$$

$$V_2 = (0.2s + 10)I_2 \qquad \text{so} \qquad z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = 0.2s + 10\,\Omega$$

P 18.19 $I_1 = y_{11}V_1 + y_{12}V_2;\qquad I_2 = y_{21}V_1 + y_{22}V_2$

Since the circuit is symmetric and reciprocal, $y_{11} = y_{22}$ and $y_{12} = y_{21}$.



$$I_1 = \frac{V_1}{(1/s)} + \frac{V_1}{s + (s/s+1)}$$

$$\frac{I_1}{V_1} = s + \frac{s+1}{s^2+2s} = \frac{s^3+2s^2+s+1}{s(s+2)}$$

$$y_{11} = y_{22} = \frac{s^3+2s^2+s+1}{s(s+2)}$$

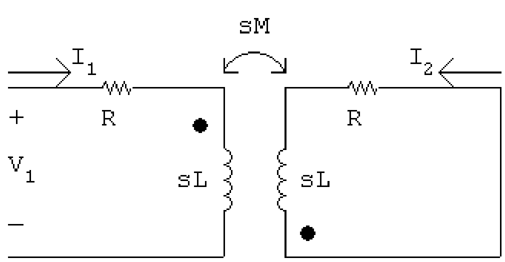
$$I_a = V_1 s; \qquad I_b = \frac{V_1(s+1)}{s(s+2)} \cdot \frac{1}{(s+1)} = \frac{V_1}{s(s+2)}$$

$$I_2 = -(I_a + I_b) = -\left[V_1 s + \frac{V_1}{s(s+2)}\right]$$

$$\frac{I_2}{V_1} = -\left[s + \frac{1}{s(s+2)}\right] = -\frac{s^3+2s^2+1}{s(s+2)}$$

$$y_{12} = y_{21} = -\frac{s^3+2s^2+1}{s(s+2)}$$

P 18.20 **[a]** $a_{12} = \left.\frac{-V_1}{I_2}\right|_{V_2=0}; \qquad a_{22} = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$



$$V_1 = (R + sL)I_1 - sMI_2$$

$$0 = -sMI_1 + (R + sL)I_2$$

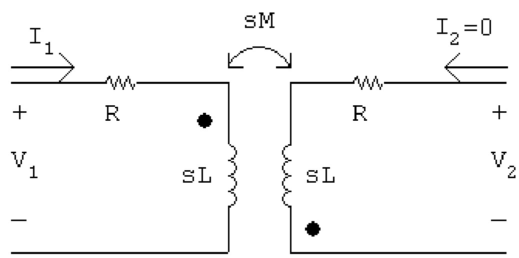
$$\Delta = \begin{vmatrix} (R+sL) & -sM \\ -sM & (R+sL) \end{vmatrix} = (R+sL)^2 - s^2M^2$$

$$N_2 = \begin{vmatrix} R+sL & V_1 \\ -sM & 0 \end{vmatrix} = sMV_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{sMV_1}{(R+sL)^2 - s^2M^2}; \qquad a_{12} = -\frac{V_1}{I_2} = \frac{(sM)^2 - (R+sL)^2}{sM}$$

$$0 = -sMI_1 + (R+sL)I_2; \qquad \therefore \quad a_{22} = \frac{-I_1}{I_2} = \frac{-(R+sL)}{sM}$$

$$a_{11} = \frac{V_1}{V_2}\Big|_{I_2=0}; \qquad a_{21} = \frac{I_1}{V_2}\Big|_{I_2=0}$$



$$V_2 = -sMI_1; \qquad I_1 = \frac{V_1}{R+sL}$$

$$V_2 = \frac{-sMV_1}{R+sL}; \qquad a_{11} = \frac{V_1}{V_2} = \frac{R+sL}{-sM}$$

$$a_{21} = \frac{I_1}{V_2} = \frac{-1}{sM}$$

[b] $a_{11} = a_{22}$ (symmetric, reciprocal)

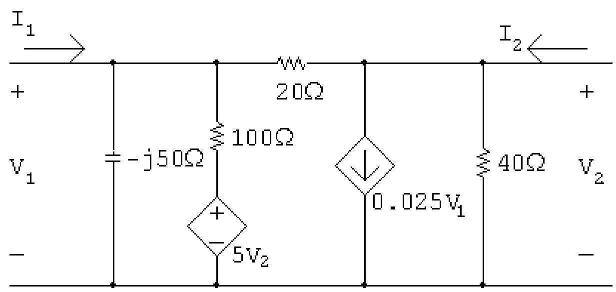
$$a_{11}a_{22} - a_{12}a_{21} = 1 \quad \text{(reciprocal)}$$

$$a_{11} = \frac{R+sL}{-sM}; \qquad a_{22} = \frac{R+sL}{-sM} \quad \text{(checks)}$$

$$a_{11}a_{22} - a_{12}a_{21} = \frac{(R+sL)^2}{(sM)^2} + \frac{(sM)^2 - (R+sL)^2}{sM} \cdot \frac{1}{sM}$$

$$= \frac{(R+sL)^2 + (sM)^2 - (R+sL)^2}{(sM)^2} = 1 \quad \text{(checks)}$$

P 18.21 For $I_2 = 0$:



$$\frac{V_2 - V_1}{20} + 0.025V_1 + \frac{V_2}{40} = 0$$

$$2V_2 - 2V_1 + V_1 + V_2 = 0 \quad \text{so} \quad 3V_2 = V_1$$

$$\therefore \qquad a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} = 3$$

$$I_1 = \frac{V_1}{-j50} + \frac{V_1 - 5V_2}{100} + \frac{V_1 - V_2}{20}$$

$$= V_1\left[\frac{j}{50} + \frac{1}{100} + \frac{1}{20}\right] - V_2\left[\frac{5}{100} + \frac{1}{20}\right]$$

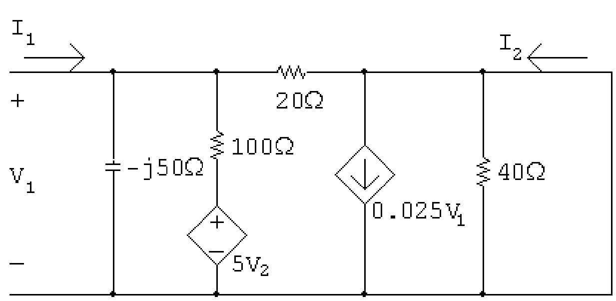
$$= V_1\left[\frac{6 + j2}{100}\right] - V_2\left[\frac{1}{10}\right]$$

But $V_1 = 3V_2$ so

$$I_1 = \left[\frac{18 + j6 - 10}{100}\right]V_2 = (0.08 + j0.06)V_2$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} = 0.08 + j0.06\,\text{S} = 80 + j60\,\text{mS}$$

For $V_2 = 0$:



$$I_1 = \frac{V_1}{-j50} + \frac{V_1}{100} + \frac{V_1}{20} = V_1\frac{(6 + j2)}{100}$$

$$I_2 = 0.025V_1 - \frac{V_1}{20} = -0.025V_1$$

$$a_{12} = -\frac{V_1}{I_2}\Big|_{V_2=0} = \frac{1}{0.025} = 40\,\Omega$$

$$a_{22} = -\frac{I_1}{I_2}\Big|_{V_2=0} = \frac{-2V_1(3 + j1)}{100(-0.025)V_1} = 2.4 + j0.8$$

Summary:

$$a_{11} = 3; \quad a_{12} = 40\,\Omega; \quad a_{21} = 80 + j60\,\text{mS}; \quad a_{22} = 2.4 + j0.8$$

P 18.22 $h_{11} = \frac{a_{12}}{a_{22}} = \frac{40}{(0.8)(3 + j1)} = 15 - j5\,\Omega$

$$h_{12} = \frac{\Delta a}{a_{22}}$$

$$\Delta a = 3(2.4 + j0.8) - 40(0.08 + j0.06) = 7.2 + j2.4 - 3.2 - j2.4 = 4$$

$$h_{12} = \frac{4}{(0.8)(3 + j1)} = 1.5 - j0.50$$

$$h_{21} = -\frac{1}{a_{22}} = \frac{-1}{(0.8)(3 + j1)} = -0.375 + j0.125$$

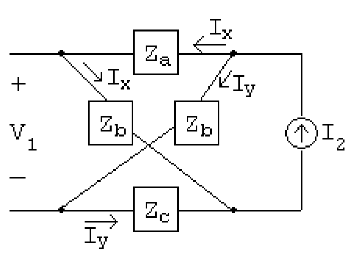
$$h_{22} = \frac{a_{21}}{a_{22}} = \frac{0.08 + j0.06}{(0.8)(3 + j1)} = 0.0375 + j0.0125 = 37.5 + j12.5\,\text{mS}$$

P 18.23 First we note that

$$z_{11} = \frac{(Z_\text{b} + Z_\text{c})(Z_\text{a} + Z_\text{b})}{Z_\text{a} + 2Z_\text{b} + Z_\text{c}} \quad \text{and} \quad z_{22} = \frac{(Z_\text{a} + Z_\text{b})(Z_\text{b} + Z_\text{c})}{Z_\text{a} + 2Z_\text{b} + Z_\text{c}}$$

Therefore $z_{11} = z_{22}$.

$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$; Use the circuit below:

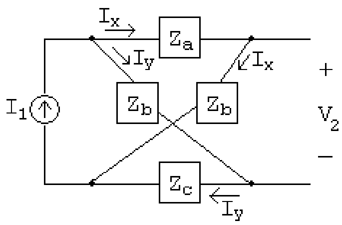


$$V_1 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_2 - I_x) = (Z_b + Z_c)I_x - Z_cI_2$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c} I_2 \qquad \text{so} \qquad V_1 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} I_2 - Z_c I_2$$

$$\therefore \quad Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_a Z_c}{Z_a + 2Z_b + Z_c}$$

$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$; Use the circuit below:



$$V_2 = Z_b I_x - Z_c I_y = Z_b I_x - Z_c(I_1 - I_x) = (Z_b + Z_c) I_x - Z_c I_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c} I_1 \qquad \text{so} \qquad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} I_1 - Z_c I_1$$

$$\therefore \quad z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_a Z_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

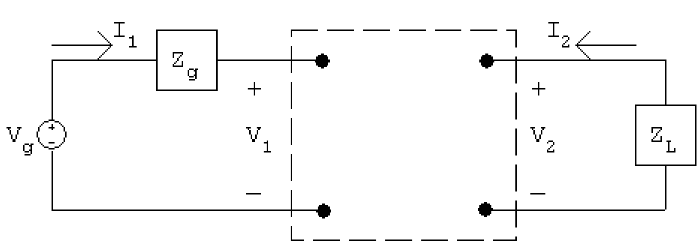
P 18.24 $V_1 = a_{11}V_2 - a_{12}I_2; \qquad V_1 = V_g - Z_g I_1$

$I_1 = a_{21}V_2 - a_{22}I_2; \qquad V_2 = -Z_L I_2$

$$V_1 = a_{11}V_2 - a_{12}\frac{V_2}{-Z_L} = \left(a_{11} + \frac{a_{12}}{Z_L}\right)V_2 = \frac{a_{11}Z_L + a_{12}}{Z_L}V_2$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{12} + a_{11}Z_L}$$

P 18.25



$I_1 = y_{11}V_1 + y_{12}V_2; \qquad V_1 = V_g - I_1 Z_g$

$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad V_2 = -Z_L I_2$$

$$V_2 = -Z_L(y_{21}V_1 + y_{22}V_2)$$

$$V_2(1 + Z_L y_{22}) = -Z_L y_{21} V_1$$

$$V_2 = \frac{-Z_L y_{21}}{1 + Z_L y_{22}} V_1$$

$$I_1 = y_{11}V_1 - \frac{y_{12}y_{21}Z_L}{1 + Z_L y_{22}} V_1$$

$$\frac{I_1}{V_1} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + Z_L y_{22}}$$

P 18.26 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad V_1 = V_g - I_1 Z_g$

$$I_2 = h_{21}I_1 + h_{22}V_2; \qquad V_2 = -Z_L I_2$$

$$I_2 = h_{21}I_1 + h_{22}(-Z_L I_2)$$

$$(1 + h_{22}Z_L)I_2 = h_{21}I_1$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

P 18.27 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_1 = V_g - I_1 Z_g$

$$V_2 = g_{21}V_1 + g_{22}I_2; \qquad V_2 = -Z_L I_2$$

$$V_{\text{Th}} = V_2 \Big|_{I_2=0}:$$

$$V_2 = g_{21}V_1; \qquad V_1 = \frac{I_1}{g_{11}} = \frac{V_g - V_1}{Z_g g_{11}}$$

$$\therefore \qquad V_1(1 + Z_g g_{11}) = V_g \qquad \text{so} \qquad V_1 = \frac{V_g}{1 + Z_g g_{11}}$$

Thus,

$$V_2 = V_{\text{Th}} = \frac{g_{21}V_g}{1 + Z_g g_{11}}$$

$$Z_{\rm Th} = \frac{V_2}{I_2}\bigg|_{V_g=0}:$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$V_1 = \frac{I_1 - g_{12}I_2}{g_{11}} = \frac{-(V_1/Z_g) - g_{12}I_2}{g_{11}} = \frac{-V_1 - g_{12}Z_gI_2}{Z_gg_{11}}$$

$$(1 + Z_gg_{11})V_1 = -g_{12}Z_gI_2$$

$$\therefore \qquad V_2 = g_{21}\left(\frac{-g_{12}Z_gI_2}{1 + Z_gg_{11}}\right) + g_{22}I_2 = \left(g_{22} - \frac{g_{12}g_{21}Z_g}{1 + Z_gg_{11}}\right)I_2$$

Thus,

$$Z_{\rm Th} = \frac{V_2}{I_2} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + Z_gg_{11}}$$

P 18.28 $V_2 = b_{11}V_1 - b_{12}I_1; \qquad V_1 = V_g - I_1Z_g$

$$I_2 = b_{21}V_1 - b_{22}I_1; \qquad V_2 = -Z_LI_2$$

$$V_g - Z_gI_1 = \frac{1}{b_{11}}V_2 + \frac{b_{12}}{b_{11}}I_1$$

$$V_g = \frac{1}{b_{11}}V_2 + \left(\frac{b_{12}}{b_{11}} + Z_g\right)I_1$$

$$I_1 = \frac{V_g - V_2/b_{11}}{Z_g + (b_{12}/b_{11})} = \frac{b_{11}V_g - V_2}{Z_gb_{11} + b_{12}}$$

$$\frac{-V_2}{Z_L} = b_{21}V_1 - b_{22}I_1 = b_{21}(V_g - Z_gI_1) - b_{22}I_1$$

$$= b_{21}V_g - (Z_gb_{21} + b_{22})I_1 = b_{21}V_g - (Z_gb_{21} + b_{22})\left[\frac{b_{11}V_g - V_2}{Z_gb_{11} + b_{12}}\right]$$

$$V_2\left(-\frac{1}{Z_L} - \frac{Z_gb_{21} + b_{22}}{Z_gb_{11} + b_{12}}\right) = \left(b_{21} - \frac{(Z_gb_{21} + b_{22})b_{11}}{Z_gb_{11} + b_{12}}\right)V_g$$

$$V_2\left(\frac{Z_gb_{11} + b_{12} + Z_gZ_Lb_{21} + b_{22}Z_L}{Z_L(Z_gb_{11} + b_{12})}\right) = \left(\frac{Z_gb_{11}b_{21} + b_{22}b_{11} - Z_gb_{11}b_{21} - b_{12}b_{21}}{Z_gb_{11} + b_{12}}\right)V_g$$

$$\frac{V_2}{V_g} = \frac{Z_L(b_{11}b_{22} - b_{12}b_{21})}{b_{12} + Z_gb_{11} + Z_Lb_{22} + Z_gZ_Lb_{21}} = \frac{Z_L\Delta b}{b_{12} + Z_gb_{11} + Z_Lb_{22} + Z_gZ_Lb_{21}}$$

P 18.29 $I_1 = g_{11}V_1 + g_{12}I_2$

$V_2 = g_{21}V_1 + g_{22}I_2$

From the first measurement:

$$g_{11} = \frac{I_1}{V_1} = \frac{100 \times 10^{-6}}{0.1} = 1\,\text{mS}$$

$$g_{21} = \frac{V_2}{V_1} = \frac{200}{0.1} = 2000$$

From the second measurement:

$$g_{12} = \frac{I_1}{I_2} = \frac{-25 \times 10^{-6}}{5 \times 10^{-3}} = -0.005$$

$$g_{22} = \frac{V_2}{I_2} = \frac{200}{5 \times 10^{-3}} = 40\,\text{k}\Omega$$

Summary:

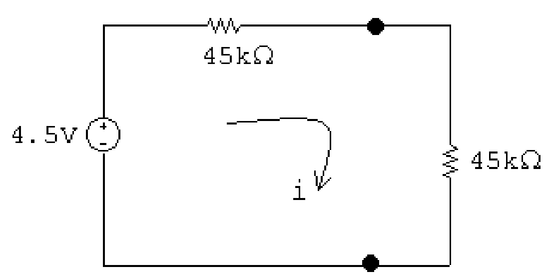
$g_{11} = 1\,\text{mS}; \quad g_{12} = -0.005; \quad g_{21} = 2000; \quad g_{22} = 40\,\text{k}\Omega$

From the circuit,

$Z_g = 1\,\text{k}\Omega; \qquad V_g = 4.5\,\text{mV}$

$$Z_{\text{Th}} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g} = 40{,}000 + \frac{10(1000)}{1+1} = 45{,}000$$

$$V_{\text{Th}} = \frac{g_{21}V_g}{1 + g_{11}Z_g} = \frac{2000(0.0045)}{1+1} = 4.5\,\text{V}$$



$$i = \frac{4.5}{90{,}000} = 50\,\mu\text{A}$$

$$P = (50 \times 10^{-6})^2(45{,}000) = 112.5\,\mu\text{W}$$

P 18.30 **[a]** $Z_{\text{Th}} = g_{22} - \dfrac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right)\left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore \quad Z_{\text{Th}} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3\,\Omega$$

$$\therefore \quad Z_L = 2.1 - j1.3\,\Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right)(2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2}(42\underline{/0^\circ}) = 5 - j6\,\text{V(rms)} = 7.81\underline{/-50.19^\circ}\,\text{V(rms)}$$

The rms value of V_2 is 7.81 V.

[b] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{Z_L} = \dfrac{-5 + j6}{2.1 - j1.3} = -3 + j1\,\text{A(rms)}$

$$P = |\mathbf{I}_2|^2(2.1) = 21\,\text{W}$$

[c] $\dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \dfrac{-g_{21}}{g_{11}Z_L + \Delta g}$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right)\left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right)\left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right)(2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore \quad g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-[(1/2) - j(1/2)]}{0.8 - j0.9}$$

$$\therefore \quad \mathbf{I}_1 = \frac{(0.8 - j0.9)\mathbf{I}_2}{-0.5 + j0.5} = \left(\frac{1.6 - j1.8}{-1 + j1}\right)\mathbf{I}_2$$

$$= (-1.7 + j0.1)(-3 + j1) = 5 - j2\,\text{A (rms)}$$

$$\therefore \quad P_g(\text{developed}) = (42)(5) = 210\,\text{W}$$

$$\%\text{ delivered } = \frac{21}{210}(100) = 10\%$$

P 18.31 [**a**] $$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(70{,}000) = -35$$

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(70 \times 10^3) = -2.5$$

$$y_{21}Z_L = (100 \times 10^{-3})(70 \times 10^3) = 7000$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{7000}{-35 - (6)(-2.5)} = \frac{7000}{-20} = -350$$

$$\mathbf{V}_2 = -350\mathbf{V}_g = -350(80) \times 10^{-3} = -28\,\text{V(rms)}$$

$$V_2 = 28\,\text{V(rms)}$$

[**b**] $$P = \frac{|\mathbf{V}_2|^2}{70{,}000} = 11.2 \times 10^{-3} = 11.20\,\text{mW}$$

[**c**] $$\mathbf{I}_2 = \frac{-28\underline{/180^\circ}}{70{,}000} = -0.4 \times 10^{-3}\underline{/180^\circ} = 400\underline{/0^\circ}\,\mu\text{A}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\Delta y = (2 \times 10^{-3})(-50 \times 10^{-6}) - (-2 \times 10^{-6})(100 \times 10^{-3})$$

$$= 100 \times 10^{-9}$$

$$\Delta y Z_L = (100)(70) \times 10^3 \times 10^{-9} = 7 \times 10^{-3}$$

$$y_{11} + \Delta y Z_L = 2 \times 10^{-3} + 7 \times 10^{-3} = 9 \times 10^{-3}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{9 \times 10^{-3}} = \frac{100}{9}$$

$$\therefore \quad 100\mathbf{I}_1 = 9\mathbf{I}_2; \qquad \mathbf{I}_1 = \frac{9(400 \times 10^{-6})}{100} = 36\,\mu\text{A(rms)}$$

$$P_g = (80)10^{-3}(36) \times 10^{-6} = 2.88\,\mu\text{W}$$

P 18.32 **[a]** $Z_{\rm Th} = \dfrac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$

From the solution to Problem 18.31

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$y_{22} + \Delta y Z_g = -50 \times 10^{-6} + 10^{-7}(2500) = 200 \times 10^{-6}$$

$$Z_{\rm Th} = \frac{6}{200} \times 10^6 = 30{,}000\,\Omega$$

$$Z_L = Z^*_{\rm Th} = 30{,}000\,\Omega$$

[b] $y_{21}Z_L = (100 \times 10^{-3})(30{,}000) = 3000$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30{,}000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20\underline{/180^\circ}\,\text{V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30{,}000} = \frac{400}{30} \times 10^{-3} = 13.33\,\text{mW}$$

[c] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{30{,}000} = \dfrac{20\underline{/0^\circ}}{30{,}000} = \dfrac{2}{3}\,\text{mA}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30{,}000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30}\,\text{mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3})\left(\frac{1}{30} \times 10^{-3}\right) = \frac{8}{3}\,\mu\text{W}$$

P 18.33 $\dfrac{V_2}{V_g} = \dfrac{\Delta b Z_{\rm L}}{b_{12} + b_{11}Z_g + b_{22}Z_{\rm L} + b_{21}Z_gZ_{\rm L}}$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21} = (25)(-40) - (1000)(-1.25) = 250$$

$$\therefore \quad \frac{V_2}{V_g} = \frac{250(100)}{1000 + 25(20) - 40(100) - 1.25(2000)} = -5$$

$$V_2 = -5(120\underline{/0^\circ}) = 600\underline{/180^\circ}\,\text{V(rms)}$$

$$I_2 = \frac{-V_2}{100} = \frac{-600/180^\circ}{100} = 6\,\text{A(rms)}$$

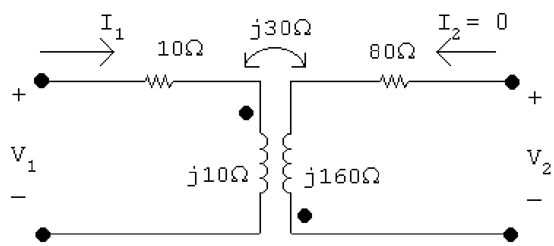
$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_\text{L}} = \frac{-250}{25 - 1.25(100)} = 2.5$$

$$\therefore \quad I_1 = \frac{I_2}{2.5} = \frac{6}{2.5} = 2.4\,\text{A(rms)}$$

$$\therefore \quad P_g = (120)(2.4) = 288\,\text{W}; \qquad P_o = 36(100) = 3600\,\text{W}$$

$$\therefore \quad \frac{P_o}{P_g} = \frac{3600}{288} = 12.5$$

P 18.34 **[a]** For $I_2 = 0$:

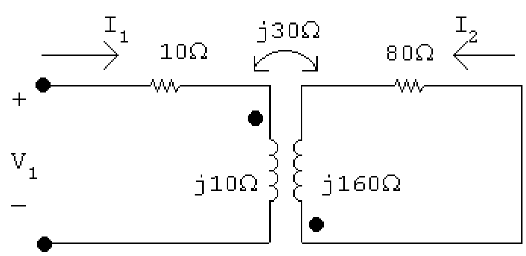


$$V_2 = -j30I_1 = -j30\frac{V_1}{10 + j10} = \frac{-j3V_1}{1 + j1}$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} = \frac{1 + j1}{-j3} = \frac{-1 + j1}{3}$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} = \frac{1}{-j30} = \frac{j}{30}\,\text{S}$$

For $V_2 = 0$:



$$V_1 = (10 + j10)I_1 - j30I_2$$

$$0 = -j30I_1 + (80 + j160)I_2$$

$$\Delta = \begin{vmatrix} 10 + j10 & -j30 \\ -j30 & 80 + j160 \end{vmatrix} = 100(1 + j24)$$

$$N_2 = \begin{vmatrix} 10 + j10 & V_1 \\ -j30 & 0 \end{vmatrix} = j30V_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{j30V_1}{100(1 + j24)}$$

$$a_{12} = \frac{-V_1}{I_2}\bigg|_{V_2=0} = -80 + j\frac{10}{3}\,\Omega$$

$$j30I_1 = (80 + j160)I_2$$

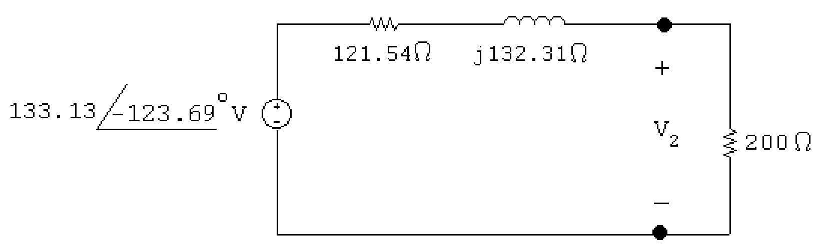
$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} = -\frac{8}{3}(2 - j1)$$

[b] $V_{\text{Th}} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{80\underline{/0^\circ}}{(-1 + j1)/3 + j5/30} = \dfrac{(80\underline{/0^\circ})30}{-10 + j10 + j5} = \dfrac{2400\underline{/0^\circ}}{-10 + j15}$

$$= 133.13\underline{/-123.69^\circ}\,\text{V}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{[-(10/3)(24 - j1)] + [(-8/3)(2 - j1)(5)]}{[(-1 + j1)/3] + [(j/30)(5)]}$$

$$= 121.54 + j132.31\,\Omega$$



[c] $V_2 = \dfrac{200}{121.54 + j132.31}(133.13\underline{/-123.69^\circ}) = 132.87\underline{/-124.16^\circ}$

$$v_2(t) = 132.87\cos(400t - 124.16^\circ)\,\text{V}$$

P 18.35 When $V_2 = 0$

$V_1 = 20\,\text{V}, \qquad I_1 = 1\,\text{A}, \qquad I_2 = -1\,\text{A}$

When $I_1 = 0$

$V_2 = 80\,\text{V}, \qquad V_1 = 400\,\text{V}, \qquad I_2 = 3\,\text{A}$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{20}{1} = 20\,\Omega$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{400}{80} = 5$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{-1}{1} = -1$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{3}{80} = 37.5\,\text{mS}$$

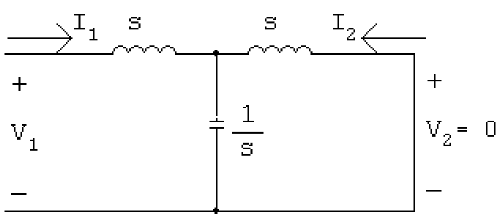
$$Z_{\text{Th}} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h} = 10\,\Omega$$

Source-transform the current source and parallel resistance to get $V_g = 240$ V. Then,

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_{\text{L}})(h_{11} + Z_g) - h_{12}h_{21}Z_{\text{L}}} = -1.5\,\text{A}$$

$$P = (-1.5)^2(10) = 22.5\,\text{W}$$

P 18.36 **[a]** $y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$; $\qquad y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$



$$V_1 = \left[s + \left(\frac{1}{s}\|s\right)\right]I_1 = \frac{s(s^2+1)+s}{s^2+1}I_1$$

$$\therefore \quad y_{11} = \frac{I_1}{V_1} = \frac{s^2+1}{s(s^2+2)}$$

$$I_2 = \frac{-(1/s)}{s+(1/s)}I_1 = \frac{-1}{s^2+1}\cdot\frac{s^2+1}{s(s^2+2)}V_1 = \frac{-1}{s(s^2+2)}V_1$$

$$\therefore \quad y_{21} = \frac{-1}{s(s^2+2)}$$

Because the two-port circuit is symmetric,

$$y_{12} = y_{21} = \frac{-1}{s(s^2+2)} \qquad \text{and} \qquad y_{22} = y_{11} = \frac{s^2+1}{s(s^2+2)}$$

[b] $$\frac{V_2}{V_g} = \frac{y_{21}Z_g}{y_{12}y_{21}Z_gZ_L - (1+y_{11}Z_g)(1+y_{22}Z_L)}$$

$$= \frac{y_{21}}{y_{12}y_{21} - (1+y_{11})(1+y_{22})}$$

$$= \frac{\dfrac{-1}{s(s^2+2)}}{\dfrac{1}{s^2(s^2+2)^2} - \left(1+\dfrac{s^2+1}{s(s^2+2)}\right)\left(1+\dfrac{s^2+1}{s(s^2+2)}\right)}$$

$$= \frac{-s(s^2+2)}{1-(s^3+s^2+2s+1)^2}$$

$$= \frac{1}{s^3+2s^2+3s+2}$$

$$= \frac{1}{(s+1)(s^2+s+2)}$$

$$\therefore \quad V_2 = \frac{10}{(s+1)(s+2)(s^2+s+2)}$$

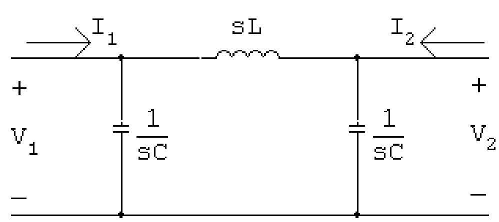
$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

$$K_1 = 5; \qquad K_2 = -2.5; \qquad K_3 = 1.34\underline{/159.3^\circ}$$

$$\therefore \quad v_2(t) = [5e^{-t} - 2.5e^{-2t} + 2.67e^{-0.5t}\cos(1.32t + 159.3^\circ)]u(t)\,\text{V}$$

P 18.37 **[a]** $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$; $\qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$



$$\frac{V_1}{I_1} = \frac{1}{sC}\Big\|\left(sL+\frac{1}{sC}\right) = \frac{[sL+(1/sC)](1/sC)}{sL+(2/sC)}$$

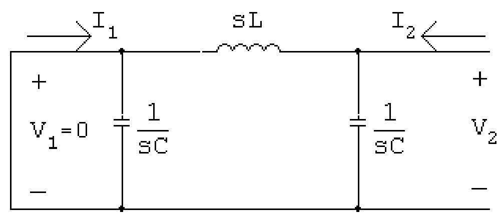
$$= \frac{sL+(1/sC)}{s^2LC+2} = \frac{s^2LC+1}{sC(s^2LC+2)} = \frac{(1/C)[s^2+(1/LC)]}{s[s^2+(2/LC)]}$$

$$\therefore \qquad g_{11} = \frac{Cs[s^2+(2/LC)])}{s^2+(1/LC)}$$

$$V_2 = \frac{(1/sC)}{sL + (1/sC)}V_1 \qquad \text{so} \qquad \frac{V_2}{V_1} = \frac{(1/sC)}{sL + (1/sC)} = \frac{1}{s^2LC + 1} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$\therefore \quad g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}; \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$



$$I_1 = \frac{-(1/sC)}{sL + (1/sC)}I_2 \qquad \text{so} \qquad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{22} = sL\|(1/sC) = \frac{sL/sC}{sL + (1/sC)} = \frac{sL}{s^2LC + 1} = \frac{(1/C)s}{s^2 + (1/LC)}$$

Summary:

$$g_{11} = \frac{Cs[s^2 + (2/LC)])}{s^2 + (1/LC)}; \qquad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}; \qquad g_{22} = \frac{(1/C)s}{s^2 + (1/LC)}$$

[b] $\frac{1}{LC} = \frac{10^9}{(0.05)(32)} = 625 \times 10^6$

$$g_{11} = \frac{32 \times 10^{-9}s(s^2 + 1250 \times 10^6)}{s^2 + 625 \times 10^6}$$

$$g_{12} = \frac{-625 \times 10^6}{s^2 + 625 \times 10^6}$$

$$g_{21} = \frac{625 \times 10^6}{s^2 + 625 \times 10^6}$$

$$g_{22} = \frac{3125 \times 10^4 s}{s^2 + 625 \times 10^6}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L} = \frac{\left(\frac{625\times10^6}{s^2+625\times10^6}\right)500}{\frac{3125\times10^4 s}{(s^2+625\times10^6)} + 500}$$

$$\frac{V_2}{V_1} = \frac{625 \times 10^6}{s^2 + 62{,}500s + 625 \times 10^6} = \frac{625 \times 10^6}{(s + 12{,}500)(s + 50{,}000)}$$

$$V_1 = \frac{4}{s}$$

$$V_2 = \frac{2500 \times 10^6}{s(s + 12{,}500)(s + 50{,}000)} = \frac{4}{s} - \frac{5.33}{s + 12{,}500} + \frac{1.33}{s + 50{,}000}$$

$$v_2 = [4 - 5.33e^{-12{,}500t} + 1.33e^{-50{,}000t}]u(t) \quad \text{V}$$

P 18.38 $a'_{11} = \frac{z_{11}}{z_{21}} = \frac{35/3}{4000/3} = 8.75 \times 10^{-3}\,\Omega$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{25 \times 10^4/3}{4000/3} = 62.5\,\Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{4000/3} = 0.75 \times 10^{-3}\,\Omega$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{10{,}000/3}{4000/3} = 2.5\,\Omega$$

$$a''_{11} = \frac{-y_{22}}{y_{21}} = \frac{-40 \times 10^{-6}}{-800 \times 10^{-6}} = 0.05\,\text{S}$$

$$a''_{12} = \frac{-1}{y_{21}} = \frac{-1}{-800 \times 10^{-6}} = 1250\,\text{S}$$

$$a''_{21} = \frac{-\Delta y}{y_{21}} = \frac{-4 \times 10^{-8}}{-800 \times 10^{-6}} = 50 \times 10^{-6}\,\text{S}$$

$$a''_{22} = \frac{-y_{11}}{y_{21}} = \frac{-200 \times 10^{-6}}{-800 \times 10^{-6}} = 0.25\,\text{S}$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (8.75 \times 10^{-3})(0.05) + (62.5)(50 \times 10^{-6}) = 3.5625 \times 10^{-3}$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (8.75 \times 10^{-3})(1250) + (62.5)(0.25) = 26.5625$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (0.75 \times 10^{-3})(0.05) + (2.5)(50 \times 10^{-6}) = 162.5 \times 10^{-6}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (0.75 \times 10^{-3})(1250) + (2.5)(0.25) = 1.5625$$

$$V_2 = \frac{Z_L V_g}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{(15{,}000)(0.03)}{[3.5625 \times 10^{-3} + (162.5 \times 10^{-6})(10)](15{,}000) + 26.5625 + (1.5625)(10)} = 3.75\,\text{V}$$

P 18.39 The a parameters of the first two port are

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{-5\times10^{-3}}{40} = -125\times10^{-6}$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-25}{40}\times10^{-6} = -625\times10^{-9}\,\text{S}$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -25\times10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4};\quad a''_{12} = \frac{3R}{4};\quad a''_{21} = \frac{3}{4R};\quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25;\quad a''_{12} = 54\,\text{k}\Omega;\quad a''_{21} = \frac{1}{96}\,\text{mS};\quad a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125\times10^{-6}(1.25) + (-25)(10^{-3}/96) = \frac{-10^{-2}}{24}$$

$$a_{12} = -125\times10^{-6}(54\times10^{3}) + (-25)(1.25) = -38\,\Omega$$

$$a_{21} = -625\times10^{-9}(1.25) + (-25\times10^{-3})(10^{-3}/96) = \frac{-10^{-4}}{96}\,\text{S}$$

$$a_{22} = -625\times10^{-9}(54\times10^{3}) + (-25\times10^{-3})(1.25) = -65\times10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11}+a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = \frac{-10^{-4}}{96}(800) = \frac{-10^{-2}}{12}$$

$$a_{11} + a_{21}Z_g = \frac{-10^{-2}}{24} + \frac{-10^{-2}}{12} = \frac{-10^{-2}}{8}$$

$$(a_{11}+a_{21}Z_g)Z_L = \frac{-10^{-2}}{8}(72{,}000) = -90$$

$$a_{22}Z_g = -65\times10^{-3}(800) = -52$$

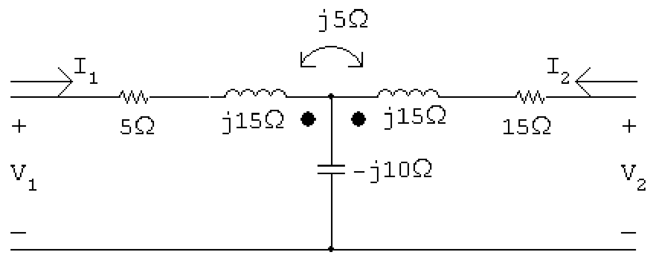
$$\frac{V_o}{V_g} = \frac{72{,}000}{-90-38-52} = -400$$

$$v_o = V_o = -400V_g = -3.6\,\text{V}$$

P 18.40 **[a]** From reciprocity and symmetry

$a'_{11} = a'_{22}, \quad \Delta a' = 1; \quad \therefore \quad 5^2 - 24a'_{21} = 1, \quad a'_{21} = 1\,\text{S}$

For network B



$$a''_{11} = \frac{\mathbf{V}_1}{\mathbf{V}_2}\bigg|_{I_2=0}$$

$$\mathbf{V}_1 = (5 + j15 - j10)\mathbf{I}_1 = (5 + j5)\mathbf{I}_1$$

$$\mathbf{V}_2 = (-j10 + j5)\mathbf{I}_1 = -j5\mathbf{I}_1$$

$$a''_{11} = \frac{5 + j5}{-j5} = -1 + j1$$

$$a''_{21} = \frac{\mathbf{I}_1}{\mathbf{V}_2}\bigg|_{I_2=0} = \frac{1}{-j5} = j0.2\,\text{S}$$

$$a''_{22} = a''_{11} = -1 + j1$$

$$\Delta a'' = 1 = (-1 + j1)(-1 + j1) - j0.2a''_{12}$$

$$\therefore \quad a''_{12} = -10 + j5$$

Summary:

$a'_{11} = 5 \qquad a''_{11} = -1 + j1$

$a'_{12} = 24\,\Omega \qquad a''_{12} = -10 + j5\,\Omega$

$a'_{21} = 1\,\text{S} \qquad a''_{21} = j0.2\,\text{S}$

$a'_{22} = 5 \qquad a''_{22} = -1 + j1$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = -5 + j9.8$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = -74 + j49\,\Omega$$

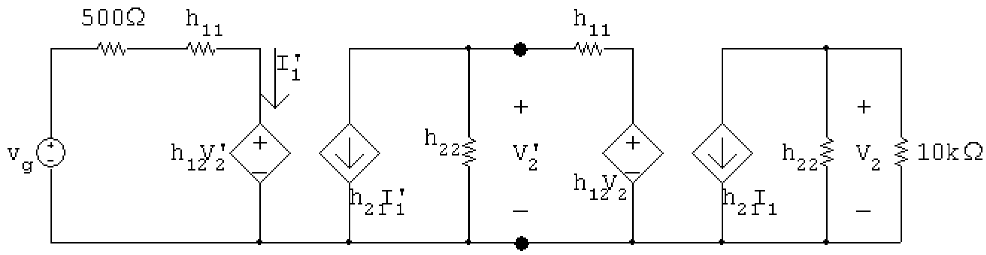
$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = -1 + j2\,\text{S}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = -15 + j10$$

$$\mathbf{I}_2 = \frac{-V_g}{a_{11}Z_\text{L} + a_{12} + a_{21}Z_gZ_\text{L}} = 0.295 + j0.279\,\text{A}$$

$$\mathbf{V}_2 = -10I_2 = -2.95 - j2.79\,\text{V}$$

P 18.41 [**a**] At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$;
At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$



[**b**] $\frac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$

therefore $I_1 = -2 \times 10^{-6}V_2$

$V_2' = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$

$100I_1' + 10^{-4}V_2' + (-2 \times 10^{-6})V_2 = 0$

therefore $I_1' = 205 \times 10^{-10}V_2$

$V_g = 1500I_1' + 15 \times 10^{-4}V_2' = 3000 \times 10^{-8}V_2$

$\frac{V_2}{V_g} = \frac{10^5}{3} = 33{,}333$

P 18.42 [**a**] $V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$

$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$

$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$

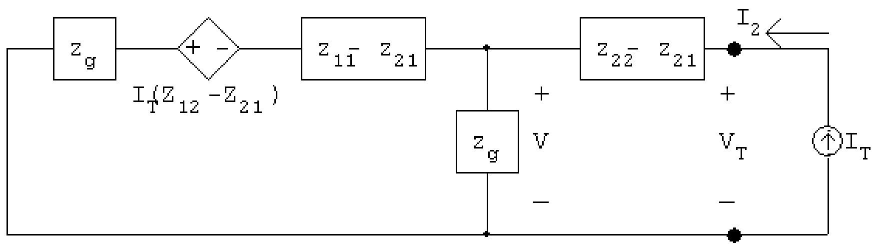
[**b**] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}}\right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

Thus $\frac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\frac{z_{12}z_{21}}{Z_g + z_{11}}\right)$



For $V_{\rm Th}$ note that $V_{\rm oc} = \dfrac{z_{21}}{Z_g + z_{11}} V_g$ since $I_2 = 0$.

P 18.43 **[a]** $V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$

$$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$$

[b] With port 2 terminated in an impedance Z_L, the two mesh equations are

$$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$$

$$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$$

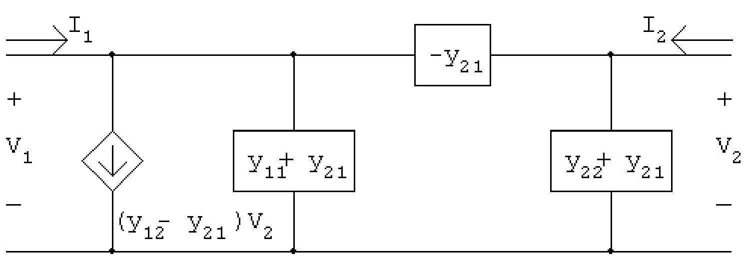
Solving for I_1:

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

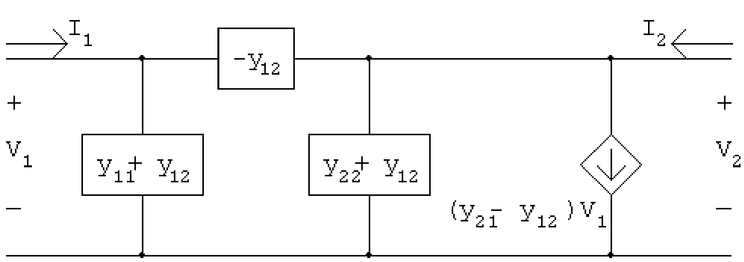
Therefore

$$Z_{\rm in} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

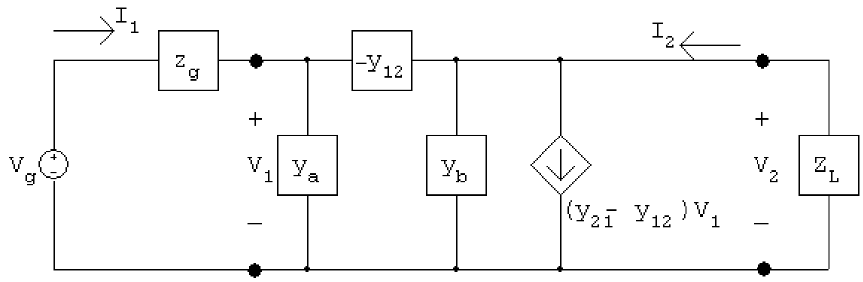
P 18.44 **[a]** $I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2;\qquad I_2 = y_{21}V_1 + y_{22}V_2$



$I_1 = y_{11}V_1 + y_{12}V_2;\qquad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$



[b] Using the second circuit derived in part [a], we have



where $y_a = (y_{11} + y_{12})$ and $y_b = (y_{22} + y_{12})$

At the input port we have

$I_1 = y_a V_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_b V_2 - y_{12}(V_2 - V_1) = 0$$

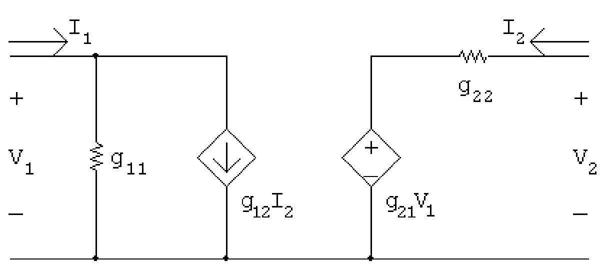
Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L}\right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_L I_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

P 18.45 **[a]** The g-parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:



[b] The g parameters for the first two port in Fig P 18.39(a) are

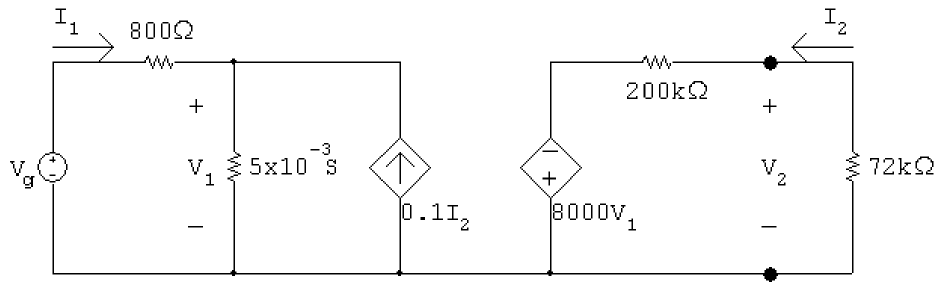
$$g_{11} = \frac{h_{22}}{\Delta h} = \frac{25 \times 10^{-6}}{5 \times 10^{-3}} = 5 \times 10^{-3}\,\text{S}$$

$$g_{12} = \frac{-h_{12}}{\Delta h} = \frac{-5 \times 10^{-4}}{5 \times 10^{-3}} = -0.10$$

$$g_{21} = \frac{-h_{21}}{\Delta h} = \frac{-40}{5 \times 10^{-3}} = -8000$$

$$g_{22} = \frac{h_{11}}{\Delta h} = \frac{1000}{5\times10^{-3}} = 200\,\text{k}\Omega$$

From Problem 3.67 $R_{ef} = 72\,\text{k}\Omega$, hence our circuit reduces to



$$V_2 = \frac{-8000V_1(72)}{272}$$

$$I_2 = \frac{-V_2}{72{,}000} = \frac{8V_1}{272}$$

$$v_g = 9\,\text{mV}$$

$$\therefore\quad \frac{V_1 - 9\times10^{-3}}{800} + V_1(5\times10^{-3}) - 0.1\frac{8V_1}{272} = 0$$

$$V_1 - 9\times10^{-3} + 4V_1 - \frac{80V_1}{34} = 0$$

$$\therefore\quad V_1 = 3.4\times10^{-3}$$

$$V_2 = \frac{-8000(72)}{272}\times3.4\times10^{-3} = -7.2\,\text{V}$$

From Problem 3.67

$$\frac{V_o}{V_2} = 0.5; \qquad \therefore\quad V_o = -3.6\,\text{V}$$

This result matches the solution to Problem 18.39.

P 18.46 **[a]** To determine b_{11} and b_{21} create an open circuit at port 1. Apply a voltage at port 2 and measure the voltage at port 1 and the current at port 2. To determine b_{12} and b_{22} create a short circuit at port 1. Apply a voltage at port 2 and measure the currents at ports 1 and 2.

[b] The equivalent b-parameters for the black-box amplifier can be calculated as follows:

$$b_{11} = \frac{1}{h_{12}} = \frac{1}{10^{-3}} = 1000$$

$$b_{12} = \frac{h_{11}}{h_{12}} = \frac{500}{10^{-3}} = 500\,\text{k}\Omega$$

$$b_{21} = \frac{h_{22}}{h_{12}} = \frac{0.05}{10^{-3}} = 50\,\text{S}$$

$$b_{22} = \frac{\Delta h}{h_{12}} = \frac{23.5}{10^{-3}} = 23{,}500$$

Create an open circuit a port 1. Apply 1 V at port 2. Then,

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{1}{V_1} = 1000 \quad \text{so} \quad V_1 = 1\,\text{mV measured}$$

$$b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{I_2}{10^{-3}} = 50\,\text{S} \quad \text{so} \quad I_2 = 50\,\text{mA measured}$$

Create a short circuit a port 1. Apply 1 V at port 2. Then,

$$b_{12} = -\frac{V_2}{I_1}\bigg|_{V_1=0} = \frac{-1}{I_1} = 500\,\text{k}\Omega \quad \text{so} \quad I_1 = -2\,\mu\text{A measured}$$

$$b_{22} = -\frac{I_2}{I_1}\bigg|_{V_1=0} = \frac{-I_2}{-2\times 10^{-6}} = 23{,}500 \quad \text{so} \quad I_2 = 47\,\text{mA measured}$$

P 18.47 [**a**] To determine y_{11} and y_{21} create a short circuit at port 2. Apply a voltage at port 1 and measure the currents at ports 1 and 2. To determine y_{12} and y_{22} create a short circuit at port 1. Apply a voltage at port 2 and measure the currents at ports 1 and 2.

[**b**] The equivalent y-parameters for the black-box amplifier can be calculated as follows:

$$y_{11} = \frac{1}{h_{11}} = \frac{1}{500} = 2\text{ mS}$$

$$y_{12} = \frac{-h_{12}}{h_{11}} = \frac{-10^{-3}}{500} = -2\,\mu\text{S}$$

$$y_{21} = \frac{h_{21}}{h_{11}} = \frac{1500}{500} = 3\text{ S}$$

$$y_{22} = \frac{\Delta h}{h_{11}} = \frac{23.5}{500} = 47\text{ mS}$$

Create a short circuit at port 2. Apply 1 V at port 1. Then,

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{I_1}{1} = 2\text{ mS} \quad \text{so} \quad I_1 = 2\,\text{mA measured}$$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{I_2}{1} = 3\text{ S} \quad \text{so} \quad I_2 = 3\,\text{A measured}$$

Create a short circuit at port 1. Apply 1 V at port 2. Then,

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{I_1}{1} = -2\,\mu\text{S} \quad \text{so} \quad I_1 = -2\,\mu\text{A measured}$$

$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = \frac{I_2}{1} = 47\text{ mS} \quad \text{so} \quad I_2 = 47\,\text{mA measured}$$