7.6 Inverse Trigonometric Fonctions

Q3. (a) $\sin^{-1}\left(-\frac{1}{2}\right)$

$= -\frac{\pi}{6}$

Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$

$\sin\theta = -\frac{1}{2}$ $\quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\theta = -\frac{\pi}{6}$

(b) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\pi}{4}$

Let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}$

(c) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$= \frac{-\pi}{3}$

let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = -\frac{\pi}{3}$

Q5 (b) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$= 3\frac{\pi}{4}$

let $\theta = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$\cos\theta = \frac{-1}{\sqrt{2}}$ $\quad \theta \in [0, \pi]$

$\theta = \frac{3\pi}{4}$

Q5 (c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$

let $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\cos(\theta) = \frac{\sqrt{3}}{2}$ $\quad \theta \in [0, \pi]$

$\theta = \frac{\pi}{6}$

$\boxed{Q_{10}}$ $\sec\left(\cos^{-1}\frac{1}{2}\right)$

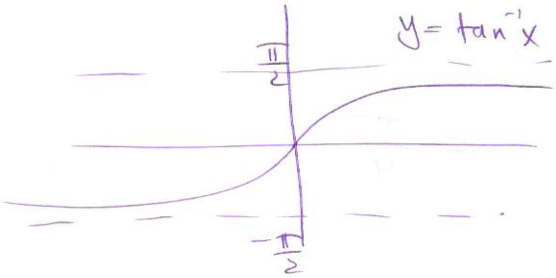
Let $\theta = \cos^{-1}\frac{1}{2}$

$\cos\theta = \frac{1}{2}$

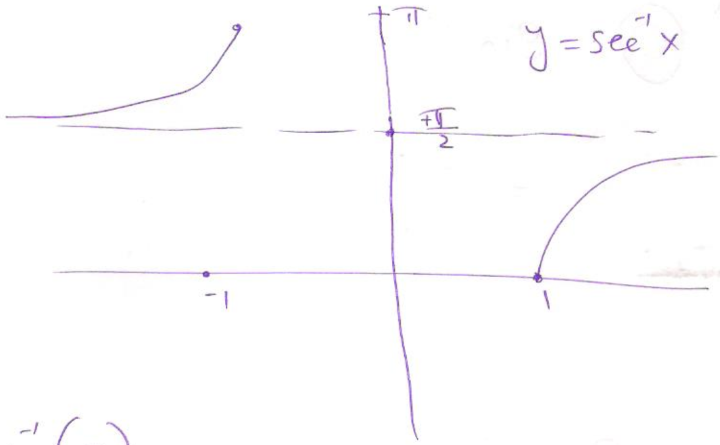
$\theta = \frac{\pi}{3}$

$\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)}$

$= \frac{1}{\frac{1}{2}} = 2$



$\boxed{Q_{16}}$ $\lim_{x\to-\infty} \tan^{-1}x = -\frac{\pi}{2}$



$\boxed{Q_{18}}$ $\lim_{x\to-\infty} \sec^{-1}x = \frac{\pi}{2}$

$\lim_{x\to-\infty} \cos^{-1}\left(\frac{1}{x}\right) = \lim \cos^{-1}(0)$

$= \frac{\pi}{2}$

$\boxed{Q_{22}}$ $\cancel{\lim}$ $y = \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x)$

$\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$. (1)

$\boxed{Q26}$ $y = \sec^{-1}(5s)$

$$\frac{dy}{ds} = \frac{1}{|5s|\sqrt{25s^2-1}}(5)$$

$$\frac{dy}{ds} = \frac{5}{5|s|\sqrt{25s^2-1}} = \frac{1}{|s|\sqrt{25s^2-1}}$$

$\boxed{Q34}$ $y = \tan^{-1}(\ln x)$

$$\frac{dy}{dx} = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x}$$

$$= \frac{1}{x\left(1+(\ln x)^2\right)}$$

$\boxed{Q35}$ $y = \csc^{-1}(e^t)$

$$\frac{dy}{dt} = \frac{-1}{|e^t|\sqrt{e^{2t}-1}}e^t$$

$f(x) = e^x$ positive function

$e^x > 0$

$$\frac{dy}{dt} = \frac{-1}{\sqrt{e^{2t}-1}}$$

$\boxed{Q_{38}}$ $y = \sqrt{x^2 - 1} - \sec^{-1} x$

$$\frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{dy}{dx} = \frac{x^2-1}{|x|\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}}{|x|}$$

$\boxed{Q_{40}}$ $y = \cot^{-1}\left(\frac{1}{x}\right) - \tan^{-1} x = \tan^{-1} x - \tan^{-1} x = 0$

$$\frac{dy}{dx} = 0$$

<u>Note</u>

* $\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right), \quad x > 0$
* $\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$

$\boxed{Q_{44}}$ $\int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{1-(\underbrace{2x}_{u})^2}} = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$

let $u = 2x$
$du = 2\,dx$

$$= \frac{1}{2}\sin^{-1}(2x) + C$$

$\boxed{Q_{46}}$ $\int \frac{dx}{9+3x^2} = \int \frac{dx}{(3)^2 + (\sqrt{3}\,x)^2}$

let $u = \sqrt{3}\,x$
$du = \sqrt{3}\,dx$

$$= \frac{1}{\sqrt{3}} \int \frac{du}{(3)^2 + (u)^2} = \frac{1}{\sqrt{3}} \cdot \frac{1}{3} \cdot \tan^{-1}\left(\frac{u}{3}\right) + C$$

$$= \frac{1}{3\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

Q50 $\displaystyle\int_0^{\frac{3\sqrt{2}}{4}} \frac{ds}{\sqrt{9-4s^2}} = \int_0^{\frac{3\sqrt{2}}{4}} \frac{ds}{\sqrt{(3)^2-(\underline{2s})^2}}$

let $U=2s$
$dU=2\,ds$

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* $S=0 \longrightarrow U=0$
$S=\frac{3\sqrt{2}}{4} \longrightarrow U=\frac{3}{\sqrt{2}}$

$$=\frac{1}{2}\int_0^{3/\sqrt{2}} \frac{dU}{\sqrt{(3)^2-(U)^2}}$$

$$=\frac{1}{2} * \left[\sin^{-1}\left(\frac{U}{3}\right)\right]_0^{\frac{3}{\sqrt{2}}}$$

$$=\frac{1}{2}\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}(0)\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{8}$$

Q52 $\displaystyle\int_{-2}^{2} \frac{dt}{4+3t^2} = \int_{-2}^{2} \frac{dt}{(2)^2+(\sqrt{3}t)^2}$

let $U=\sqrt{3}\,t$
$dU=\sqrt{3}\,dt$

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* $t=-2 \longrightarrow U=-2\sqrt{3}$
$t=2 \longrightarrow U=2\sqrt{3}$

$$=\frac{1}{\sqrt{3}}\int_{-2\sqrt{3}}^{2\sqrt{3}} \frac{du}{(2)^2+(U)^2}$$

$$=\frac{1}{\sqrt{3}}\,\frac{1}{2}\,\tan^{-1}\left(\frac{U}{2}\right)\Big]_{-2\sqrt{3}}^{2\sqrt{3}}$$

$$=\frac{1}{2\sqrt{3}}\left[\tan^{-1}(\sqrt{3}) - \tan^{-1}(-\sqrt{3})\right]$$

$$I = \frac{1}{2\sqrt{3}}\left[\tan^{-1}(\sqrt{3}) - \tan^{-1}(-\sqrt{3})\right]$$

$$= \frac{1}{2\sqrt{3}}\left[\frac{\pi}{3} - -\frac{\pi}{2}\right]$$

let $\theta = \tan^{-1}\sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$$= \frac{1}{2\sqrt{3}} \quad \frac{2\pi}{3}$$

$$= \frac{\pi}{3\sqrt{3}}.$$

$\boxed{Q_{59}}$ $$\int \frac{dx}{\underbrace{(2x-1)}_{u}\sqrt{\underbrace{(2x-1)^2}_{u^2} - \underbrace{4}_{a^2}}}$$

let $u = 2x-1$

$du = 2\,dx$

$$= \frac{1}{2}\int \frac{du}{u\,(\sqrt{u^2 - (2)^2}}$$

$$= \frac{1}{2}\,\frac{1}{2}\sec^{-1}\left|\frac{u}{2}\right| + c = \frac{1}{4}\sec^{-1}\left|\frac{2x-1}{2}\right| + C$$

$$= \frac{1}{4}\sec^{-1}\left|\frac{2x-1}{2}\right| + C.$$

$\boxed{Q64}$ $\displaystyle\int_1^{e^{\pi/4}} \frac{4\,dt}{t\left(1+\underbrace{\ln^2 t}_{U^2}\right)}$ $\quad 1+ \quad U^2$

$U = \ln t$

$dU = \frac{1}{t}\,dt$

$t = 1 \longrightarrow U = \ln 1 = 0$

$t = e^{\pi/4} \longrightarrow U = \frac{\pi}{4}$

$$\int_0^{\pi/4} \frac{4\cancel{t}\,dU}{\cancel{t}\left(1+U^2\right)}$$

$$4\int_0^{\pi/4} \frac{dU}{1+U^2} = 4\ \tan^{-1}(U)\Big]_0^{\pi/4}$$

$$= 4\left[\tan^{-1}\left(\frac{\pi}{4}\right) - \tan^{-1}(0)\right]$$

$$= 4\left[\tan^{-1}\left(\frac{\pi}{4}\right)\right]$$

$\boxed{Q68}$ $\displaystyle\int \frac{dx}{\sqrt{2x - x^2}}$

let $U = \sqrt{x}$

$dU = \frac{1}{2\sqrt{x}}\,dx$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{x}\ \sqrt{2-(\sqrt{x})^2}}$$

$$= \int \frac{2\sqrt{x}\ dU}{\sqrt{x}\ \sqrt{2-U^2}}$$

$$= \int \frac{2\,dU}{\sqrt{2-U^2}} = 2\int \frac{dU}{\sqrt{(\sqrt{2})^2 - U^2}}$$

$$I = 2\int \frac{du}{\sqrt{(\sqrt{2})^2 - u^2}}$$

$$= 2 \sin^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$= 2 \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}}\right) + C$$

$$= 2 \sin^{-1}\sqrt{\frac{x}{2}} + C$$

$\boxed{Q74}$ $$\int_2^4 \frac{2\,dx}{x^2 - 6x + 10}$$

Complete the square:- $\left(\frac{b}{2a}\right)^2$

$$x^2 - 6x + 10 = x^2 - 6x + \underline{9 - 9} + 10$$
$$= (x-3)^2 + 1$$

$$\int_2^4 \frac{2\,dx}{(x-3)^2 + 1}$$

Note:

- To complete the square we add & subtract $\left(\frac{b}{2a}\right)^2$

let $u = x - 3$

$du = dx$

* $x = 2 \longrightarrow u = -1$

$x = 4 \longrightarrow u = 1$

$$\int_{-1}^{1} \frac{2\,du}{u^2 + 1}$$

$$= 2 \tan^{-1}(u)\Big]_{-1}^{1}$$

$$= 2\left[\tan^{-1}(+1) - \tan^{-1}(-1)\right]$$

$$2\left[\tan^{-1}(1) + \tan^{-1}(1)\right] = 4\tan^{-1}(1) = 4 \cdot \frac{\pi}{4} = \boxed{\pi}$$

$\boxed{Q_{79}}$

$$\int \frac{dx}{(x+1)\sqrt{x^2+2x}}$$

Complete the square

$x^2+2x = x^2+2x+1-1$
$= (x+1)^2-1$

$$\int \frac{dx}{(x+1)\sqrt{(x+1)^2-1}}$$

let $u = x+1$
$du = dx$

$$\int \frac{du}{u\sqrt{u^2-1}}$$

$$= \sec^{-1}|u| + C$$

$$= \sec^{-1}|x+1| + C$$

$\boxed{Q_{84}}$

$$\int \frac{\sqrt{\tan^{-1}x}\; dx}{1+x^2}$$

$u = \tan^{-1} x$

$\frac{du}{dx} = \frac{1}{1+x^2}$

$$\int \frac{\sqrt{u}\;\cancel{(1+x^2)}\, du}{\cancel{(1+x^2)}}$$

$$\int u^{1/2}\, du = \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}\left(\tan^{-1}x\right)^{3/2} + C$$

$\boxed{Q_{86}}$

$$\int \frac{dy}{(\sin^{-1} y)\sqrt{1-y^2}}$$

$U = \sin^{-1} y$

$dU = \frac{1}{\sqrt{1-y^2}} \, dy$

$$\int \frac{\sqrt{1-y^2} \, dU}{U \sqrt{1-y^2}}$$

$$\int \frac{1}{U} \, dU = \ln|U| + C$$

$$= \ln|\sin^{-1} y| + C$$

$\boxed{Q_{90}}$

$$\int \frac{e^x \sin^{-1} e^x}{\sqrt{1-e^{2x}}} \, dx$$

$U = \sin^{-1} e^x$

$dU = \frac{1}{\sqrt{1-e^{2x}}} \cdot e^x \, dx$

$$= \int U \, dU$$

$$= U^2/2 + C$$

$$= \frac{(\sin^{-1} e^x)^2}{2} + C$$