

### Exercise 10.1:

Q2: Consider the following hypothesis test  $H_0: \mu_1 - \mu_2 \leq 0$   
 $H_1: \mu_1 - \mu_2 > 0$

The following results are for two independent samples taken from the two populations

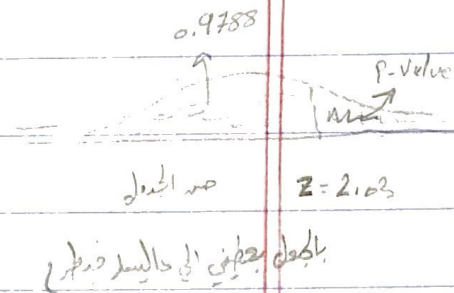
sample 1	sample 2
$n_1 = 40$	$n_2 = 50$
$\bar{x}_1 = 25.2$	$\bar{x}_2 = 22.8$
$s_1 = 5.2$	$s_2 = 6.0$

(1) What is the value of the test statistic?

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(25.2 - 22.8) - 0}{\sqrt{\frac{(5.2)^2}{40} + \frac{36}{50}}} = 2.03$$

(2) What is the p-value? By Ztable.

$$\begin{aligned} \text{p-value} &= 1 - 0.9788 \\ &= 0.0212 \end{aligned}$$



(3) With  $\alpha = 0.05$ , what is your hypothesis testing conclusion?

Reject  $H_0$  if p-value  $\leq \alpha$

$$0.0212 \leq 0.05 \quad \checkmark$$

So we Reject  $H_0$ .

Q3: Consider the following hypothesis test  $H_0: \mu_1 - \mu_2 = 0$  Two tailed test  
 $H_1: \mu_1 - \mu_2 \neq 0$

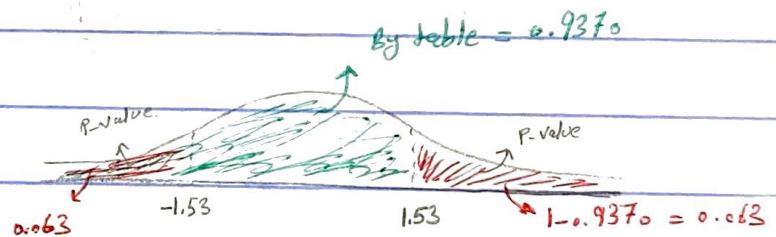
The following result are for two independent samples taken from the two value pop.

sample 1	sample 2
$n_1 = 80$	$n_2 = 70$
$\bar{x}_1 = 104$	$\bar{x}_2 = 106$
$s_1 = 8.4$	$s_2 = 7.6$

(a) What is the value of the test statistic?

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{104 - 106}{\sqrt{\frac{70.56}{80} + \frac{57.76}{70}}} = -1.53$$

(b) What is the p-value?  
By z-table.



$$\begin{aligned} \text{p-value} &= 2(1 - 0.9370) \\ &= (0.063 + 0.063) \\ &= \underline{\underline{0.126}} \end{aligned}$$

(c) With  $\alpha = 0.05$ , what is your hypothesis testing conclusion?

$$\text{p-value} \quad \square \quad \alpha$$

$0.126 > 0.05$  so we don't reject  $H_0$  ( $\alpha = 0.05$ ).

Q4:

Men

Women

$$n_1 = 100$$

$$n_2 = 85$$

$$\bar{x}_1 = 14.9 \text{ years}$$

$$\bar{x}_2 = 10.3 \text{ years}$$

$$s_1 = 5.2 \text{ years}$$

$$s_2 = 3.8 \text{ years}$$

(a) What is the point estimate of the difference between the two pop. mean?

point estimate for  $\mu_1 - \mu_2 = \bar{x}_1 - \bar{x}_2 = 14.9 - 10.3 = 4.6 \text{ years}$ .

(b) At 95% confidence, what is the margin of error? E

$$E = Z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \quad 1-\alpha = 0.95 \rightarrow \alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

$$= Z_{0.025} \sqrt{\frac{27.04}{100} + \frac{14.44}{85}} =$$

$$= 1.96 \sqrt{11} = 1.3$$

(c) What is the 95% CI estimate of the difference between the two pop. mean?

$$(1-\alpha) \text{ CI} = (\bar{x}_1 - \bar{x}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad \left\{ \begin{array}{l} \text{الحدود} \\ \text{ثقة} \end{array} \right.$$

$$= 4.6 \pm 1.3$$

$$= (3.3, 5.9) \quad \leftarrow \text{الحدود الموثوقة}$$



Q6: male:  $n_1 = 40$  female:  $n_2 = 30$   
 $s_1 = 35$   $s_2 = 20$   
 $\bar{x}_1 = 135.67$   $\bar{x}_2 = 68.64$

a. point estimator for  $\mu_1 - \mu_2 = \bar{x}_1 - \bar{x}_2 = 67.03$

b. 99% CI, what is the margin of error:

$$E = Z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= Z_{0.005} \sqrt{\frac{35^2}{40} + \frac{20^2}{30}}$$

$$= 2.576 \times \sqrt{43.95833333}$$

$$= 17.08$$

$1 - \alpha = 0.99$   
 $1 - 0.99 = \alpha$   
 $\alpha = 0.01 \rightarrow \frac{\alpha}{2} = 0.005$   
 $Z_{0.005} = 2.576$   
 By t-table  
 df =  $\infty$  with 0.005

c. 99% CI for  $\mu_1 - \mu_2$ :

$$99\% \text{ CI} = (\bar{x}_1 - \bar{x}_2) \pm E$$

$$= 67.03 \pm 17.08$$

$$= (49.95, 84.11)$$