Indeterminate forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$

Apply L'Hopital Rule

اشتق البسط والمقام

$\infty \cdot 0$, $\infty - \infty$

إعادة ترتيب حتى نصل
الى $\frac{0}{0}$, $\frac{\infty}{\infty}$
مثل توحيد مقامات

0^0 , ∞^0 , 1^∞

indeterminate Power

موجب

$$\lim f(x) = e^{\lim \ln f(x)}$$

cont.

Exp

① $\lim_{x \to -3} \frac{x+3}{x^2-9} = \lim_{x \to -3} \frac{1}{2x} = \frac{1}{-6}$ $\qquad \frac{0}{0}$

or $\lim_{x \to -3} \frac{\cancel{x+3}}{(x-3)\cancel{(x+3)}} = \lim_{x \to -3} \frac{1}{x-3} = \frac{1}{\boxed{-3}-3} = \frac{1}{-6}$

② $\lim_{x \to -1} \frac{x-2}{x^2-4} = \frac{\boxed{-1}-2}{(-1)^2-4} = \frac{-3}{1-4} = \frac{-3}{-3} = 1$

③ $\lim_{x \to 0} \frac{x - \sin x}{x^3} = \lim_{x \to 0} \frac{1-\cos x}{3x^2}$ $\qquad \frac{0}{0}$ $\qquad \frac{1-1}{0} = \frac{0}{0}$

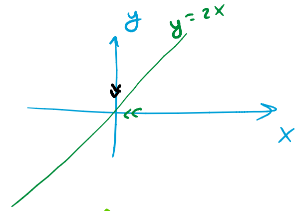
$= \lim_{x \to 0} \frac{\sin x}{6x} = \lim_{x \to 0} \frac{\cos x}{6} = \frac{1}{6}$ $\qquad \frac{0}{0}$

④ $$\lim_{x\to 0} \frac{1-\cos x}{x^2+x} = \lim_{x\to 0} \frac{\sin x}{2x+1} = \frac{0}{0+1} = \frac{0}{1} = 0$$

$\frac{0}{0}$

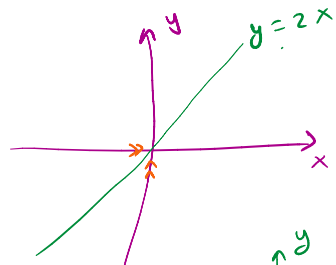
⑤ $$\lim_{x\to 0^+} \frac{\sin x}{x^2} = \lim_{x\to 0^+} \frac{\cos x}{2x} = \frac{1}{\text{small} +} = +\infty$$

$\frac{0}{0}$



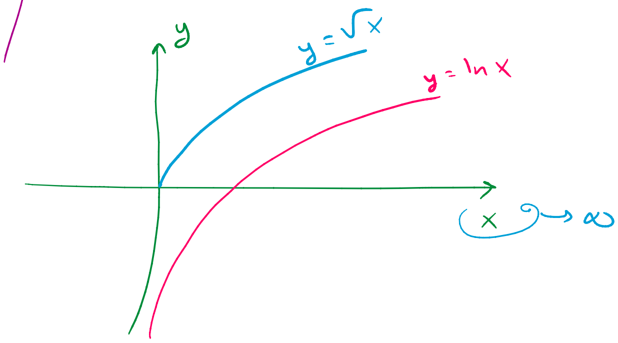
⑥ $$\lim_{x\to 0^-} \frac{\sin x}{x^2} = \lim_{x\to 0^-} \frac{\cos x}{2x} = \frac{1}{\text{small} -} = -\infty$$

$\frac{0}{0}$



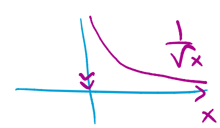
⑦ $$\lim_{x\to \infty} \frac{\ln x}{\sqrt{x}}$$

$\frac{\infty}{\infty}$

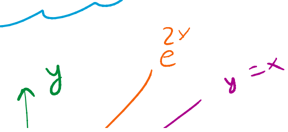


$$\lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{1}{x} \cdot \frac{2\cancel{\sqrt{x}}}{1}$$

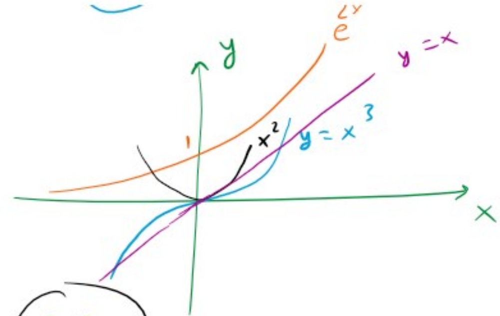
$x \to \sqrt{x}\cancel{\sqrt{x}}$



$$= \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 2 \lim_{x\to\infty} \frac{1}{\sqrt{x}} = 2\cdot(0) = 0$$



8) $\lim\limits_{x\to\infty} \dfrac{e^{2x}}{x^3}$ $\qquad \dfrac{\infty}{\infty}$

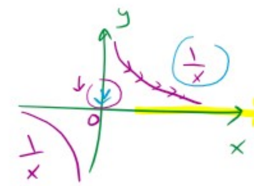


$$\lim_{x\to\infty} \frac{e^{2x}(2)\overset{1}{\ln e}}{3x^2} = \lim_{x\to\infty} \frac{2\,e^{2x}}{3\,x^2} \qquad \frac{\infty}{\infty}$$

$$= \frac{2}{3}\lim_{x\to\infty} \frac{e^{2x}\cancel{(2)}\overset{1}{\ln e}}{\cancel{2}x} = \frac{2}{3}\lim_{x\to\infty} \frac{e^{2x}}{x}$$

$$= \frac{2}{3}\lim_{x\to\infty} \frac{e^{2x}(2)\overset{1}{\ln e}}{1} = \frac{4}{3}\lim_{x\to\infty} e^{2x} = \frac{4}{3}\cdot\infty = \infty$$

9) $\lim\limits_{x\to\infty} \left(x \sin\dfrac{1}{x} \right)$

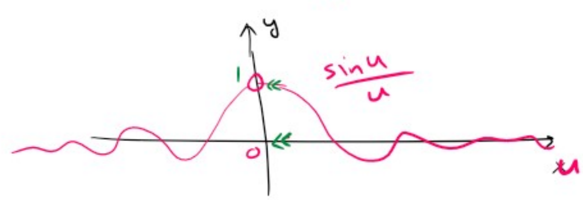


$\infty \cdot \sin 0$

$\boxed{\infty \cdot 0}$

$u = \dfrac{1}{x}$

$$= \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}}$$



$$= \lim_{u\to \boxed{0^+}} \frac{\sin u}{u} \qquad \frac{0}{0}$$

$$= \lim_{u\to 0^+} \frac{\cos u}{1} = \cos 0 = 1$$

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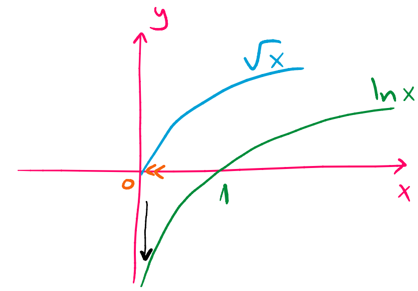
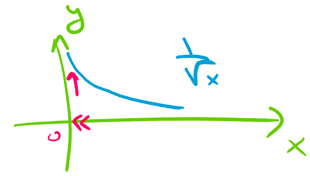
10 $u \to 0^+$

$$\lim_{x \to 0^+} \sqrt{x}\,\ln x$$

$$0 \cdot (-\infty) = -(0 \cdot \infty)$$



$$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}} \qquad \frac{-\infty}{\infty}$$



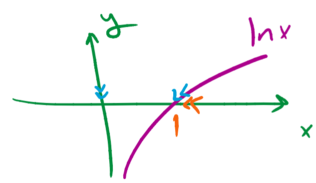
$$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

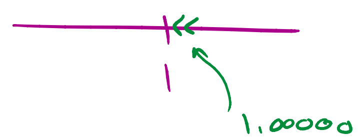
$$\lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}\, x^{-\frac{3}{2}}} = -2 \lim_{x \to 0^+} x^{\frac{3}{2}}\, x^{-1} = -2 \lim_{x \to 0^+} x^{\frac{1}{2}}$$

$$= -2 \lim_{x \to 0^+} \sqrt{x} = 2(0) = 0$$

11

$$\lim_{x \to 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right)$$





$$\frac{1}{\text{small}+} - \frac{1}{\text{small}+}$$

$$\boxed{\infty - \infty}$$

$$\lim_{x \to 1^+} \frac{\ln x - (x-1)}{(x-1)(\ln x)} \qquad \frac{0 - (1-1)}{(1-1)\;0} = \frac{0}{0}$$

$$\lim_{x\to 1^+} \frac{\frac{1}{x} - 1}{\frac{x-1}{x} + \ln x} = \lim_{x\to 1^+} \frac{\frac{1}{x} - 1}{1 - \frac{1}{x} + \ln x} \qquad \frac{1-1}{1-1+0} \quad \frac{0}{0}$$

$$\lim_{x\to 1^+} \frac{-\frac{1}{x^2}}{\frac{1}{x^2} + \frac{1}{x}} = \frac{-1}{1+1} = \frac{-1}{2}$$

Th (L'Hopital Rule) Assume $f(a) = g(a) = 0$ or $f(a) = g(a) = \infty$

Assume f, g diff such that $g'(x) \neq 0$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

$$\lim_{x\to a} \frac{f'(x)}{g'(x)}$$

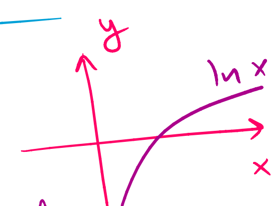
Exp 56 (1)

$$\lim_{x\to\infty} \frac{\frac{1}{\ln x}}{x}$$

$f(x)$

$$\frac{\frac{1}{\infty}}{\infty} = \frac{0}{\infty}$$

indeterminant power



$f(x)$

$$f(x) = e^{\ln f(x)}$$

$$\lim_{x\to\infty} e^{\ln x^{\frac{1}{\ln x}}} = \lim_{x\to\infty} e^{\cancel{\frac{1}{\ln x}}\,\cancel{\ln x}} = \lim_{x\to\infty} e = e$$

$$e^{\ln 10} = 10 \quad , \quad e^{\ln\sqrt{5-x}} = \sqrt{5-x}$$

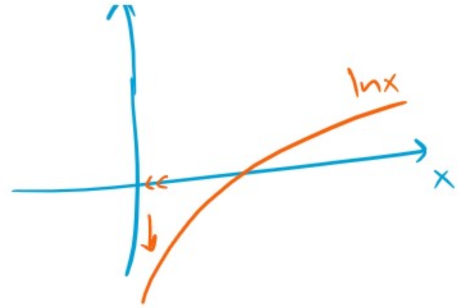
$$e^{\ln \text{جمل.}} = \text{جمل.} \quad , \quad e^{\ln x^{\frac{1}{\ln x}}} = x^{\frac{1}{\ln x}}$$

∞ / $-\infty$ $\left.\right\}$ $\dfrac{\text{رقم غير الصفر}}{\text{صفر}} = \dfrac{\text{البسط}}{\text{المقام}}$

صفر $= \dfrac{\text{صفر}}{\text{رقم غير الصفر}} =$

Exp $\lim \dfrac{x}{\quad}$ $\qquad$ $\dfrac{0}{\quad}$ $\qquad$ y $\qquad$ $\ln x$

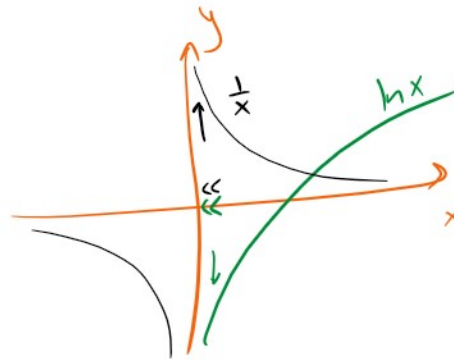
Exp $\displaystyle \lim_{x\to 0^+} \frac{x}{\ln x}$ $\qquad \frac{0}{\infty}$



$$\left(\lim_{x\to 0^+} x\right)\left(\lim_{x\to 0^+} \frac{1}{\ln x}\right) \to -\frac{1}{\infty} \Rightarrow 0$$

$$0 \cdot 0 = 0$$

Exp $\displaystyle \lim_{x\to 0^+} \frac{\ln x}{x}$ $\qquad \frac{-\infty}{0}$

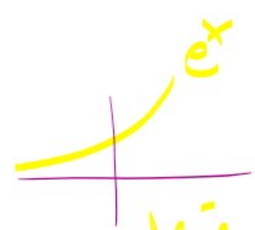


$$\left(\lim_{x\to 0^+} \frac{1}{x}\right)\left(\lim_{x\to 0^+} \ln x\right)$$

$$= \boxed{\infty} \cdot \boxed{-\infty} = -\infty$$

③ $\displaystyle \lim_{x\to 0^+} (1+x)^{\frac{1}{x}}$ $\qquad (1+0)^{\frac{1}{\text{small}\,+}} = 1^{\infty}$

جمل.



متصل

$$\lim \frac{\ln(1+x)}{\ldots} = \lim e^{\frac{1}{x}\ln(1+x)}$$

$$\lim_{x\to 0^+} e^{\ln(1+x)} = \lim_{x\to 0^+} e$$

$$\frac{\ln 1}{0} = \frac{0}{0}$$

$$\lim_{x\to 0^+} \frac{\ln(1+x)}{x}$$

$$= e^{\lim_{x\to 0^+} \frac{\ln(1+x)}{x}} = e^{\lim_{x\to 0^+} \frac{\frac{1}{1+x}}{1}} \to 1$$

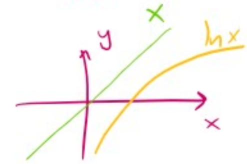
$$= e^{1} = e$$

4) $$\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} e^{\ln x^{\frac{1}{x}}}$$

$$\infty^{\frac{1}{\infty}} = \infty^{0}$$

$$= \lim_{x\to\infty} e^{\frac{1}{x}\ln x} = e^{\lim_{x\to\infty} \frac{\ln x}{x}}$$

$$\frac{\infty}{\infty}$$



$$= e^{\lim_{x\to\infty} \frac{\frac{1}{x}}{1}} = e^{\lim_{x\to\infty} \frac{1}{x}} = e^{0} = 1$$

68) $$\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$$

$$\frac{0}{0}$$

$f(x) = \ln x$

$= e^{x}$

$\neq \sqrt{x}$

$\lim e^{x} = e^{\lim_{x\to\infty} x}$ ∞

$$\lim_{x\to 0^+} \frac{\frac{1}{\cancel{2}\sqrt{x}}}{\frac{\cos x}{\cancel{2}\sqrt{\sin x}}}$$

$$\lim_{x\to 0^+} \frac{1}{\cos x}\sqrt{\frac{\sin x}{x}}$$

$$\lim_{x\to 0^+} \frac{1}{\cos x} \quad \lim_{x\to 0^+}\sqrt{\frac{\sin x}{x}}$$

$$\frac{1}{1}\sqrt{\lim_{x\to 0^+}\frac{\sin x}{x}} \quad \frac{0}{0} \quad = \sqrt{1} = 1$$

$$\lim_{x\to\infty} e^{x} = e^{\lim_{x\to\infty} x} = e^{\infty} = \infty$$

$$\lim_{x\to\infty} \sqrt{x} = \sqrt{\lim_{x\to\infty} x} = \sqrt{\infty} = \infty$$

$$\lim_{x\to\infty} \ln x = \ln \lim_{x\to\infty} x = \ln \infty = \infty$$

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}} = \lim_{x\to 0^+}\sqrt{\frac{x}{\sin x}}$$

$$= \sqrt{\lim_{x\to 0^+}\frac{x}{\sin x}} \quad \frac{0}{0} \quad = \sqrt{\lim_{x\to 0^+}\frac{1}{\cos x}} = \sqrt{1} = 1$$