Question (10 points)

Consider the following two random independent samples taken from two normal populations.

Sample 1	10	7	13	7	9	8
Sample 2	8	7	8	4	6	9

Provide a 95% confidence interval estimate for the difference between the two population means.

Sample 1 : $n_1 = 6$, $\bar{X}_1 = 9$, $S_1 = 2.28$.

Sample 2 : $n_2 = 6$, $\bar{X}_2 = 7$, $S_2 = 1.79$.

95% CI for $\mu_1 - \mu_2 = (\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

$= (9-7) \pm (2.262) \sqrt{\frac{2.28^2}{6} + \frac{1.79^2}{6}}$

$= 2 \pm 2.68 = [-0.68, 4.68]$.

$df = \left[\frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right)^2}{\left(\frac{1}{n_1-1} \right) \left(\frac{S_1^2}{n_1} \right)^2 + \left(\frac{1}{n_2-1} \right) \left(\frac{S_2^2}{n_2} \right)^2} \right] = \lfloor 9.4667 \rfloor = 9$.

$1 - \alpha = 95\% \Rightarrow \frac{\alpha}{2} = 0.025$, $df = 9 \Rightarrow t_{\frac{\alpha}{2}} = 2.262$.

Question (10 points)

Consider the following two random independent samples taken from two normal populations.

Sample 1	5	6	5	3	3	5
Sample 2	6	5	4	2	7	6

Test if the two population variances are equal using a 5% level of significance.
1) Write the hypotheses.
2) Calculate the test statistic.
3) What is your conclusion? Use the critical value approach.
4) What is your conclusion? Use the p-value approach.

$\alpha = 0.05$.

1- $H_0 : \sigma_1^2 = \sigma_2^2$.

$H_1 : \sigma_1^2 \neq \sigma_2^2$.

2- $F = \dfrac{S_1^2}{S_2^2} = \dfrac{3.20}{1.50} = 2.13$, $df_1 = 6-1 = 5$, $df_2 = 5$.

$S_2^2 = 1.50$, $S_1^2 = 3.20$.

3- $F_{\frac{\alpha}{2}} = 7.15$.

$F < F_{\frac{\alpha}{2}} \Rightarrow$ Don't Reject H_0 $(\alpha = 0.05)$.

$\Rightarrow \sigma_1^2 = \sigma_2^2$. $(\alpha = 0.05)$.

4- p-value $> 0.10(2) = 0.20$.

$\Rightarrow$ p-value $> \alpha = 0.05$

$\Rightarrow$ Don't Reject H_0 $(\alpha = 0.05)$.

$\Rightarrow \sigma_1^2 = \sigma_2^2$ $(\alpha = 0.05)$.

Question (10 points)

The following table shows the data obtained for a study regarding the gender of individuals and their selected field of study. We want to determine if the selected field of study and gender are independent using a 10% level of significance.

$\alpha = 0.1$.

Field of study	Male		Female		Total
Medicine	30	(30)	20	(20)	50
Business	30	(36)	30	(24)	60
Engineering	60	(54)	30	(36)	90
Total	120		80		200

1) Write the hypotheses.
2) Calculate the test statistic.
3) What is your conclusion? Use the critical value approach.
4) What is your conclusion? Use the p-value approach.

1) H_0: The selected field of study and geneder are independent.
H_1: The selected field of study and geneder are **not** independent.

2) $x^2 = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{(f_{ij} - e_{ij})^2}{e_{ij}} = 4.17$, $df = (n-1)(m-1) = (2)(1) = 2$

$n=3, m=2$.

all $e_{ij} \geq 5$. ✓

3) $x_{\alpha}^2 = 4.605$.

$x^2 < x_{\alpha}^2 \implies$ Don't Reject H_0 $(\alpha = 0.10)$
$\implies$ The selected field of study and geneder are independent. $(\alpha = 0.10)$.

4) p-value $\in (0.10, 0.90)$.

P-value $> \alpha \implies$ Dont Reject H_0 $(\alpha = 0.10)$.
$\implies$ The selected field of study and geneder are independent. $(\alpha = 0.10)$.