

key

Birzeit University
Department of Mathematics
Quiz 1

Math 234

February 20, 2019

Second Semester 2018/2019

Section: 4

Name: _____

Number: _____

Q1 [4 points]. Solve the following system by Gauss-Jordan elimination.

$$5x_1 - 2x_2 + 6x_3 = 0$$

$$-2x_1 + x_2 + 3x_3 = 1$$

Q2 [6 points]. For which values of α will the following system have no solution? Exactly one solution? Infinitely many solutions?

$$x_1 + 2x_2 - 3x_3 = 4$$

$$3x_1 - x_2 + 5x_3 = 2$$

$$4x_1 + x_2 + (\alpha^2 - 14)x_3 = \alpha + 2$$

Ans. Q1) the augmented matrix is

$$\left[\begin{array}{ccc|c} 5 & -2 & 6 & 0 \\ -2 & 1 & 3 & 1 \end{array} \right] \xrightarrow{2R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 12 & 2 \\ -2 & 1 & 3 & 1 \end{array} \right]$$

$$\xrightarrow{2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 12 & 2 \\ 0 & 1 & 27 & 5 \end{array} \right] \Rightarrow \begin{cases} x_1 = 2 - 12x_3 \\ x_2 = 5 - 27x_3 \end{cases}$$

$$\text{let } x_3 = t \Rightarrow x_1 = 2 - 12t, \quad x_2 = 5 - 27t$$

$$\text{the solution set} = \left\{ (2 - 12t, 5 - 27t, t) : t \in \mathbb{R} \right\}$$

Q2) the augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & \alpha^2 - 14 & \alpha + 2 \end{array} \right] \xrightarrow{\substack{-3R_1+R_2 \\ -4R_1+R_3}} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & \alpha^2 - 2 & \alpha - 14 \end{array} \right]$$

Good Luck

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & \textcircled{1} & -2 & \frac{10}{7} \\ 0 & -7 & \alpha^2 - 2 & \alpha - 14 \end{array} \right]$$

$$\begin{array}{l} \rightarrow \\ 7R_2 + R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & \alpha^2 - 16 & \alpha - 4 \end{array} \right]$$

The system has infinitely many solutions if $\alpha = 4$

" " " no solution if $\alpha = -4$

" " " unique solution if $\alpha \neq \pm 4$