

Exercise#1 [3 marks]. Find all $x \in \mathbb{R}$ that satisfy the inequality

$$4 < |x + 2| + |x - 1| < 5.$$

Exercise#2 [3 marks]. Let a and b be any two real numbers such that $a < b$.

- (a) Let p be a fixed positive irrational number. Show that there is a rational number q such that $a < pq < b$.
- (b) Show that pq is irrational.

Exercise#3 [3 marks]. Let E be a nonempty subset of \mathbb{R} that is bounded above, and let $a \in \mathbb{R}$ be any real number. Define the set $a + E = \{a + e : e \in E\}$. Prove that $\sup\{a + E\} = a + \sup E$.