BIRZEIT UNIVERSITY

Numbering Systems

Computer Science Department

Comp 1310

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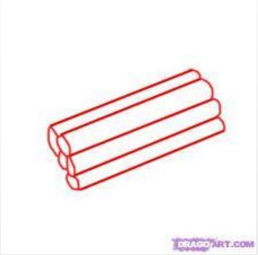
Outline

- History.
- Decimal System.
- Binary System.
- Octal System.
- Hexadecimal System.
- Converting from one System to another system & back.
- Binary Addition
- Signed Numbers
- Summary

History

- Long ago, humans used sticks to count.

- Later learned how to draw pictures of sticks in the ground and eventually on paper.



- Using symbols to represent the numbers instead of sticks. 5 v 

Decimal System

- Most People Use decimal representation to count.

- In decimal there are **10** digits

  0,1,2,3,4,5,6,7,8,9
- The base is **10**
- We can Represent any value for these digits

  Ex: 754 , 123 , 889 , 345

Decimal System

Ex: 754

$$7.10^2 + 5.10^1 + 4.10^0 = 700+50+4=754$$

base

Digit position

123 ???

Binary System

- Computer is not smart as a human .
- Easy to make an electronic machine with two states: on and off , or 1 and 0.
- In Binary there are **2** digits

0,1

The base is **2**

Binary System

- Each digit in binary number called **BIT.**

  1 0 1 0 , 4 digits, **How many bits ?**

  answer : 4 bits

- 4 bits form a **NIBBLE**.
- 8 bits form a byte.
- 1 0 1 0 0 0 1 1 **, How many Bits, Nibbles and Bytes?**

Answer :8 bits ,2 Nibbles and 1 byte

Binary System

❖ Two bytes form a **WORD** and two words form a **DOUBLE WORD (rarely used) .**

EX:

0000 1111 1010 1010 : 16 bits , WORD

Octal System

- Uses 8 digits
  0,1,2,3,4,5,6,7

- The base is **8**

- **EX** $(123)_8$, $(156)_8$

Hexadecimal System

❖ Uses 16 digits
  0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

❖ The base is **16**

❖ **EX: 123h , 456h 0E120h**

❖ **Suppose we need to develop new system with base 5,7 or 3?**

Base 5 : 0,1,2,3,4

Base 7 : 0,1,2,3,4,5,6

Base 3: 0,1,2

Decimal To any other System

Rule: Divide the number by the target base and take the remainders in reverse order

Let see examples

Decimal to Binary

$(\mathbf{22})_{10} = (\ )_2$

Input	Result	Remainder
22/2	11	0
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

$(\mathbf{22})_{10} = (\mathbf{10110})_2$

Decimal to Binary

$(13)_{10} = (1101)_2$

$(220)_{10} = (11011100)_2$

$(21)_{10} = (\quad)_2$ H.W

$(15)_{10} = (\quad)_2$ H.W

Decimal to Octal

Let's convert the value $(39)_{10}$ to Octal

Input	Result	Remainder
39/8	4	7
4/8	0	4

$(39)_{10} = (47)_8$

Decimal to Hexadecimal

Let's convert the value $(39)_{10}$ to Hexadecimal

Input	Result	Remainder
39/16	2	7
2/16	0	2

$(39)_{10}$ = (27h)

Any other system to Decimal

Rule

Ex: 754

$7.10^2 + 5.10^1 + 4.10^0 = 700+50+4=754$

base

Digit position

123 ???

Binary to Decimal

- 10110b

  $1*2^4+0*2^3+1*2^2+1*2^1+0*2^0=$

  $16+0+4+2= (22)_{10}$

1010b =?? , 0010b = ?? , 101b=??

Answer: **1010b=(10)** $_{10}$
0010b=(2) $_{10}$
101b=(5) $_{10}$

Octal to decimal

- $(47)_8$

  $4*8^1 + 7*8^0 = 32 + 7 = (39)_{10}$

Hexadecimal to decimal

- $(27)_8$

$2*16^1 + 7*16^0 = 32 + 7 = (39)_{10}$

H.W

Covert the following numbers to **decimal**

a. $(72)_8 = (58)_{10}$

b. $(72)_{16} = (114)_{10}$

c. $(DE1)_{16} = (3553)_{10}$

From Binary to other system or other System to binary

- You can first convert to decimal then to the target system but this is a long way.

Octal	binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Binary to Octal

100101010b = ( )$_8$

100 101 010 = (452) $_8$

111000111b= ( )$_8$

111 000 111 = (707)$_8$

Binary to Octal

- ❖ 100101011b = $(453)_8$
- ❖ 101101011b = $(\quad)_8$ **H.W**
- ❖ 100101001b = $(\quad)_8$ **H.W**

Binary to Hexadecimal

Hexadecimal	Binary	Hexadecimal	Binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Binary to Hexadecimal

10010101b = ( )$_h$

1001 0101 =(95h)

11100011b= (E3h) H.W

Extra Exercises

Using pen and paper , solve the following questions :

a. $(AB)_{16} = (\quad)_2$

b. $(23)_4 = (\quad)_8$

c. $(35)_7 = (\quad)_8$

d. $(72E)_{16} = (\quad)_8$

Binary Addition

0	0	1	1
+ 0	+ 1	+ 0	+ 1
--------	-------	------	------
0	1	1	10

1
+ 1
+ 1

11

Binary Addition

01111+00110 =

```
    1    1  1
    0  1 1 1 1
+   0  0 1 1 0
  ---------------
     1  0 1 0 1
```

Binary Addition

11010011+01010110=

```
         1   1   1  1
         1 1 0 1  0  0  1  1      →(211)₁₀
+        0 1 0 1  0 1   1   0     →(86)₁₀
        --------------------------
       1 0 0  1  0 1  0   0  1    = (297)₁₀
```

$(211)_{10}$, $(86)_{10}$, $(297)_{10}$

Signed Numbers

Our study of binary arithmetic, we have only considered positive numbers .

What about negative numbers?

Signed Numbers

➢ Signed Magnitude

add an extra digit to the front of our binary number to indicate whether the number is positive or negative.

this digit called sign bit.

0 for positive

1 for negative

Signed Numbers

Example:

$(5)_{10} = (101)_2$

Positive 5 is **0** 1 0 1

Negative 5 is **1** 1 0 1

The Problem : We need to specify how many bits in our numbers so we can be certain which bit is representing the sign !!!

Signed Numbers

1 1 0 1 is 13 or -5

- **One's Complement**

Representing a signed number with 1's

Complement is done by changing all the bits that are 1 to 0 and all bits that are 0 to 1.

Signed Numbers

- ❑ Represent -5 in 1's complement by using 4-bit arithmetic?

0101 → 1010

- ❑ Represent -1 in 1's complement ?

0001 → 1110

Signed Numbers

➢ Two's Complement

2's comp=1'comp +1

❑ Represent -5 in 2's complement by using 4-bit arithmetic?

(101)1's → 1010

2's + 1

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1 0 1 1 =(-5)

Summary

Decimal System.
Binary System.
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Converting from one System to another system & back.
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