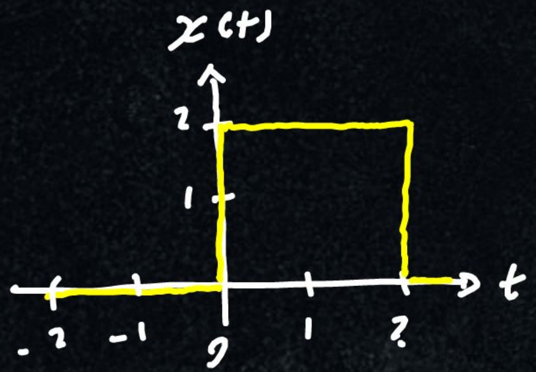
1.5) Time and Amplitude Transformations

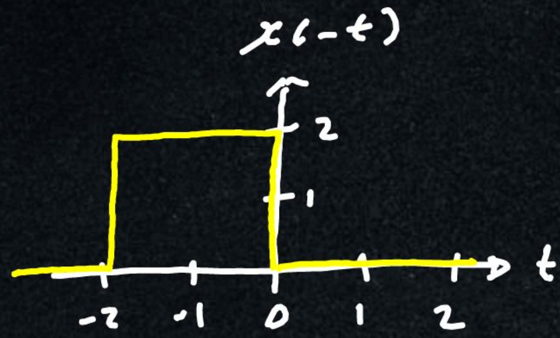
1.5.1) Time Transformation

① Time Reversal (Folding or Reflection)

$$x(t) \xrightarrow{T.R} x(-t)$$



$\xrightarrow{T.R}$

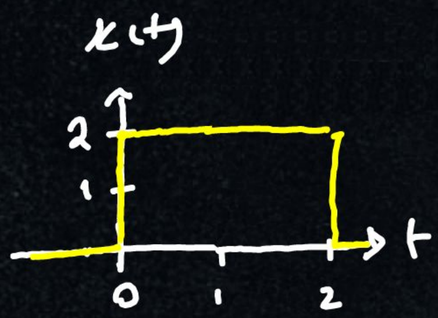


$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 < t < 2 \\ 0 & t > 2 \end{cases} \xrightarrow{T.R} x(-t) = \begin{cases} 0 & t > 0 \\ 2 & -2 < t < 0 \\ 0 & t < -2 \end{cases}$$

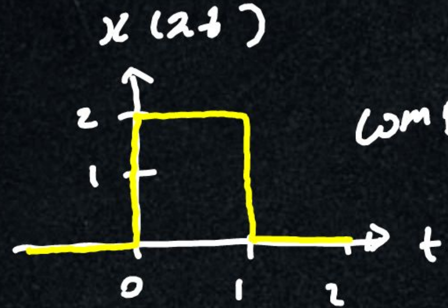
② Time Scaling ⇝ compression or expation of $x(t)$ in time

$$x(t) \xrightarrow{T.Sc} x(\alpha t) \qquad \alpha \neq 0$$

Case I : $|\alpha| > 1 \Rightarrow \alpha \in (-\infty, 1) \cup (1, \infty)$



$$\xrightarrow[\alpha = 2]{T.Sc}$$

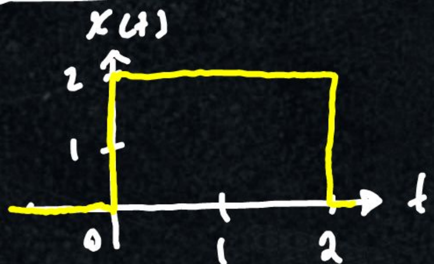


compression of $x(t)$ in time

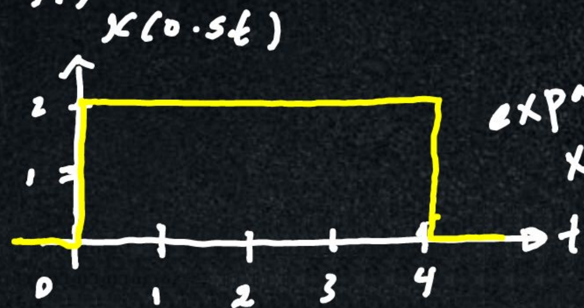
$x(t) = 2, \; 0 < t < 2$

$x(2t) = 2, \; 0 < 2t < 2 = 2, \; 0 < t < 1$

Case II : $|\alpha| < 1 \Rightarrow \alpha \in (-1, 0) \cup (0, 1)$



$$\xrightarrow[\alpha = 0.5]{T.Sc}$$



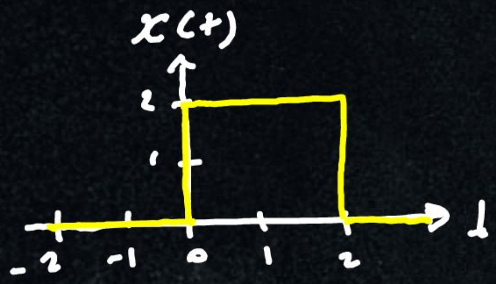
expansion of $x(t)$ in time

③ Time shifting

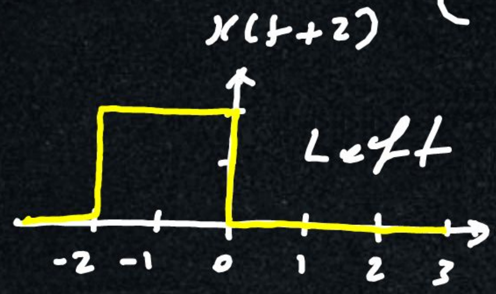
$$x(t) \xrightarrow{T.S} x(t+k)$$

<u>Case I</u> : $k > 0$

$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 < t < 2 \\ 0 & t > 2 \end{cases} \xrightarrow[k=2]{T.S} x(t+2) = \begin{cases} 0 & t < -2 \\ 2 & -2 < t < 0 \\ 0 & t > 0 \end{cases}$$

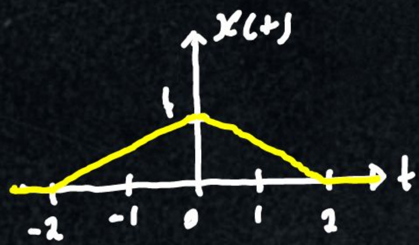


$$\xrightarrow[k=2]{T.S}$$

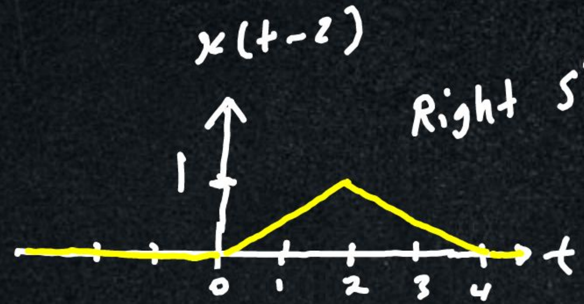


Left shifting

Case II : $k < 0$



$$\xrightarrow[k=-2]{T.S}$$



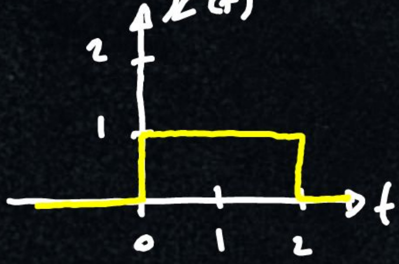
Right shifting

1.5.2) Amplitude Transformation

① Amplitude Scaling

$$x(t) \xrightarrow{A\cdot Sc} y(t) = \beta x(t)$$

Case I: $|\beta| > 1 \Rightarrow \beta \in (-\infty, 1) \cup (1, \infty)$

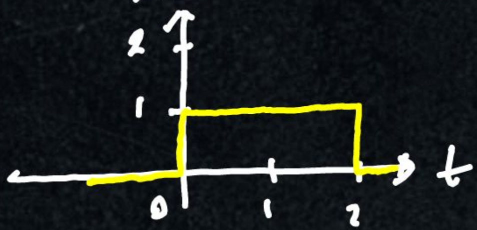


$$\xrightarrow[\beta = 2]{A.Sc}$$

y(t) = 2x(t)

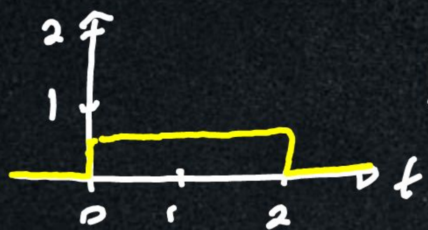
Amplification

Case II: $|\beta| < 1 \Rightarrow \beta \in (-1, 0) \cup (0, 1)$



$$\xrightarrow[\beta = 0.5]{A\cdot Sc}$$

y(t) = 0.5 x(t)

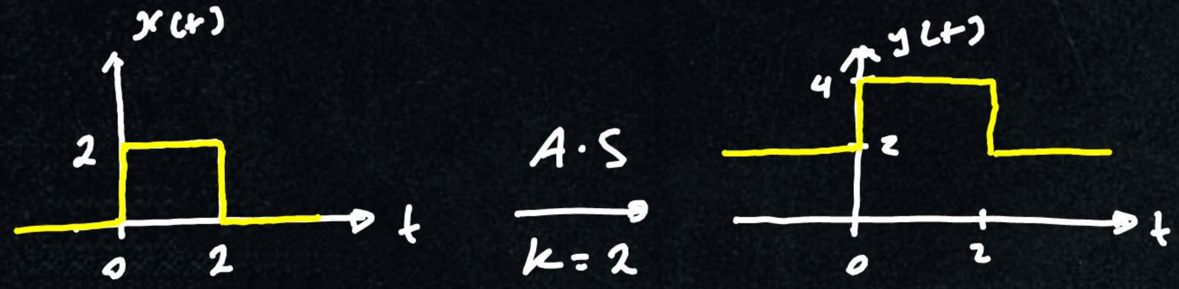


Reduction

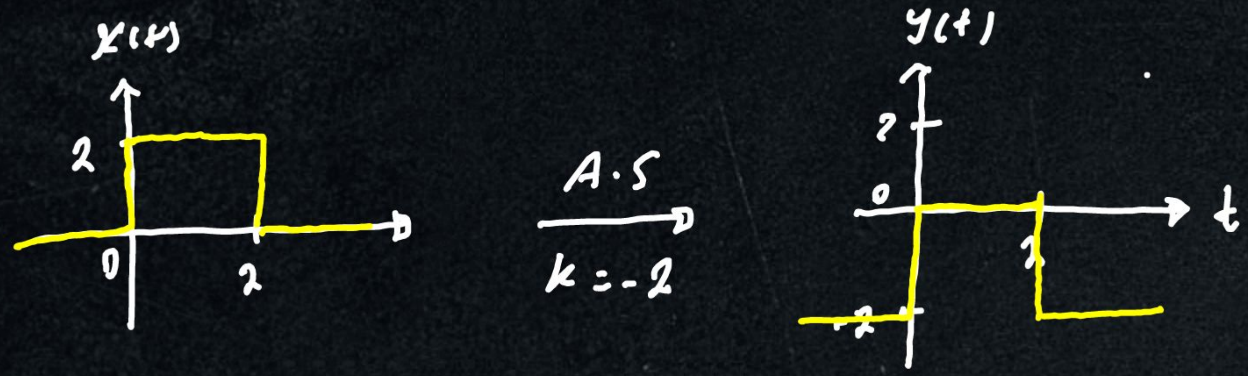
② Amplitude shifting

$$x(t) \xrightarrow{A\cdot S} y(t) = x(t) + k$$

Case I : $k > 0$ upward shifting



Case II : $k < 0$ Downward shifting



Multiple Transformation

$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t) = A x(-at - t_0) + K$

A.Sc T.Sc T.R T.S A.S

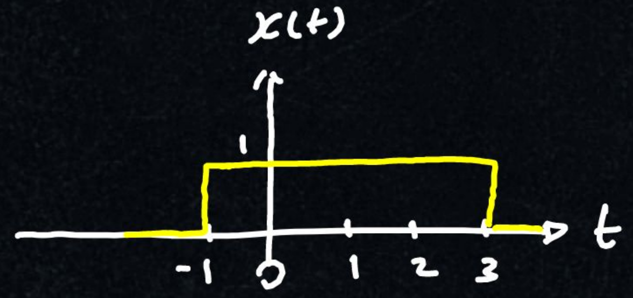
step 1: plot $A x(t) + B$

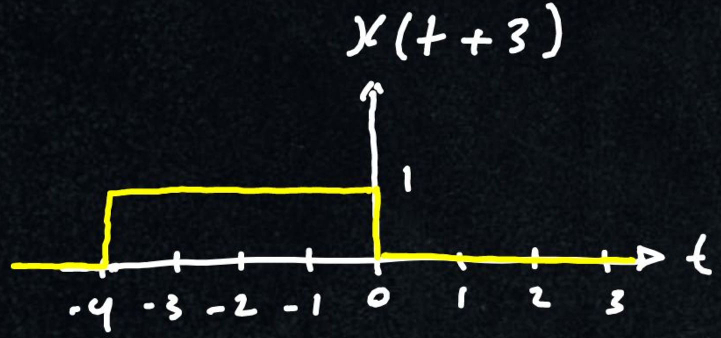
step 2: plot $A x(t - t_0) + K$

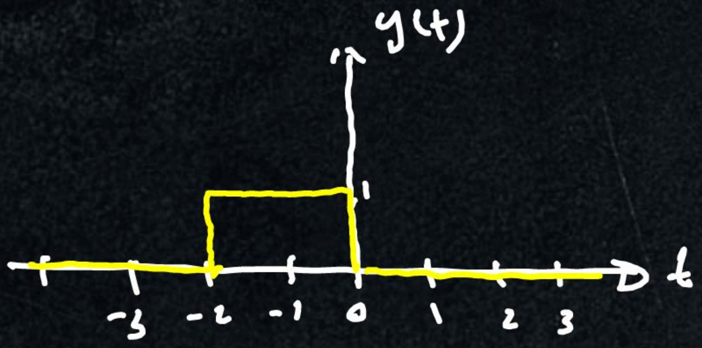
step 3: plot $A x(at - t_0) + K$

step 4: plot $A x(-at - t) + K$

Ex $x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t) = x(2t+3)$

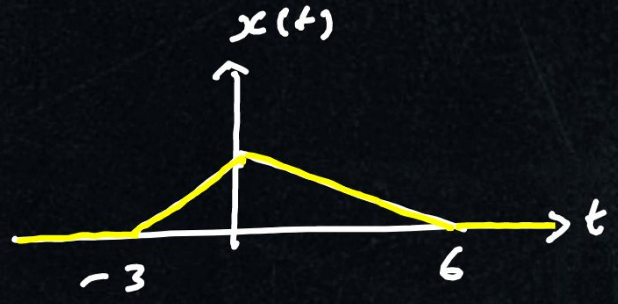


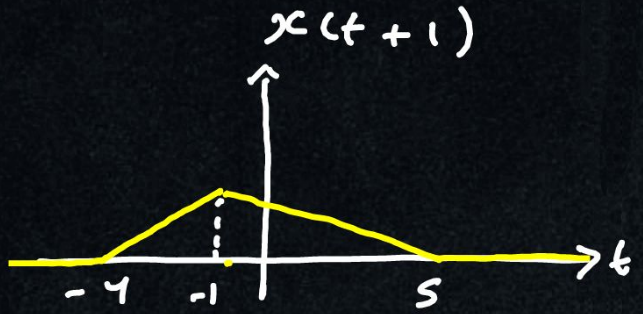


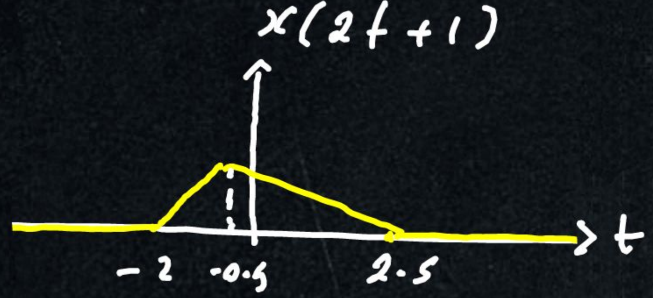


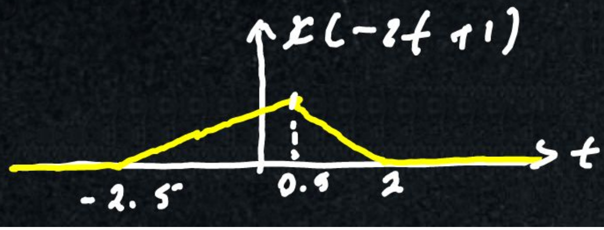
EX :-

$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t) = x(-2t+1)$







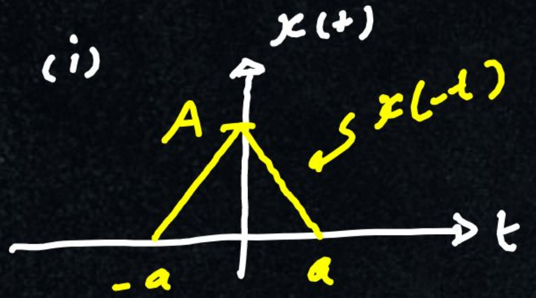


Even and Odd Signals

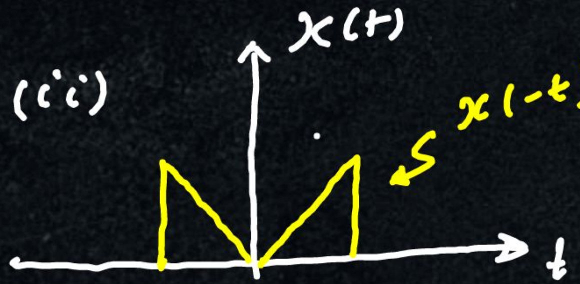
Even signals : Remain identical under reversing operation

$$x(-t) = x(t)$$

ex : (i)

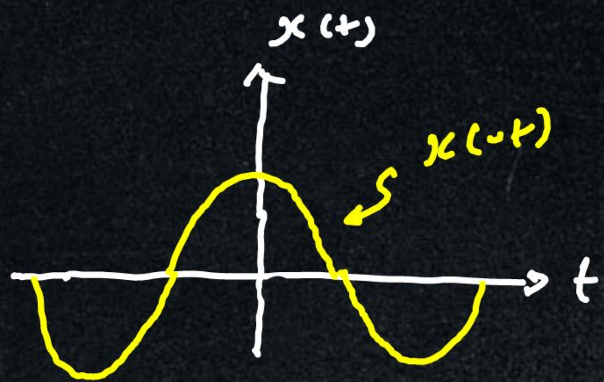


(ii)



(iii) $x(t) = \cos(\omega t)$

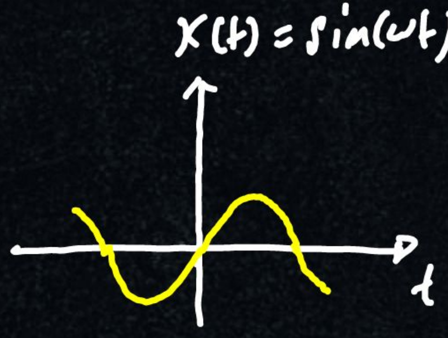
$$x(-t) = \cos(-\omega t) = \cos(\omega t) = x(t)$$

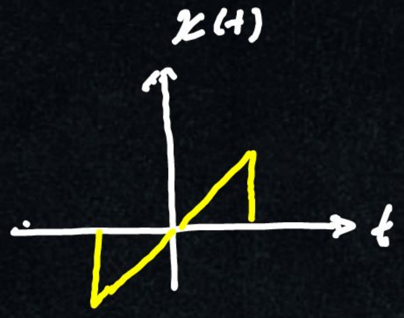


Odd signals :- Doesn't remain identical under reversing operation

$$x(-t) \neq x(t)$$

$$x(-t) = -x(t) \quad \text{or} \quad x(t) = -x(-t)$$





$t = 0$

$$x(0) = -x(-0) = -x(0)$$

$$x(0) + x(0) = 0$$

$$2x(0) = 0 \Rightarrow x(0) = 0$$

properties of odd signal,

1) At $t = 0 \Rightarrow x(0) = 0$

2) Average / mean / or DC value of odd signal $= 0$

Even and odd components of a signal

$$x(t) = x_e(t) + x_o(t)$$

$x(t)$ → general signal

$x_e(t)$ → even signal

$x_o(t)$ → odd signal

$$\left.\begin{aligned} x(t) &= x_e(t) + x_o(t) \\ x(-t) &= x_e(t) - x_o(t) \end{aligned}\right] \Rightarrow \begin{aligned} x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \\ x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right] \end{aligned}$$

properties of even and odd signals

1) $E + E = E$

2) $E + O$ = Neither E nor O

3) $E \times E = E$

4) $E \times O = O$

5) $O \times O = E$

6) $\frac{d}{dt}(E) = O$ except DC value

$\frac{d}{dt}(\text{constant}) = 0$

7) $\frac{d}{dt}(O) = E$

8) $\int E\,dt = O$

9) $\int O\,dt = E$

10) $\frac{1}{O} = O$

11) $\frac{1}{E} = E$

$\underline{EX}$:- Find the even and odd components of the following signals

1) $x(t) = \cos t + \sin t + 2$

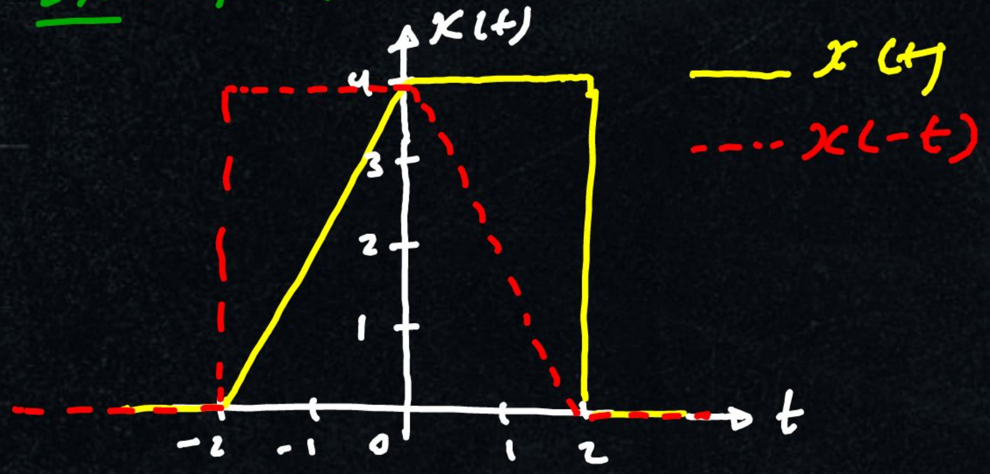
$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] = \frac{1}{2}\left[\cos t + \sin t + 2 + \cos t - \sin t + 2\right]$$

$$\boxed{x_e(t) = \cos t + 2}$$

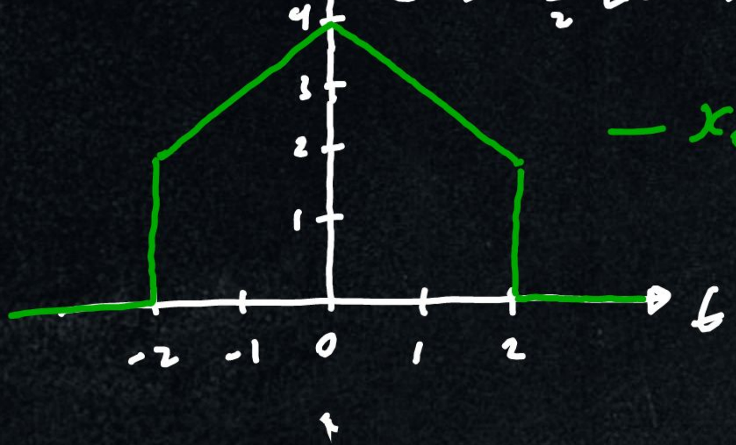
$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right] = \frac{1}{2}\left[\cos t + \sin t + 2 - \cos t + \sin t - 2\right]$$

$$\boxed{x_o(t) = \sin t}$$

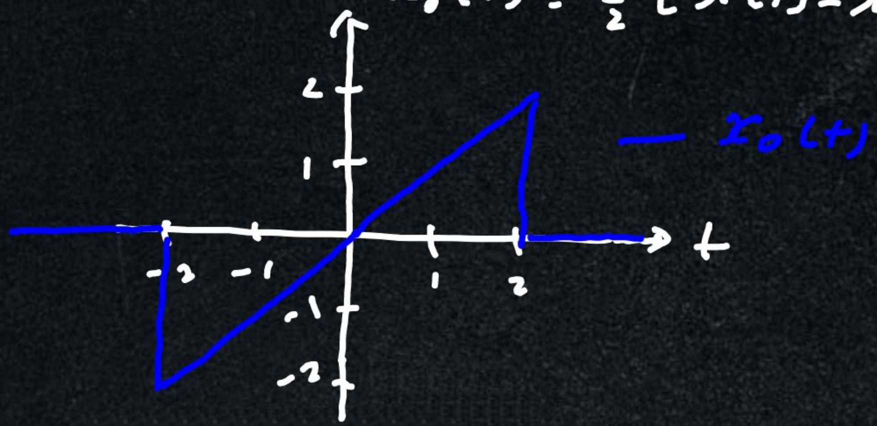
EX :- plot the even and odd components of $x(t)$



$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



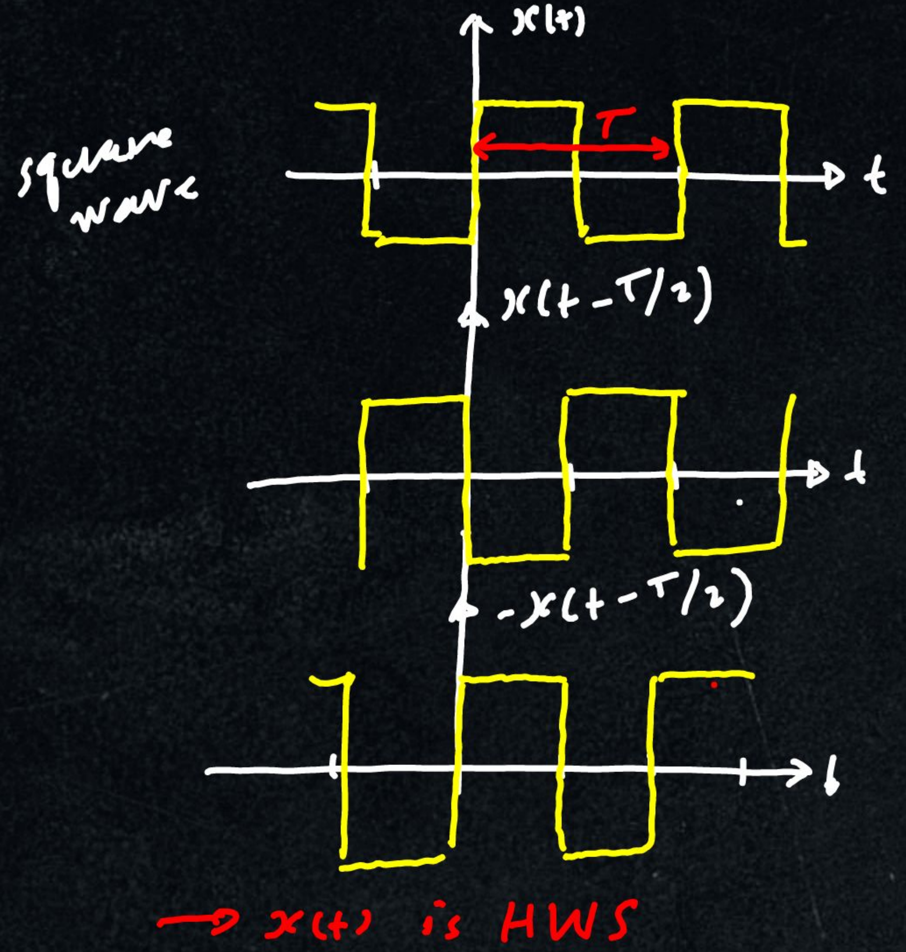
$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

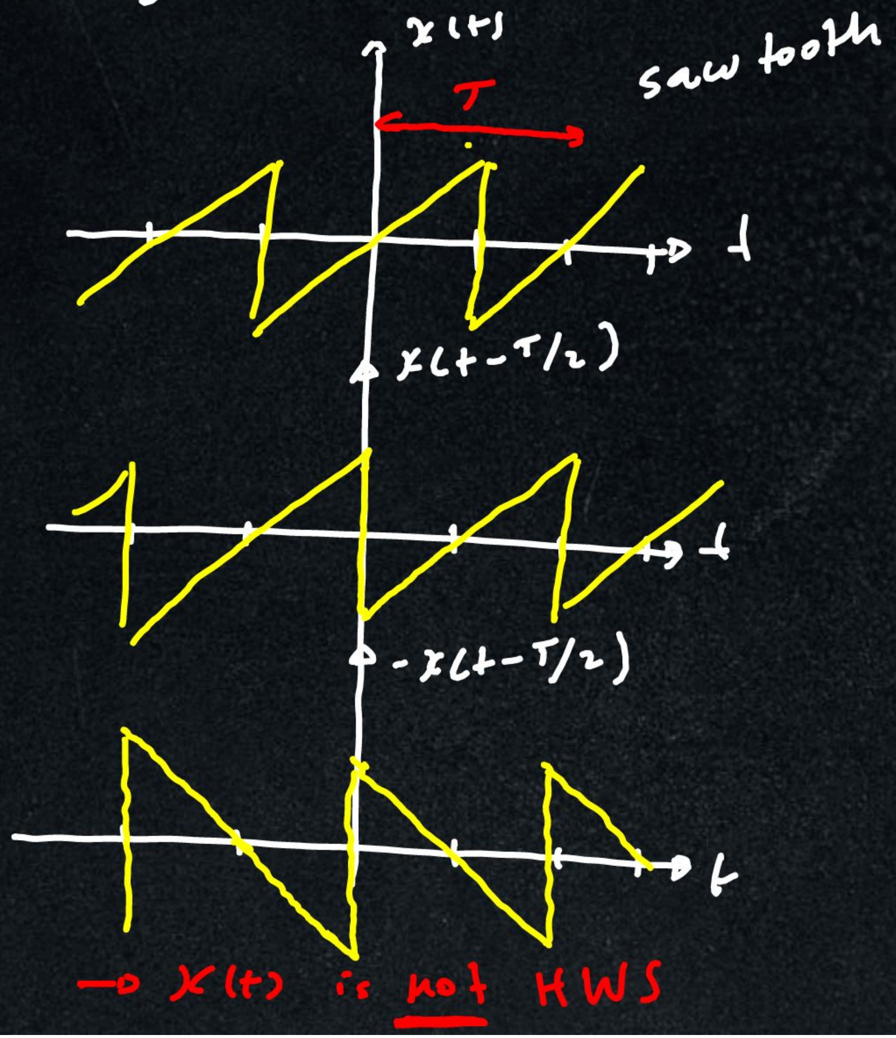


Half-Wave Symmetric Signals

$$x(t) = -x(t \mp T/2)$$

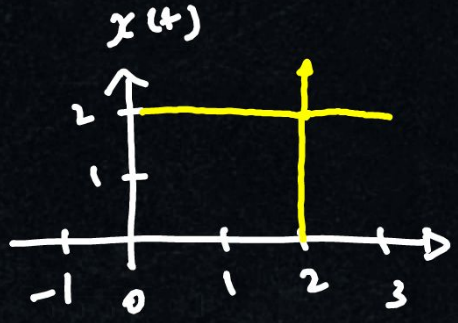
where T is the fundamental period





EX : Sketch the following signals

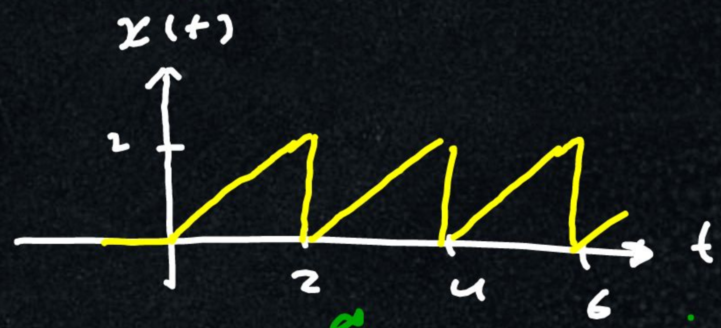
① $x(t) = 2u(t) + \delta(t-2)$



② $x(t) = 2u(t)\,\delta(t-2)$

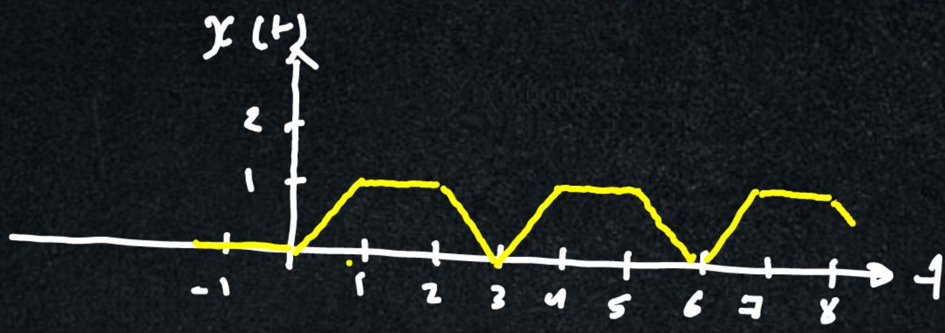
③ $x(t) = \sum_{n=0}^{\infty} x_a(t-2n)$

$x_a(t) = r(t)\,u(2-t)$

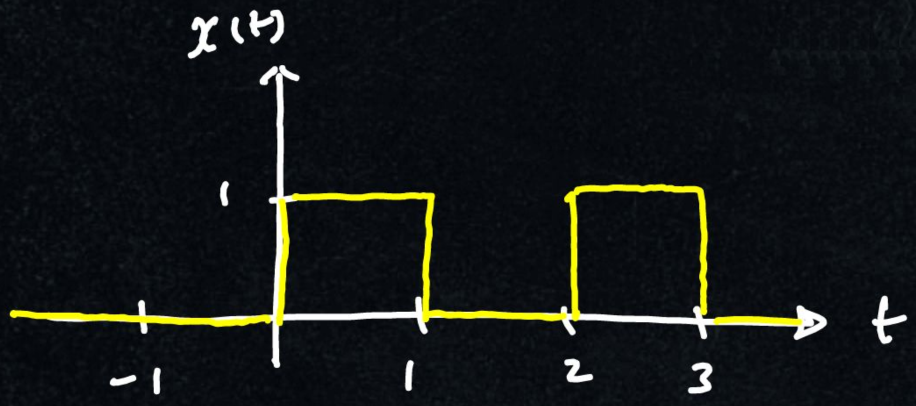


④ $x(t) = \sum_{n=0}^{\infty} x_a(t-3n)$

$x_a(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$

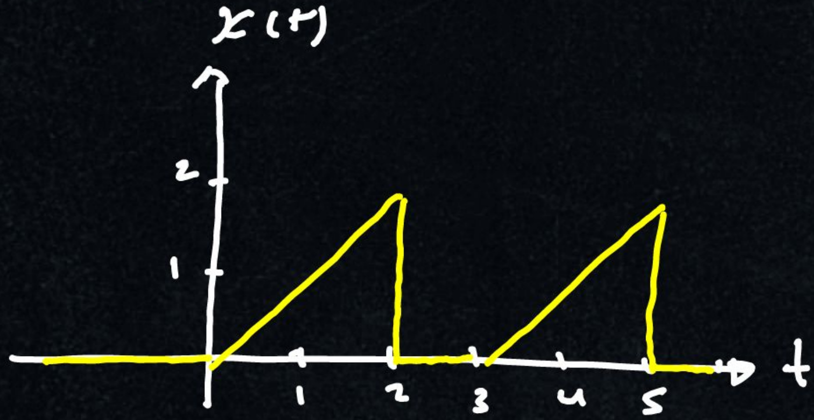


⑤ $x(t) = \sum_{n=0}^{\infty} u(t-2n)\,u(1+2n-t)$



EX: Express the signal shown in terms of singularity functions

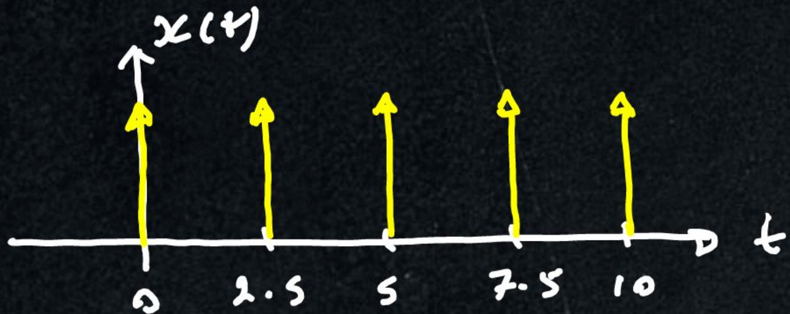
①



$$x(t) = r(t)\,u(2-t) + r(t-3)\,u(5-t) + \cdots$$

$$= \sum_{n=0}^{\infty} r(t-3n)\,u(2+3n-t)$$

②



$$x(t) = \delta(t) + \delta(t-2.5) + \delta(t-5) + \cdots$$

$$= \sum_{n=0}^{\infty} \delta(t-2.5n)$$