Chapter 8

Potential Energy and Conservation of Energy

8-1 Potential Energy

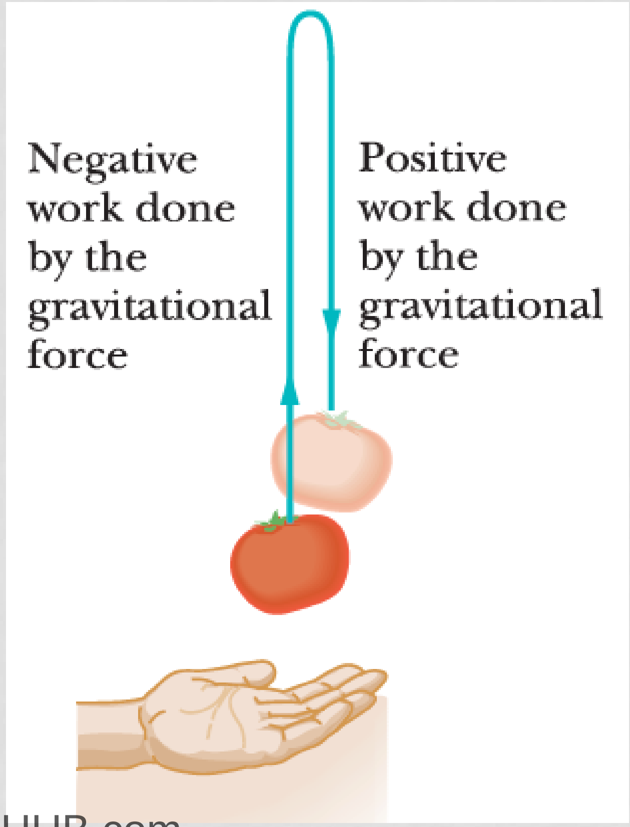
- **Potential energy** *U* is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper

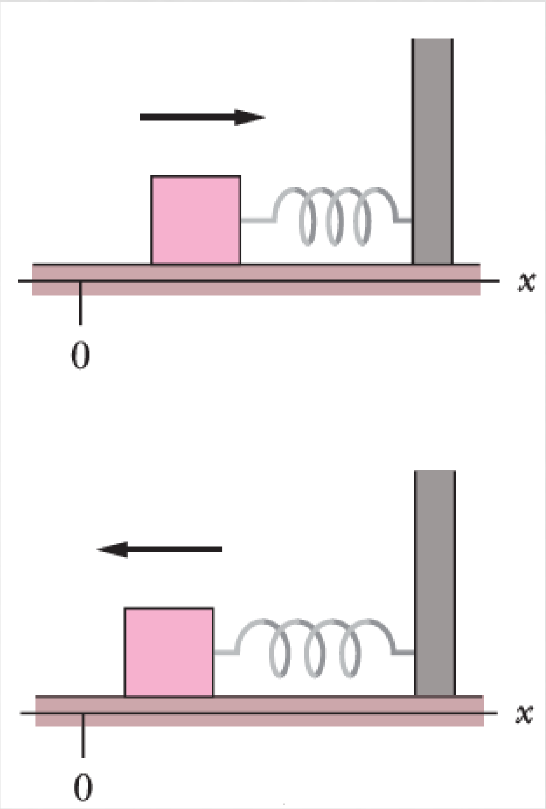
  - **Gravitational potential energy** accounts for kinetic energy increase during the fall
  - **Elastic potential energy** accounts for deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy

- For an object being raised or lowered:

$$\Delta U = -W$$

- The change in gravitational potential energy is the negative of the work done
- This also applies to an elastic block-spring system





- Key points:
  1. The *system* consists of two or more objects
  2. A *force* acts between a particle (tomato/block) and the rest of the system
  3. When the configuration changes, the force does *work* W_1, changing kinetic energy to another form
  4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2
- Thus the kinetic energy of the tomato/block becomes potential energy, and then kinetic energy again

- **Conservative forces** are forces for which $W_1 = -W_2$ is always true
  - Examples: gravitational force, spring force
  - Otherwise we could not speak of their potential energies

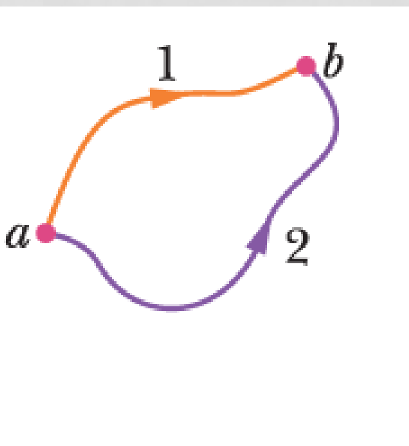
- **Nonconservative forces** are those for which $W_1 = -W_2$ is false
  - Examples: kinetic friction force, drag force
  - Kinetic energy of a moving particle is transferred to heat by friction
  - Thermal energy cannot be recovered back into kinetic energy of the object via the friction force
  - Therefore the force is not conservative, thermal energy is not a potential energy

- When only conservative forces act on a particle, we find many problems can be simplified:

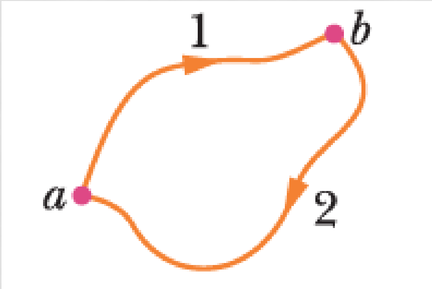
The net work done by a conservative force on a particle moving around any closed path is zero.

- A result of this is that:

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

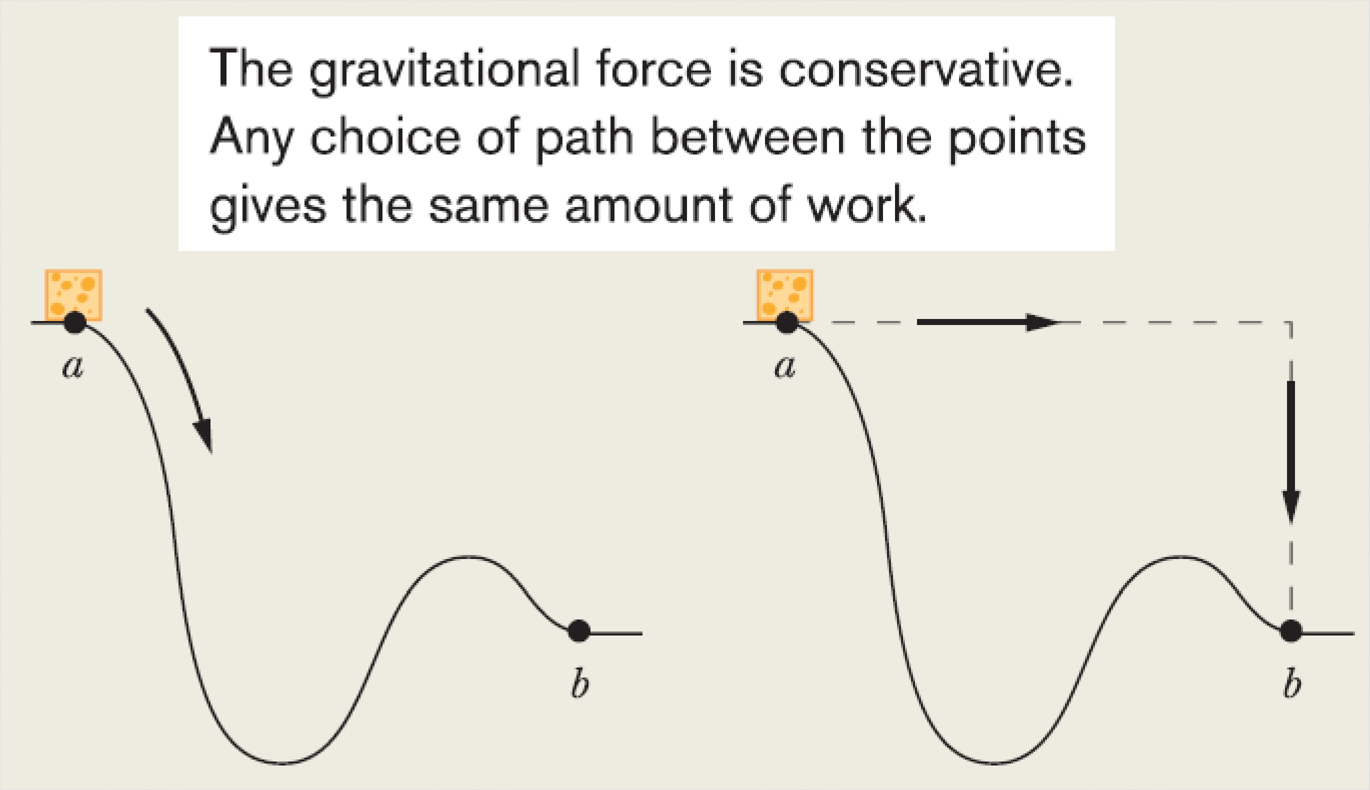


The force is conservative. Any choice of path between the points gives the same amount of work.



And a round trip gives a total work of zero.

- Mathematically: $$W_{ab,1} = W_{ab,2}$$
- This result allows you to substitute a simpler path for a more complex one if only conservative forces are involved

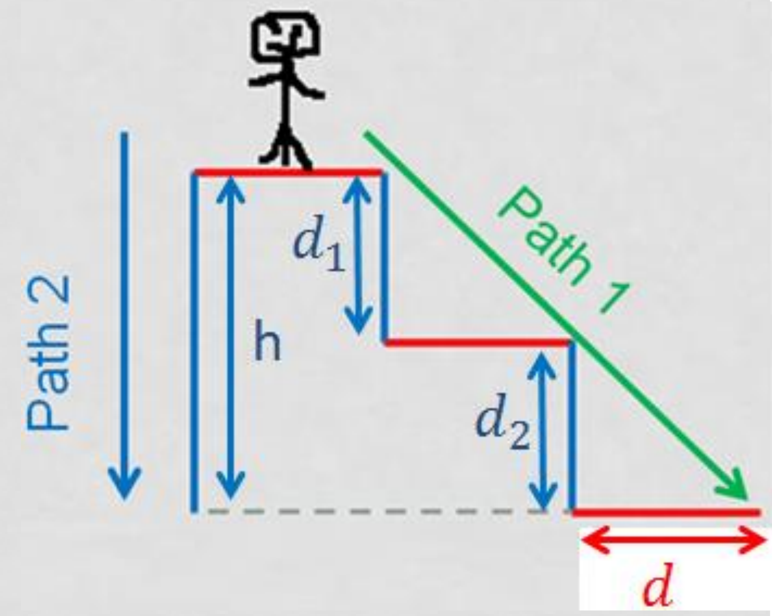


Gravitational energy is conservative one:

$$W_{ab,1} = W_{ab,2}$$

$$W_g = mgd\ cos\emptyset$$

In which $\emptyset$ is the angle between $\vec{F}_g$ and $\vec{d}$



- Work done by the gravitational force in path 1:

$$W_g = mgd\ \cos 90^\circ + mgd_1\ \cos 0^\circ + mgd\ \cos 90^\circ + mgd_2 \cos 0^\circ$$

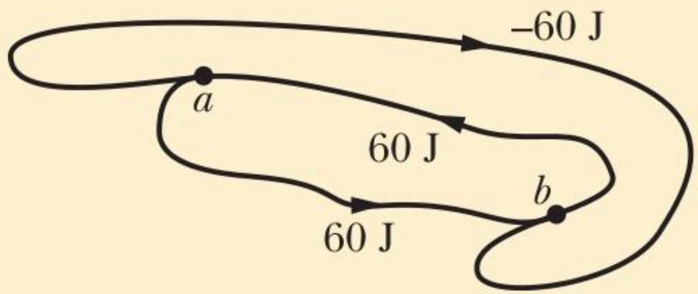
$$W_g = mgh$$

- Work done by the gravitational force in path 2:

$$W_g = mgh\ \cos 0^\circ = mgh$$

Checkpoint 1

The figure shows three paths connecting points *a* and *b*. A single force $\vec{F}$ does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force $\vec{F}$ conservative?



Answer: No. The paths from *a* → *b* have different signs. One pair of paths allows the formation of a zero-work loop. The other does not.

- For the general case, we calculate work as: $$W = \int_{x_i}^{x_f} F(x)\,dx$$

- So we calculate potential energy as: $$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

- **Gravitational PE**, relative to a reference configuration with reference point $y_i = 0$:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\,dy = mg\int_{y_i}^{y_f} dy = mg\Big[y\Big]_{y_i}^{y_f}$$

$$\Delta U = mg(y_f - y_i) = mg\,\Delta y$$

$$U(y) = mgy$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

Elastic PE:

Spring force: $F_x = -kx$

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \tfrac{1}{2}k\Big[x^2\Big]_{x_i}^{x_f}$$

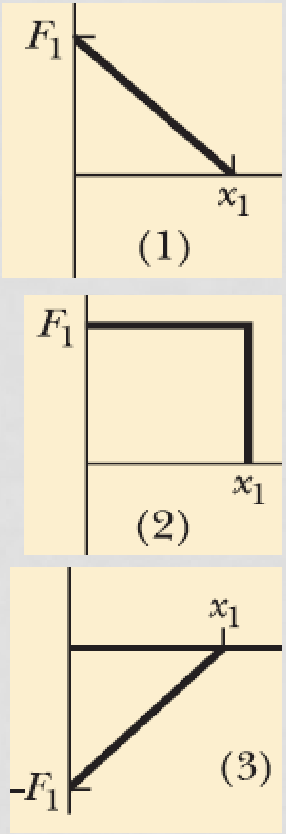
$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2$$

- With reference point $x_i = 0$ for a relaxed spring:

$$U(x) = \tfrac{1}{2}kx^2$$

Checkpoint 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x. The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.



Answer: (3), (1), (2); a positive force does positive work, decreasing the PE.

a negative force (in figure 3) does negative work, increasing the PE

8-2 Conservation of Mechanical Energy

- The mechanical energy of a system is the sum of its potential energy *U* and kinetic energy *K*: $$E_{mec} = K + U$$

- Work done by conservative forces increases *K* and decreases *U* by that amount, so: $$\Delta K = -\Delta U$$

- Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1$$

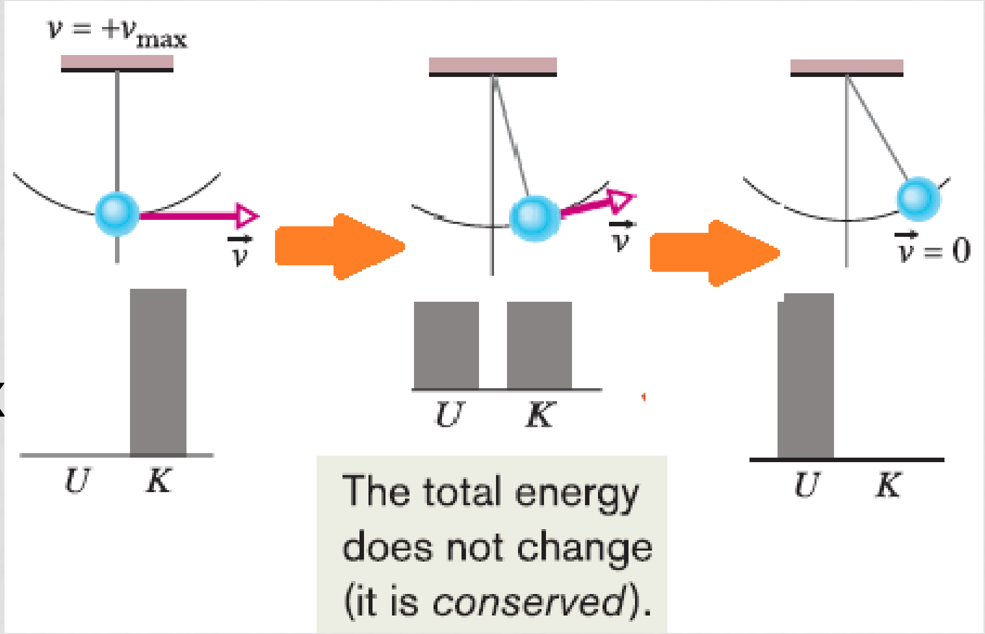
In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

The principle of the Conservation of Mechanical Energy:

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

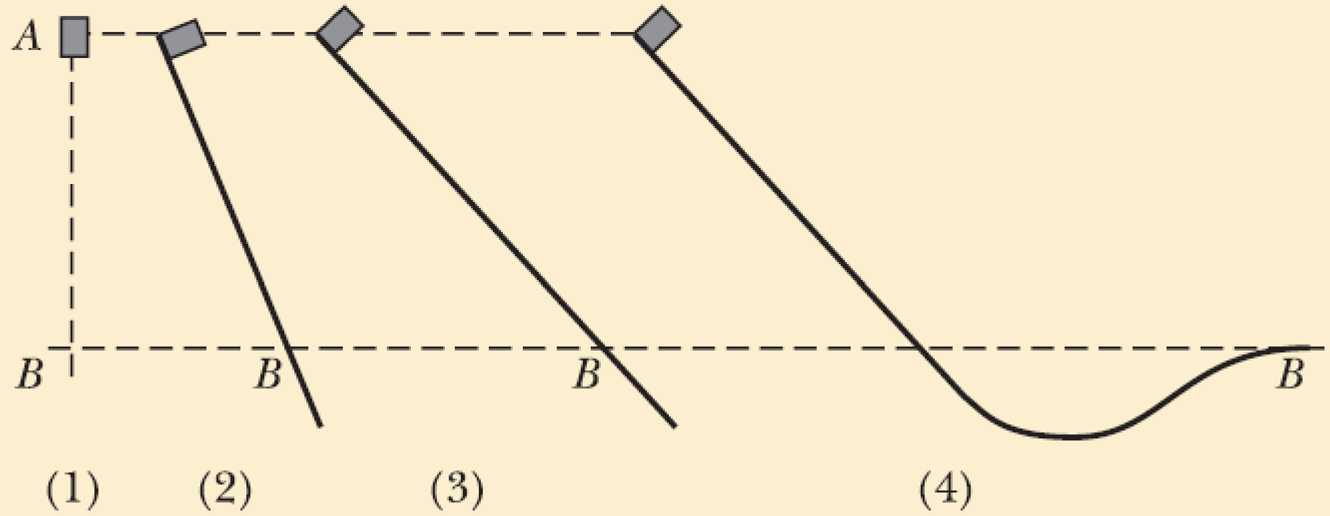
When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved.*

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point U = max and K = min



Checkpoint 3

The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point B, greatest first. (b) Rank them according to the speed of the block at point B, greatest first.



Answer: Since there are no nonconservative forces, all of the difference in potential energy must go to kinetic energy. Therefore all are equal in (a). Because of this fact, they are also all equal in (b).

•13 SSM A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring. (a) What is the change ΔU_g in the gravitational potential energy of the marble–Earth system during the 20 m ascent? (b) What is the change ΔU_s in the elastic potential energy of the spring during its launch of the marble? (c) What is the spring constant of the spring?

a) Change in gravitational potential energy:

$$\Delta U_g = mg\Delta y = (0.005\,kg)(9.8\,m/s^2)(20\,m\,) = 0.98\,\text{J}$$

a) Change in elastic potential energy:

By conservation of mechanical energy (Kinetic energy is zero at the release point and highest one) $\Delta E_{mec} = 0 \;\rightarrow \Delta U_g + \Delta U_s + \Delta K = 0$

$$\Delta U_s = -\Delta U_g = -0.98\,J$$

c) The change in elastic potential energy by the compressed spring is equal to work done by the spring. $\Delta U_s = -W_s = -\frac{1}{2}k\,x^2$

$$-0.98\,J = -\frac{1}{2}k\,(0.08m)^2 \qquad \rightarrow \rightarrow \qquad k = 306.1N/m$$

••25 At $t = 0$ a 1.0 kg ball is thrown from a tall tower with $\vec{v} = (18 \text{ m/s})\hat{i} + (24 \text{ m/s})\hat{j}$. What is ΔU of the ball–Earth system between $t = 0$ and $t = 6.0$ s (still free fall)?

From chapter 4:

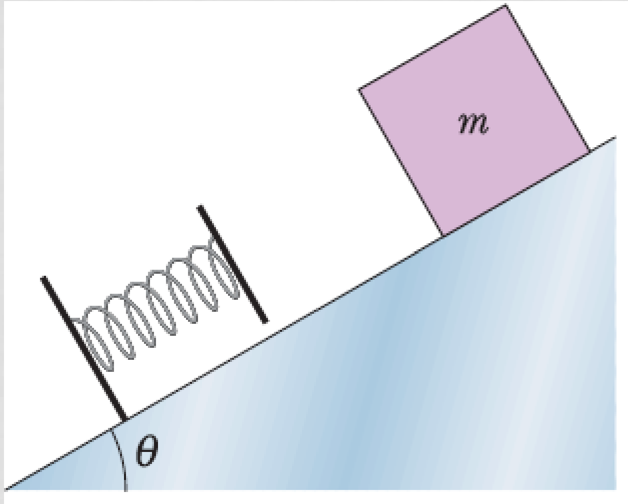
$$\Delta y = v_{oy}t + \frac{1}{2}gt^2$$

To find the vertical displacement of the particle between t= 0 to 6 sec:

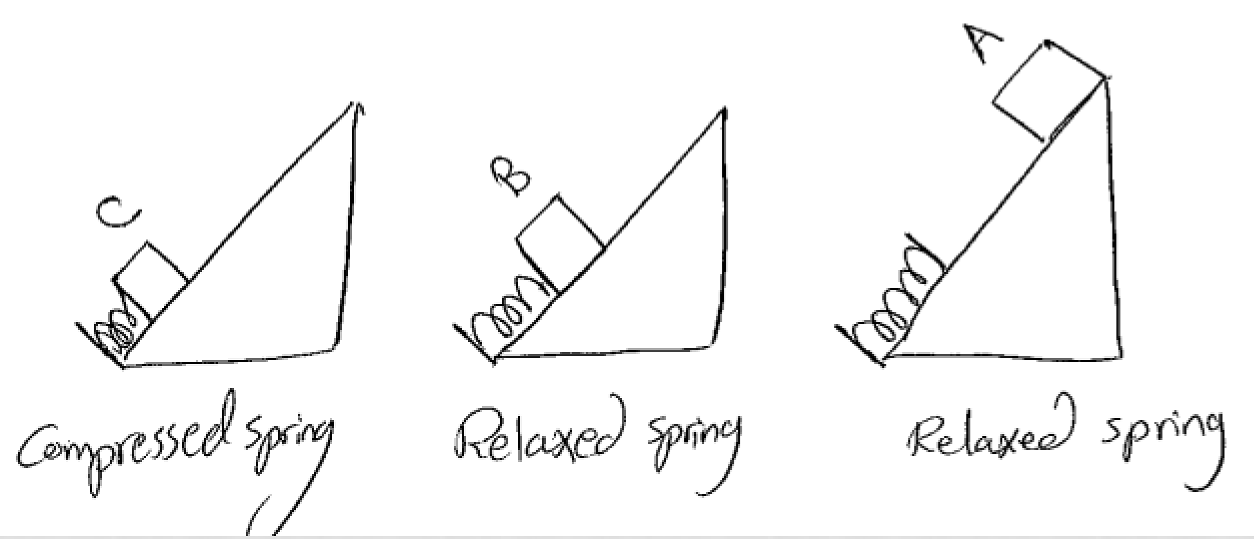
$$\Delta y = (24m/s)(6\ s) + \frac{1}{2}(-10\ m/s^2)(6\ s)^2 = -36 \text{ m}$$

$$\Delta U = mg\ \Delta y = (1.0\ kg)(10\ m/s^2)(-36 \text{ m }) = -360\ J$$

••29 SSM WWW In Fig. 8-42, a block of mass $m = 12$ kg is released from rest on a frictionless incline of angle $\theta = 30°$. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm. (a) How far does the block move down the incline from its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring?



Spring constant: $F_a = kx \rightarrow\rightarrow k = \frac{F_a}{x} = \frac{270\ N}{0.02\ m} = 13.5 \times 10^3 N/m$



a) By conservation of mechanical energy(A and C):

$$\Delta E_{mec} = 0 \rightarrow \Delta U_g + \Delta U_s + \Delta K = 0$$

$$\Delta U_g = -\Delta U_s = -\frac{1}{2}k\,x^2 = -\frac{1}{2}\left(13.5 \times \frac{10^3 N}{m}\right)(0.055\,m)^2 = -20.4\,J$$

$$\Delta U_g = mg\Delta y \rightarrow \Delta y = \frac{-20.4\,J}{(12\,kg)(9.8\,m/s^2)} = 0.174\,m$$

The total distance travelled by the block $= \frac{0.174\,m}{sin30^{\circ}} = 0.35m$

b) The speed of the block just as it touches the spring:
By conservation of mechanical energy(A and B):

$$\Delta E_{mec} = 0 \rightarrow \Delta U_g + \Delta K = 0$$

$$\Delta K = -\Delta U_g = mg\Delta y = mg(0.35m - 0.055m)\sin 30^{\circ} = mg(0.1475m)$$

$$\Delta K = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.8\,m/s^2)(0.1475m)} = 1.7\,m/s$$

Energy is conserved in the process. The total energy of the block at position *B* is:

$$E_B = \frac{1}{2} m v_B^2 + m g h_B$$

$$E_B = \frac{1}{2}(12\ kg)(1.7 m/s)^2 + (12\ kg)(9.8\ m/s^2)(0.0275\ m) = 20.5\text{J}$$

8-3 Reading a Potential Energy Curve

- For one dimension, force and potential energy are related (by work) as:

$$F(x) = -\frac{dU(x)}{dx}$$

- Therefore we can find the force *F*(*x*) from a plot of the potential energy *U*(*x*), by taking the derivative (slope)
- If we write the mechanical energy out:

$$U(x) + K(x) = E_{\text{mec}}$$

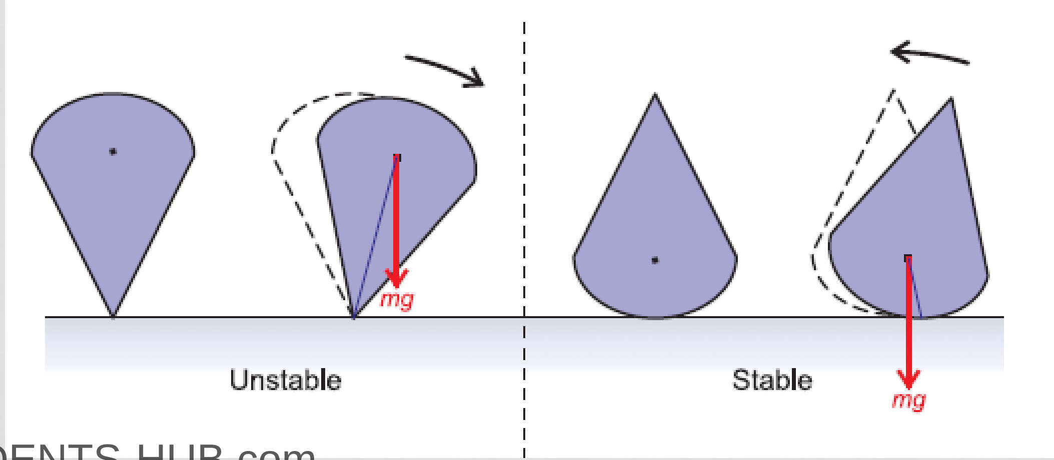
- We see how *K*(*x*) varies with *U*(*x*):

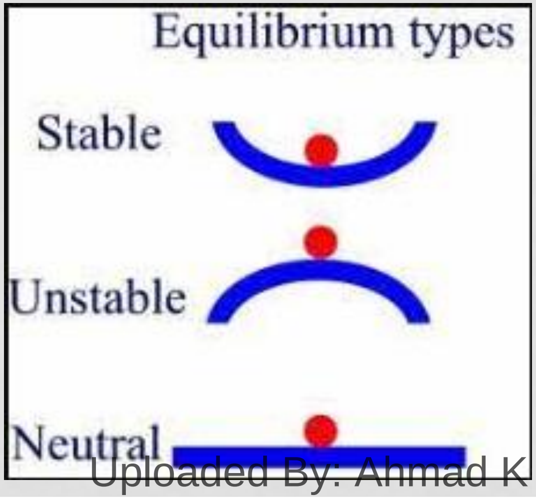
$$K(x) = E_{\text{mec}} - U(x)$$

Notes:

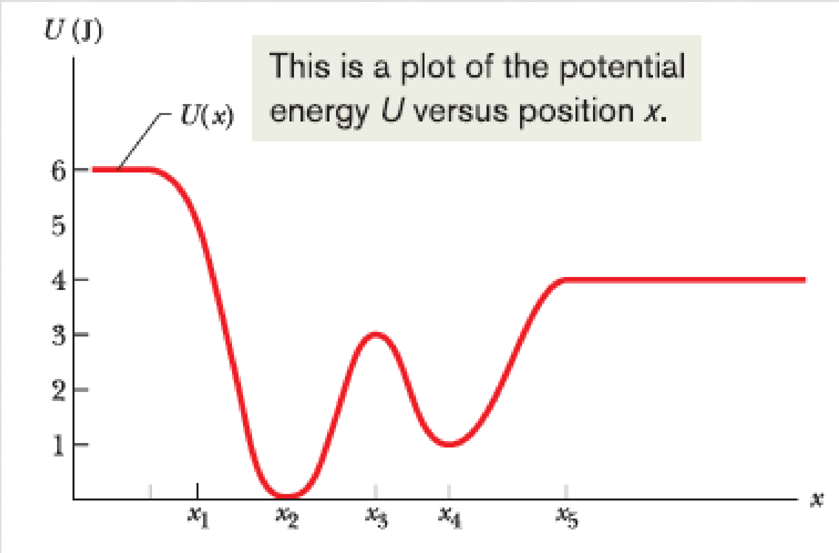
- To find $K(x)$ at any place, take the total mechanical energy (constant) and subtract $U(x)$
- Places where $K = 0$ are **Turning points**
  - There, the particle changes direction (K cannot be negative)
- At equilibrium points, the slope of $U(x)$ is 0
- A particle in **Neutral equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it would remain in its new position
  - Example: a marble on a flat tabletop

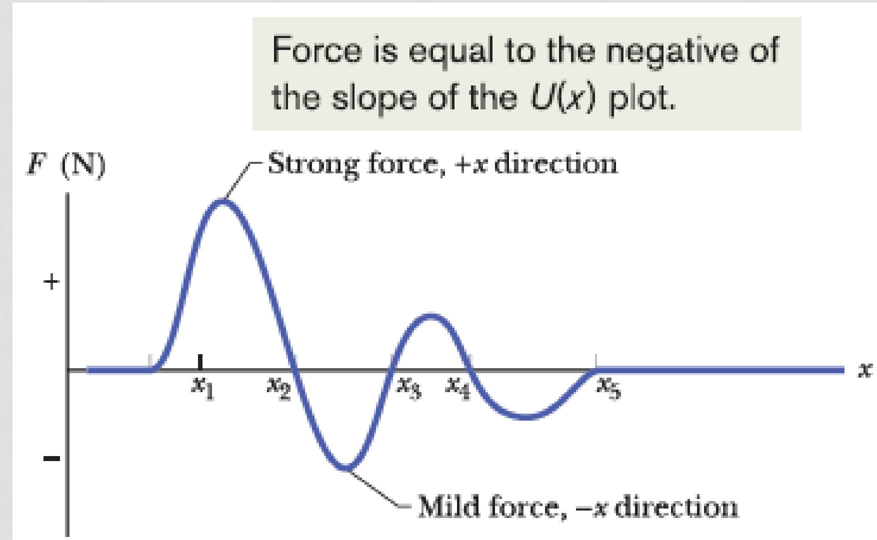
- A particle in **Unstable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced slightly to one direction, it will feel a force propelling it in that direction
  - Example: a marble balanced on a bowling ball
- A particle in **Stable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it will feel a force returning it to its original position
  - Example: a marble placed at the bottom of a bowl

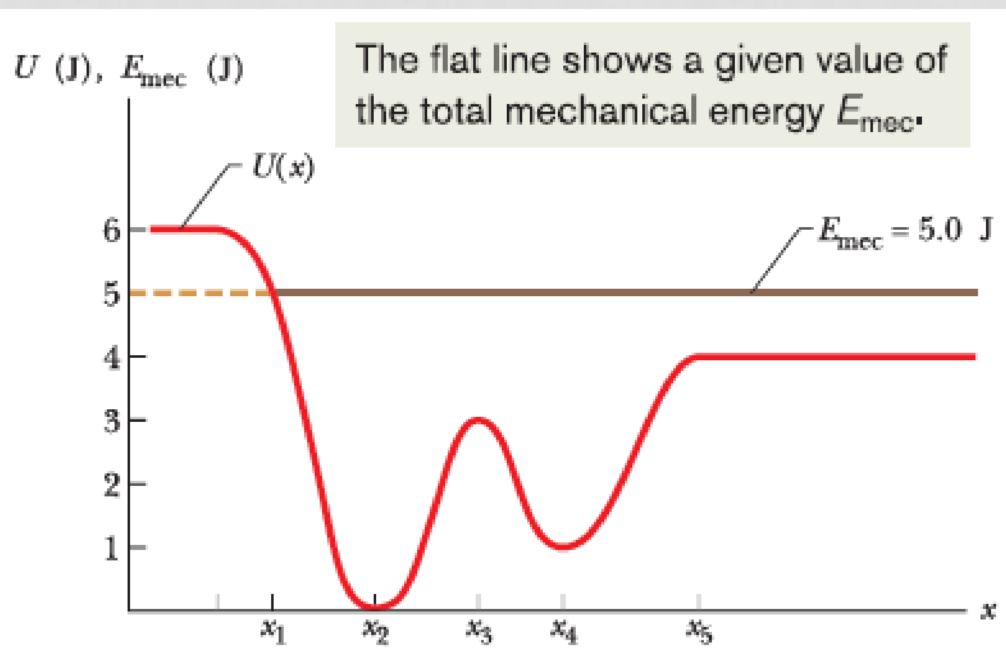


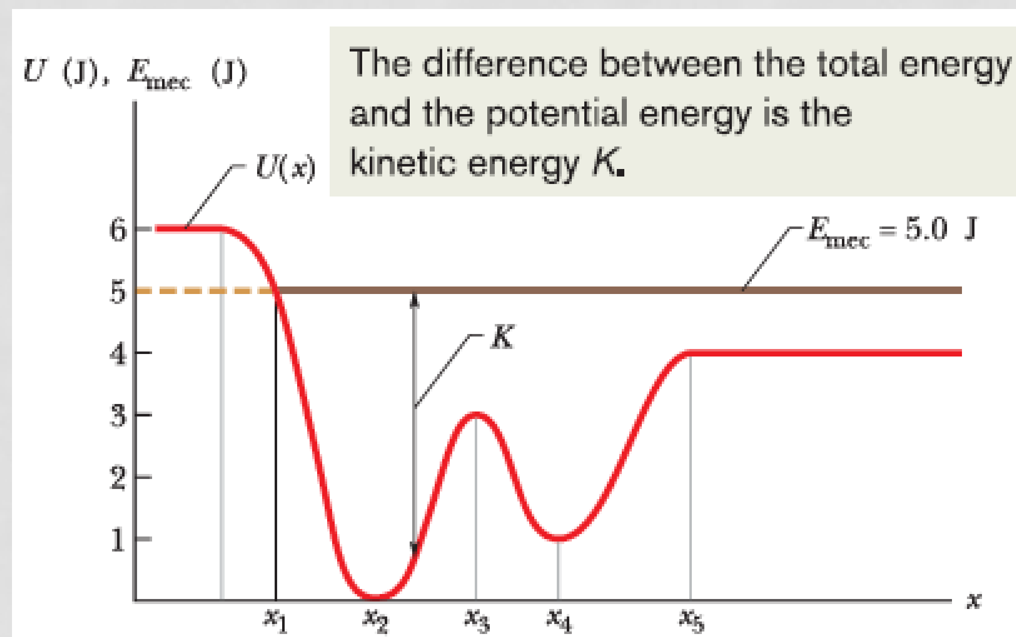


$$F(x) = -\frac{dU(x)}{dx}$$

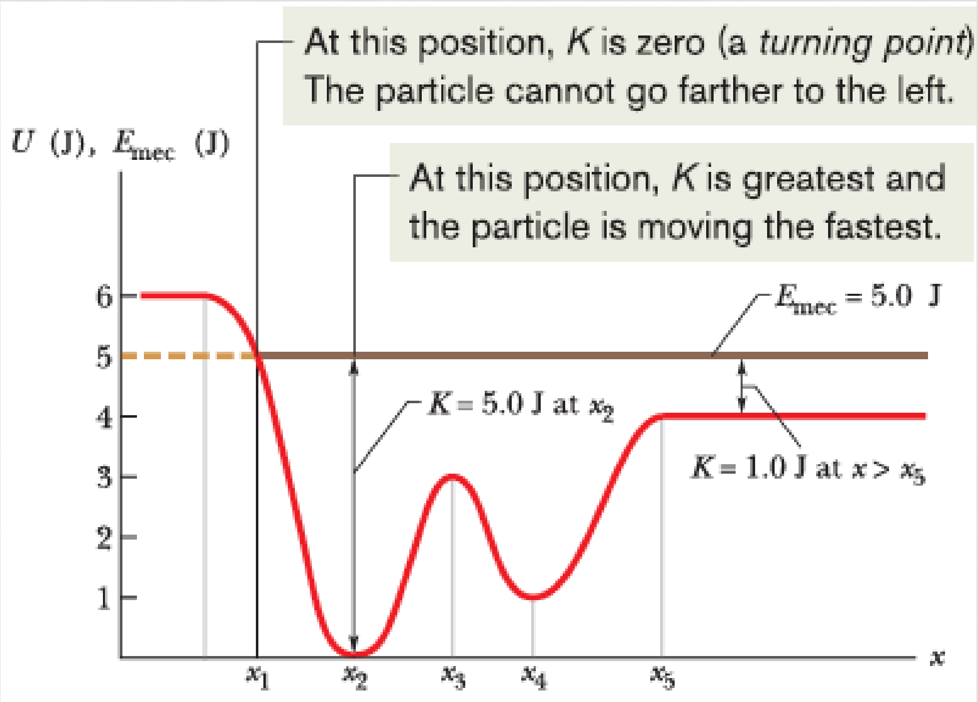


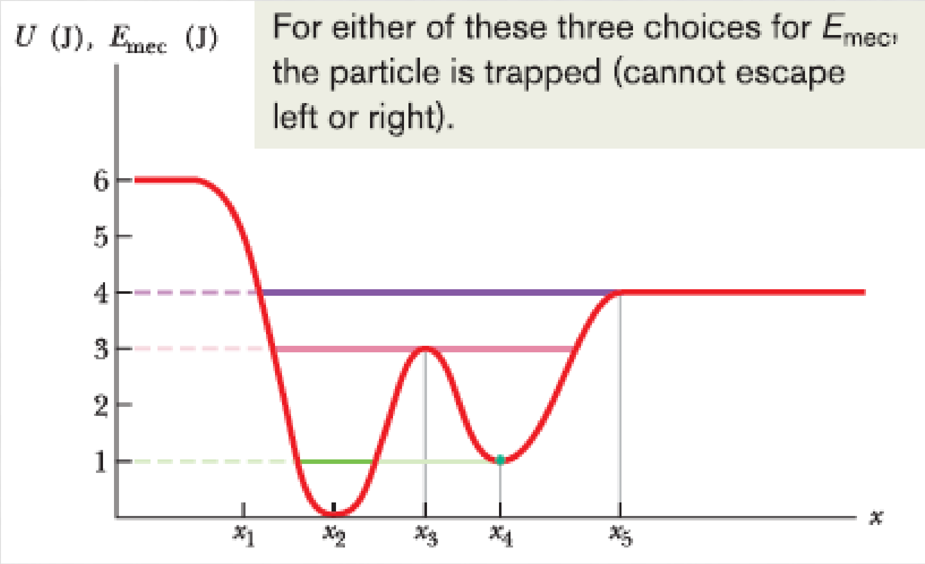






$$K(x) = E_{mec} - U(x)$$

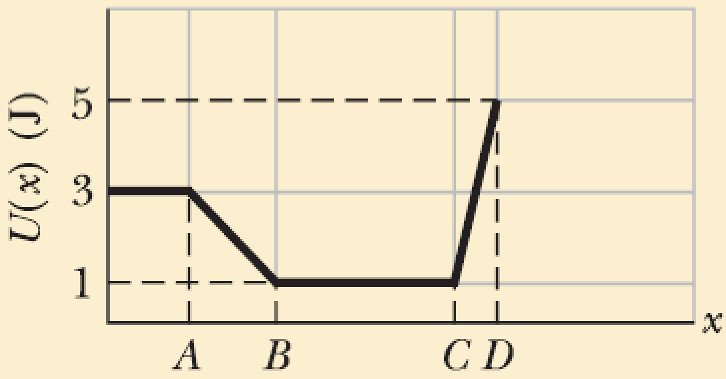




$x < x_1$ is forbidden for the E_{mec} = 5 J: the particle does not have the energy to reach those points

Checkpoint 4

The figure gives the potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion. (a) Rank regions AB, BC, and CD according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region AB?

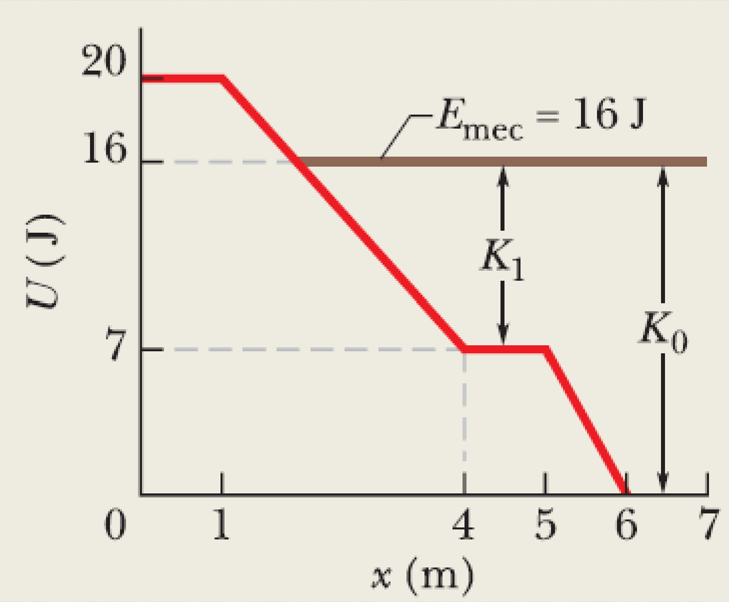


Answer: (a) *CD*, *AB*, *BC* (b) to the right

$$F(x) = -\frac{dU(x)}{dx}$$

Sample Problem 8.04 Reading a potential energy graph

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10a. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U. At $x = 6.5$ m, the particle has velocity $\vec{v}_0 = (-4.00 \text{ m/s})\hat{\text{i}}$.



(a) From Fig. 8-10a, determine the particle's speed at $x_1 = 4.5$ m.

At $x = 6.5$ m, the particle has kinetic energy

$$K_0 = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(2.00 \text{ kg})(4.00 \text{ m/s})^2 = 16.0 \text{ J}.$$

$$E_{mec} = K_0 + U_0 = 16.0 \text{ J} + 0 = 16.0 \text{ J}.$$

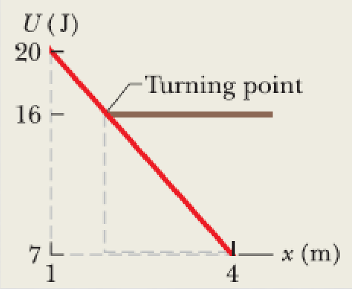
From that figure we see that at $x = 4.5$ m, the potential energy is $U_1 = 7.0$ J. The kinetic energy K_1 is the difference between E_{mec} and U_1:

$$K_1 = E_{mec} - U_1 = 16.0 \text{ J} - 7.0 \text{ J} = 9.0 \text{ J}.$$

$$K_1 = \tfrac{1}{2}mv_1^2 \qquad v_1 = 3.0 \text{ m/s}.$$

(b) Where is the particle's turning point located?

particle momentarily has $v = 0$ and thus $K = 0$.



A straight line has constant slope

$$(1, 20), (x, 16), (4, 7) \qquad \frac{20 - 16}{1 - x} = \frac{20 - 7}{1 - 4} \rightarrow\rightarrow x = 1.9\ m$$

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0 \text{ m}$.

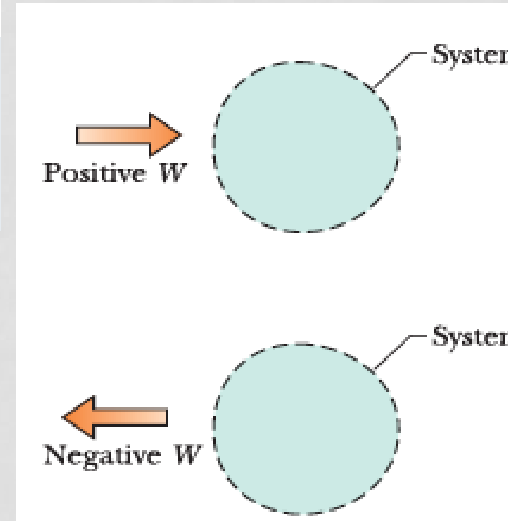
$$F = -\frac{20 \text{ J} - 7.0 \text{ J}}{1.0 \text{ m} - 4.0 \text{ m}} = 4.3 \text{ N}$$

To the right

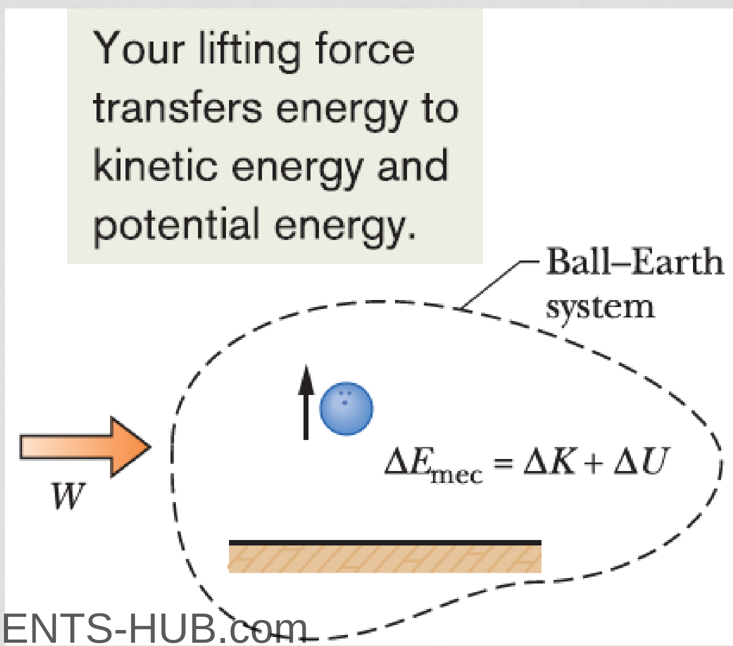
8-4 Work Done on a System by an External Force

- We can extend work on an object to work on a system:

Work is energy transferred to or from a system by means of an external force acting on that system.



- For a system of more than 1 particle, work can change both *K* and *U*, or other forms of energy of the system
- For a frictionless system:



$$W = \Delta K + \Delta U$$

$$W = \Delta E_{mec}$$

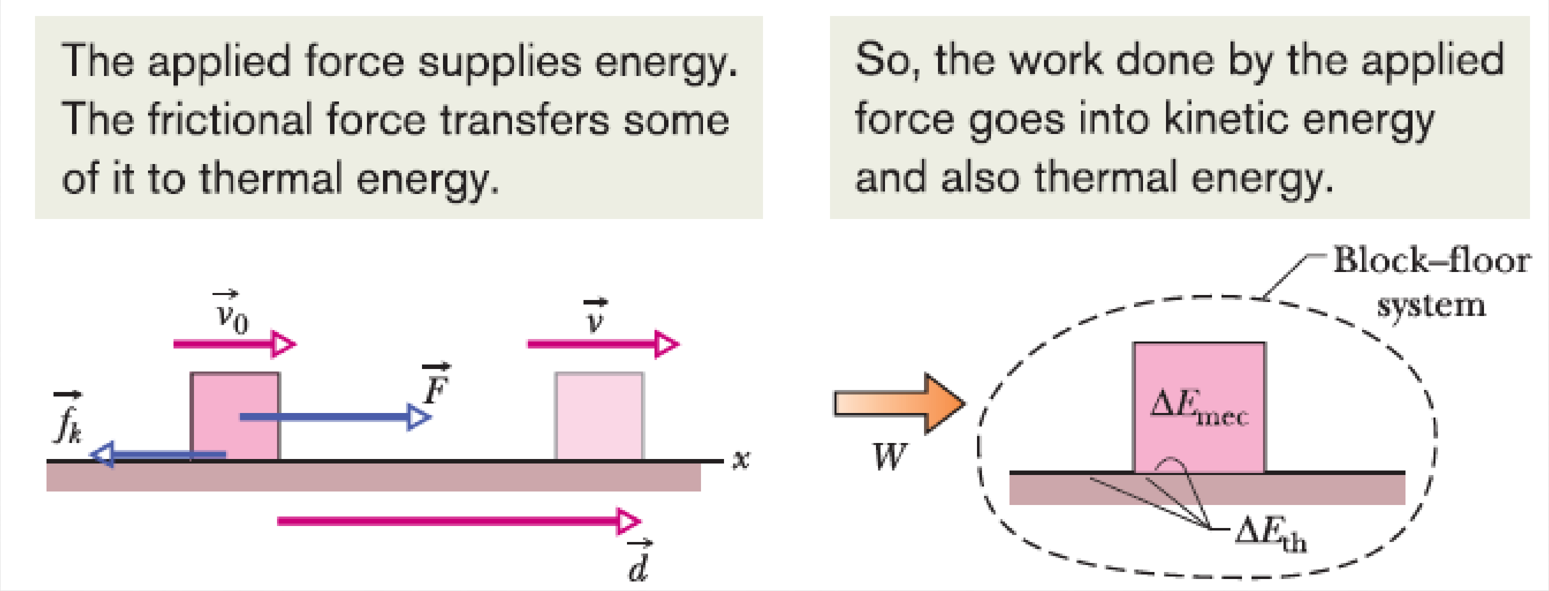
(work done on system, no friction involved)

- For a system with Friction:

$$\Delta E_{th} = f_k d \quad \text{(increase in thermal energy by sliding).}$$

$$W = \Delta E_{mec} + \Delta E_{th}$$

- The thermal energy comes from the forming and breaking of the welds between the sliding surfaces



Checkpoint 5

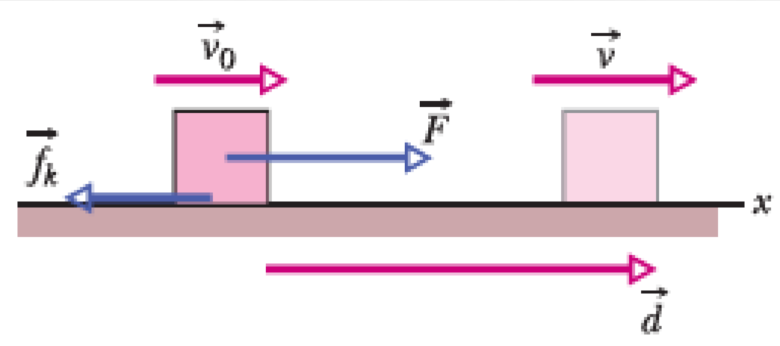
In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless, as in Fig. 8-13*a*. The magnitudes F of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance d. Rank the three trials according to the change in the thermal energy of the block and floor that occurs in that distance d, greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	increases

Answer: All trials result in equal thermal energy change. The value of f_k is the same in all cases.

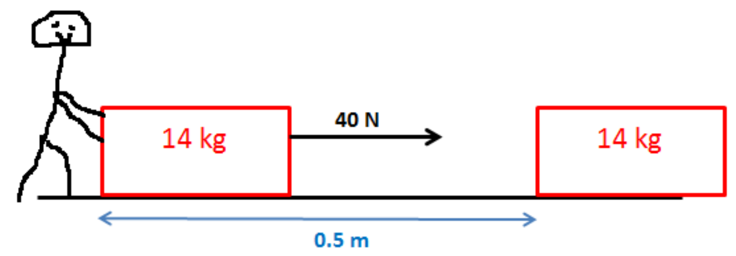
$$\Delta E_{th} = f_k d$$

$$f_k = \mu_k F_N = \mu_k mg$$



Sample Problem 8.05 Work, friction, change in thermal energy, cabbage heads

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14$ kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50$ m, the speed of the crate decreases from $v_0 = 0.60$ m/s to $v = 0.20$ m/s.



(a) How much work is done by force $\vec{F}$, and on what system does it do the work?

$$W = Fd\cos\phi = (40\text{ N})(0.50\text{ m})\cos 0^\circ$$

$$= 20\text{ J}.$$

To determine the system on which the work is done, let's check which energies change:
Crate: change in its kinetic energy ΔK.
There is a friction between the crate and the floor since the applied force in the same direction of the crate velocity but the crate is slowing ΔE_{th}.
Crate-floor system, because both energy changes occur in that system.

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

$$W = \Delta E_{mec} + \Delta E_{th}$$

$$\Delta E_{mec} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

$$\Delta E_{th} = W - (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2) = W - \tfrac{1}{2}m(v^2 - v_0^2)$$
$$= 20\text{ J} - \tfrac{1}{2}(14\text{ kg})[(0.20\text{ m/s})^2 - (0.60\text{ m/s})^2]$$
$$= 22.2\text{ J} \approx 22\text{ J}.$$

8-5 Conservation of Energy

- Energy transferred between systems can always be accounted for
- The **law of conservation of energy** concerns
  - The **total energy** *E* of a system
  - Which includes mechanical, thermal, and other internal energy

The total energy *E* of a system can change only by amounts of energy that are transferred to or from the system.

- Considering only energy transfer through work:

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

- An isolated system is one for which there can be no *external* energy transfer

The total energy E of an isolated system cannot change.

- Energy transfers may happen internal to the system
- We can write: $$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0 \quad \text{(isolated system)}$$
- Or, for two instants of time:

$$E_{mec,2} = E_{mec,1} - \Delta E_{th} - \Delta E_{int}$$

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

- We can expand the definition of power
- In general, power is the rate at which energy is transferred by a force from one type to another
- If energy ΔE is transferred in time Δt, the **average power** is:

$$P_{avg} = \frac{\Delta E}{\Delta t}$$

- And the **instantaneous power** is:

$$P = \frac{dE}{dt}$$

54 A child whose weight is 267 N slides down a 6.1 m playground slide that makes an angle of 20° with the horizontal. The coefficient of kinetic friction between slide and child is 0.10. (a) How much energy is transferred to thermal energy? (b) If she starts at the top with a speed of 0.457 m/s, what is her speed at the bottom?

a) $\Delta E_{th} = f_k d$

$$f_k = \mu_k F_N = \mu_k \, mg \cos\theta = 0.1(267\ N)\cos 20^{\circ} = 25.1\ \text{N}$$

$$\Delta E_{th} = f_k d = (25.1N)(6.1\ m) = 153\ J$$

b) $\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$ (isolated system)

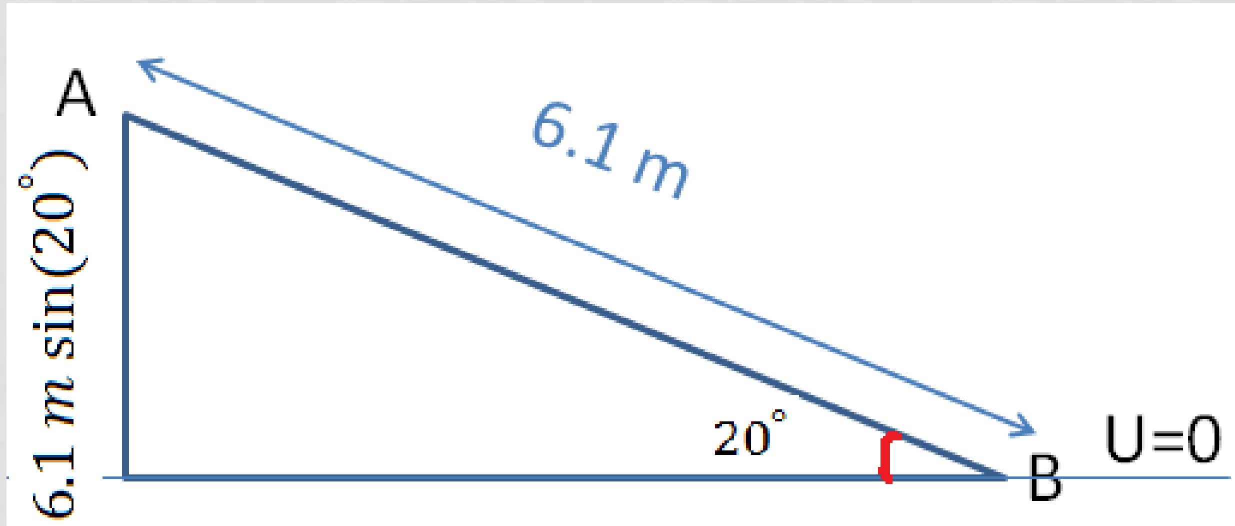
$$K_f = K_i - \Delta U - \Delta E_{th}$$

$$\Delta U = mg\,\Delta y = (267N)\,(-6.1\ m)\sin 20^{\circ} = -557\ \text{J}$$

$$K_i = \frac{1}{2} m v_i^{\ 2} = \frac{1}{2}\left(\frac{267\ N}{10\ m/s^2}\right)(0.457 m/s)^2 = 2.8\ J$$

$$K_f = 2.8J + 557J - 153\ J = 406.8J$$

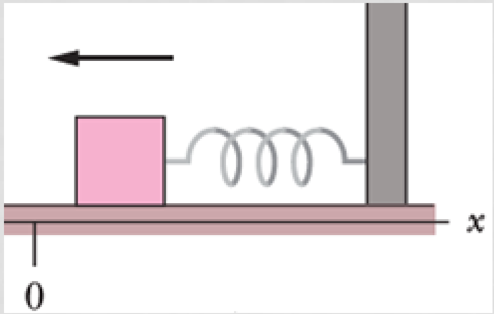
$$v_f = \sqrt{2K_f/m} = 5.5\ \text{m/s}$$



$$K_i + U_i = K_f + U_f + \Delta E_{th}$$
$$2.8\,J + 557\,J = K_f + 0 + 153\,J$$

$$K_f = 2.8J + 557J - 153\,J = 406.8J$$

••56 You push a 2.0 kg block against a horizontal spring, compressing the spring by 15 cm. Then you release the block, and the spring sends it sliding across a tabletop. It stops 75 cm from where you released it. The spring constant is 200 N/m. What is the block–table coefficient of kinetic friction?



$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad \text{(isolated system)}$$

$$\Delta E_{\text{th}} = f_k d$$

$$\Delta E_{th} = -\Delta E_{mec} = K_i - K_f + U_i - U_f = 0 - 0 + \frac{1}{2}kx^2 - 0$$

$$\mu_k\, mgd = \frac{1}{2}kx^2$$

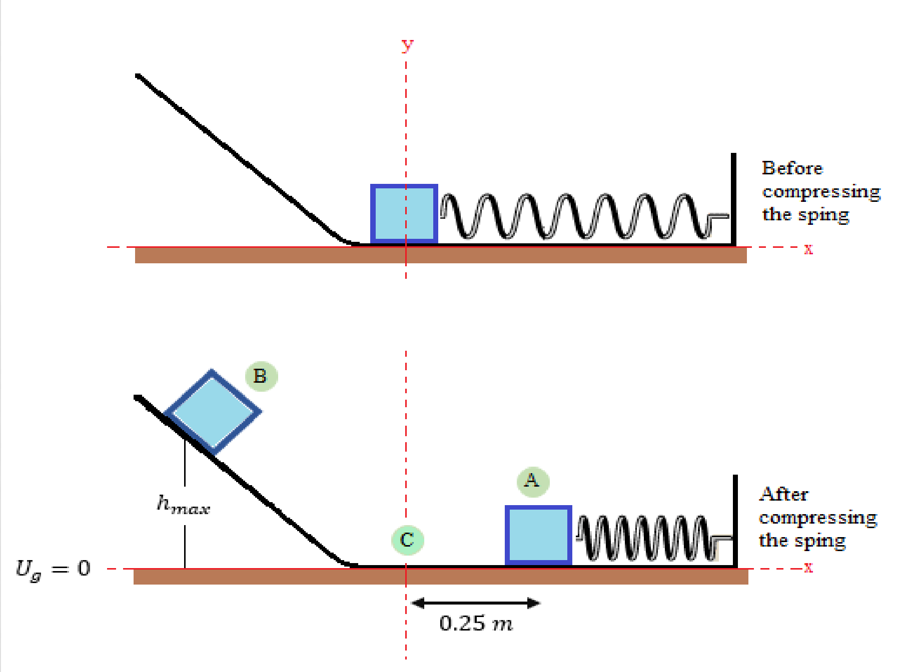
$$\mu_k\,(2\,kg)(10m/s^2)(0.75\,m) = \frac{1}{2}\left(200\,\frac{N}{m}\right)(0.15m)^2$$

$$\mu_k = 0.15$$

Example: As shown in the below figure, the right end of a spring is fixed to a wall. A 1.00 Kg block is then pushed against the free end so that the spring is compressed by 0.25m. After the block is released, it slides along a horizontal floor and (after leaving the spring) up an incline; both floor and incline are frictionless. Its maximum (vertical) height on the incline is 5.00m. What are (a) the spring constant and (b) the maximum speed? (c) If the angle of the incline is increased, What happens to the maximum (vertical) height?

a) By using conservation of mechanical energy when the block from point A to point B

Isolated system with No friction



$$E_{mec,A} = E_{mec,B}$$
$$K_A + U_{s,A} + U_{g,A} = K_B + U_{s,B} + U_{g,B}$$
$$0 + \frac{1}{2}k(x)^2 + 0 = 0 + 0 + mgh_{max}$$
$$k = \frac{2mgh_{max}}{(x)^2} = \frac{2(1kg)(10m/s^2)(5m)}{(0.25m)^2} = 1600\ N/m$$

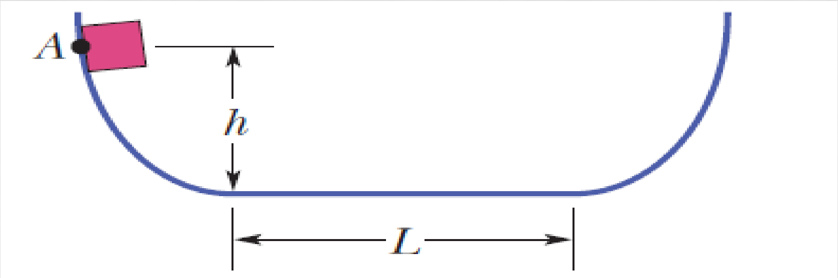
b) The maximum speed of the block is just when it leaves the spring (point C)

Using conservation of mechanical energy:

$$E_{mec,A} = E_{mec,C}$$
$$K_A + U_{s,A} + U_{g,A} = K_c + U_{s,c} + U_{g,c}$$
$$0 + \frac{1}{2}k(x)^2 + 0 = \frac{1}{2}mv_{max}^2 + 0 + 0$$
$$\frac{1}{2}k(x)^2 = \frac{1}{2}mv_{max}^2$$
$$v_{max} = \sqrt{\frac{k(x)^2}{m}} = \sqrt{\frac{(1600N/m)(0.25m)^2}{1kg}} = 10\ m/s$$

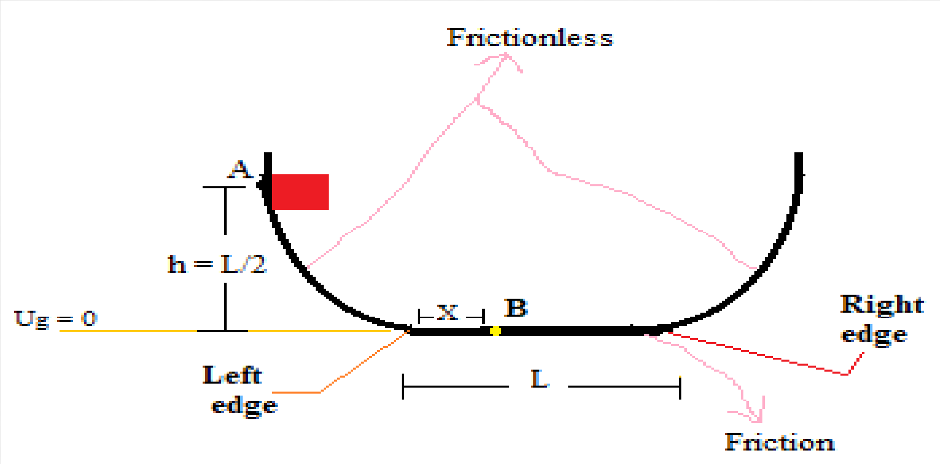
c) The maximum height depends only on the mechanical energy of the block, thus the block will reach the same vertical height regardless the angle of the incline. This is clear in the figure below.

8-65) A particle can slide along a track with elevated ends and a flat central part, as shown in the below figure. The flat part has length *L* =40 cm. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is μ_k =0.20.The particle is released from rest at point *A*, which is at height *h*= *L*/2. How far from the left edge of the flat part does the particle finally stop?



Isolated system with friction

Assuming that the block will stop at point B, which is at distance x from the left end.
The block may pass the flat part many times before it stops. Let's assume that the block will pass the total flat area N times



$$\Delta E_{mec} + \Delta E_{th} = 0$$

$$\Delta E_{mec} = -\Delta E_{th}$$

$$\Delta U_g + \Delta K = -N(\mu_k mgL)$$

$$U_{g,B} - U_{g,A} + K_B - K_A = -N(\mu_k mgL)$$

$$0 - U_{g,A} + 0 - 0 = -N(\mu_k mgL)$$

$$-U_{g,A} = -N(\mu_k mgL)$$

$$mg\frac{L}{2} = N(\mu_k mgL)$$

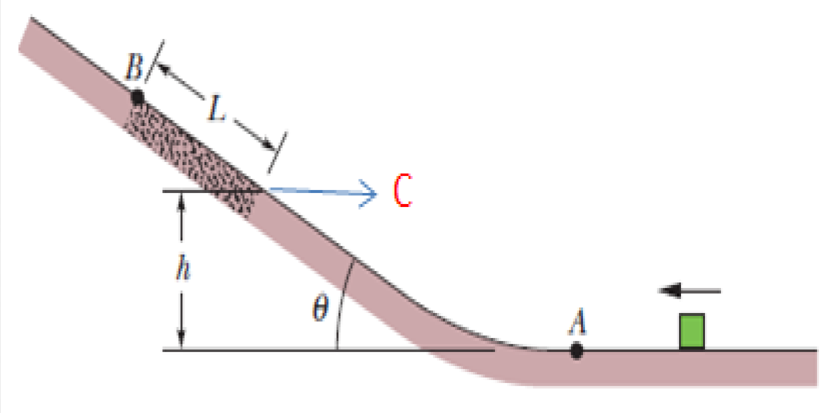
$$\frac{1}{2} = N\mu_k$$

$$N = \frac{0.5}{\mu_k} = \frac{0.5}{0.2} = 2.5$$

This means that the block will pass the flat area two times and in the third time the block stops in the middle of the flat area, thus

$$\boldsymbol{x = 20\ cm}$$

8-62) In the below figure, a block slides along a path that is without friction until the block reaches the section of length L = 0.65 m, which begins at height h = 2.0 m on a ramp of angle $\theta = 30°$. In that section, the coefficient of kinetic friction is 0.40. The block passes through point A with a speed of 8.0 m/s. If the block can reach point B (where the friction ends), what is its speed there, and if it cannot, what is its greatest height above A?



By using the conservation of a mechanical energy, (A and C)

$$\frac{1}{2} M v_A^2 = \frac{1}{2} M v_C^2 + Mgh$$

$$K_C = \frac{1}{2} M v_C^2 = M\left(\frac{1}{2} v_A^2 - gh\right) = M\big(0.5\,(8\,m/s)^2 - (9.8\,m/s^2)(2m)\big) = 12.4\,M$$

Recall, $F_N = Mg\,cos\theta, y = d\,sin\theta$ when the block on the ramp.

If $d < L \rightarrow$ The block does not reach point B and all its kinetic energy will turn entirely into thermal and potential energy.

$$K_C = Mgy + f_k d \quad \rightarrow 12.4\,M = Mgd\,sin\theta + \mu_k Mgd\,cos\theta$$

$$d = \frac{12.4}{g\,sin\theta + \mu_k g\,cos\theta} = 1.49\,m > L$$

So the block will reach point B: (d replaced by L)

$$K_C = mgy + f_k d + K_B \quad \rightarrow 12.4\,M = MgL\,sin\theta + \mu_k MgL\,cos\theta + K_B$$

$$K_B = 12.4\,M - MgL\,sin\theta - \mu_k MgL\,cos\theta = \frac{1}{2} M v_B^2$$

$$v_B = \sqrt{-2gL(sin\theta + \mu_k\,cos\theta) + 2(12.4)} = 3.74\,m/s$$