

- 30 a. $E(Y) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$
 b. It is an estimate of the probability that a customer that does not have a Simmons credit card will make a purchase.

Logistic Regression Table

Predictor	Coef	SE Coef	z	P	Odds Ratio	Lower	Upper
Constant	-0.9445	0.3150	-3.00	0.003			
Card	1.0245	0.4235	2.42	0.016	2.79	1.21	6.39

Log-Likelihood = 64.265

Test that all slopes are zero: G = 6.072, DF = 1, P-Value = 0.014

Thus, the estimated logit is $\hat{g}(x) = -0.9445 + 1.0245x$.

- d. For customers that do not have a Simmons credit card ($x = 0$)

$$\hat{g}(0) = -0.9445 + 1.245(0) = 0.9445$$

and

$$\hat{y} = \frac{e^{\hat{g}(0)}}{1 + e^{\hat{g}(0)}} = \frac{e^{0.9445}}{1 + e^{0.9445}} = \frac{0.3889}{1 + 0.3889} = 0.28$$

$$t_{0.025} = 2.776$$

($n - p - 2 = 8 - 2 - 2 = 4$ degrees of freedom)

Since none of the studentized deleted residuals is less than -2.776 or greater than 2.776, we conclude that there are no outliers in the data.

For customers that have a Simmons credit card ($x = 1$)

Logistic Regression Table

Predictor	Coef	SE Coef	z	P	Odds Ratio	Lower	Upper
Constant	-2.6335	0.7985	-3.30	0.001			
Balance	0.22018	0.09002	2.45	0.014	1.25	1.04	1.49

Log-Likelihood = 25.813

Test that all slopes are zero: G = 9.460, DF = 1, P-Value = 0.002

Thus, the estimated logistic regression equation is

$$E(y) = \frac{e^{2.6355 + 0.22018x}}{1 + e^{2.6355 + 0.22018x}}$$

- c. Significant result: the p -value corresponding to the G test statistic is 0.0002.

- d. For an average monthly balance of €1000, $x = 10$

$$E(y) = \frac{e^{2.6355 + 0.22018x}}{1 + e^{2.6355 + 0.22018x}} = \frac{e^{2.6355 + 0.22018(10)}}{1 + e^{2.6355 + 0.22018(10)}} = \frac{e^{4.8373}}{1 + e^{4.8373}} = \frac{0.6494}{1 + 0.6494} = 0.39$$

Thus, an estimate of the probability that customers with an average monthly balance of €1000 will sign up for direct payroll deposit is 0.39.

- c. A portion of the Minitab binary logistic regression output follows:

$$\hat{g}(1) = -0.9445 + 1.245(1) = 0.0800$$

and

$$\hat{y} = \frac{e^{\hat{g}(1)}}{1 + e^{\hat{g}(1)}} = \frac{e^{0.08}}{1 + e^{0.08}} = \frac{1.0833}{1 + 1.0833} = 0.52$$

- e. Using the Minitab output shown in part (c), the estimated odds ratio is 2.79. We can conclude that the estimated odds of making a purchase for customers who have a Simmons credit card are 2.79 times greater than the estimated odds of making a purchase for customers that do not have a Simmons credit card.

32 a. $E(Y) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$

- b. A portion of the Minitab binary logistic regression output follows:

- e. Repeating the calculations in part (d) using various values for x , a value of $x = 12$ or an average monthly balance of approximately €1200 is required to achieve this level of probability.

- f. Using the Minitab output shown in part (b), the estimated odds ratio is 1.25. Because values of x are measured in hundreds of euros, the estimated odds of signing up for payroll direct deposit for customers that have an average monthly balance of €600 is 1.25 times greater than the estimated odds of signing up for payroll direct deposit for customers that have an average monthly balance of €500. Moreover, this interpretation is true for any €100 increment in the average monthly balance.

Chapter 16

Solutions

- 2 a. The MINITAB output is shown below:

The regression equation is

$$Y = 9.32 + 0.424 X$$

Predictor	Coef	SE Coef	T	P
Constant	9.315	4.196	2.22	0.113
X	0.4242	0.1944	2.18	0.117

S = 3.531 R-sq = 61.4% R-sq(adj) = 48.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	1	59.39	59.39	4.76	0.117
Residual Error	3	37.41	12.47		
Total	4	96.80			

The high p -value (0.117) indicates a weak relationship; note that 61.4 per cent of the variability in y has been explained by x .

- b. The MINITAB output is shown below:

The regression equation is

$$Y = -8.10 + 2.41 X - 0.0480 XSQ$$

Predictor	Coef	SE Coef	T	P
Constant	-8.101	4.104	-1.97	0.187
X	2.4127	0.4409	5.47	0.032
XSQ	-0.04797	0.01050	-4.57	0.045

S = 1.279 R-sq = 96.6% R-sq(adj) = 93.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	93.529	46.765	28.60	0.034
Residual Error	2	3.271	1.635		
Total	4	96.800			

At the 0.05 level of significance, the relationship is significant; the fit is excellent.

c. $\hat{y} = -8.101 + 2.4127(20) - 0.04797(20)^2 = 20.965$

- 4 a. The MINITAB output is shown below:

The regression equation is

$$Y = 943 + 8.71 X$$

Predictor	Coef	SE Coef	T	P
Constant	943.05	59.38	15.88	0.000
X	8.714	1.544	5.64	0.005

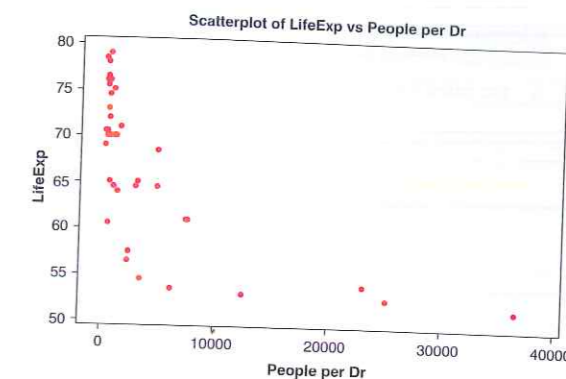
S = 32.29 R-sq = 88.8% R-sq(adj) = 86.1%

Analysis of Variance

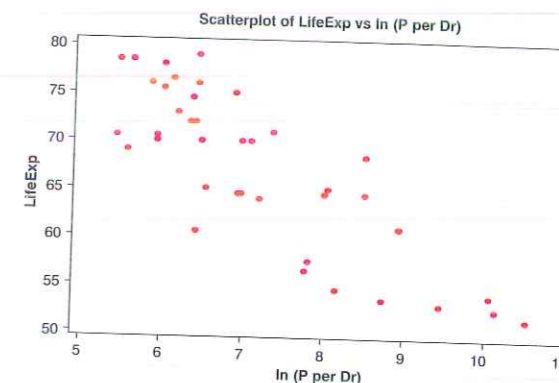
SOURCE	DF	SS	MS	F	P
Regression	1	33223	33223	31.86	0.005
Residual Error	4	4172	1043		
Total	5	37395			

- b. The p -value of $0.005 < \alpha = 0.01$; reject H_0

- 6 a. The scatter diagram for LifeExp against People per Dr suggests the existence of a possible nonlinear relationship between the two variables:



- b. However when the People per Dr variable is replaced by its logarithm in the scatter diagram, a linear model now seems plausible:



- c. The situation is exactly analogous for the scatter diagram of LifeExp with People per TV variables.

Correspondingly we have the two simple regression models:

$$\hat{LifeExp} = 77.887 - 4.26 \ln(P \text{ per TV}) \quad R^2 = 0.731$$

$$\hat{LifeExp} = 102.873 - 4.974 \ln(P \text{ per Dr}) \quad R^2 = 0.693$$

Neither of these relationships is causal but the first with the $\ln(P \text{ per TV})$ predictor has a slightly better R^2 value which might favour it in this instance.

- 8 a. The scatter diagram is shown below:

