(Example 1) $f(x) = \dfrac{x^2}{x+1}$ $\quad f'(x) = \dfrac{x^2+2x}{(x+1)^2}$ $\quad f''(x) = \dfrac{2}{(x+1)^3}$

(1) Domain: $\mathbb{R}\backslash\{-1\}$

(2) Horizontal asymptotes: None

(3) Vertical asymptotes: $x = -1$

(4) Oblique asymptotes: $y = x - 1$ → $\overline{x^2}$

(5) $\lim_{x\to\infty} f(x) = \infty$

(6) $\lim_{x\to-\infty} f(x) = -\infty$] NO H.A.

(7) $\lim_{x\to-1^+} f(x) = \infty$

(8) $\lim_{x\to-1^-} f(x) = -\infty$] $x = -1$ is V.A.

(9) Critical points: $x = 0$ and $x = -2$

++ — — ++
-2 -1 0

(10) Increasing intervals: $(-\infty, -2] \cup [0, \infty)$

(11) Decreasing intervals: $[-2, -1) \cup (-1, 0]$

(12) Local maximum values: $f(-2) = -4$

(13) Local minimum values: $f(0) = 0$

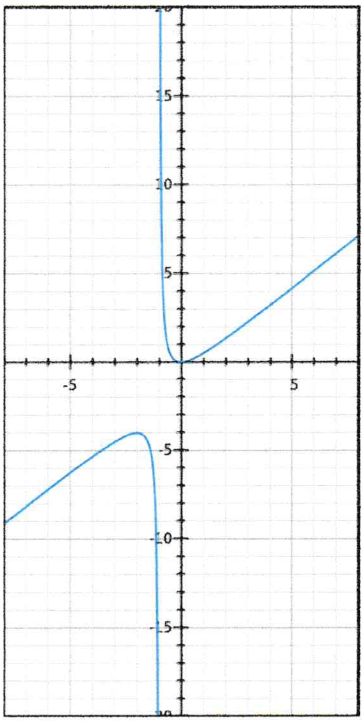
(14) Absolute maximum values: None

(15) Absolute minimum values: None

(16) Concave up intervals: $(-1, \infty)$

(17) Concave down intervals: $(-\infty, -1)$ → y'' — — ++
-1

(18) Inflection points: None



(Example 2) $f(x) = \dfrac{x^2}{x^2-1}$ $\quad f'(x) = \dfrac{-2x}{(x^2-1)^2}$ $\quad f''(x) = \dfrac{6x^2+2}{(x^2-1)^3}$

(1) Domain: $\mathbb{R}\backslash\{\pm 1\}$

(2) Horizontal asymptotes: $y = 1$ → since $\lim\limits_{x\to\infty} f(x) = 1$ and $\lim\limits_{x\to-\infty} f(x) = 1$

(3) Vertical asymptotes: $x = 1\ ,\ x = -1$ → $\lim\limits_{x\to 1^+} f(x) = \infty$, $\lim\limits_{x\to 1^-} f(x) = -\infty$ so $x=1$ is V.A. ; $\lim\limits_{x\to -1^+} f(x) = -\infty$, $\lim\limits_{x\to -1^-} f(x) = \infty$ so $x=-1$ is V.A.

(4) Oblique asymptotes: None

(5) $\lim\limits_{x\to\infty} f(x) = 1$

(6) $\lim\limits_{x\to-\infty} f(x) = 1$ → $y=1$ is H.A.

(7) $\lim\limits_{x\to 1^+} f(x) = \infty$

(8) $\lim\limits_{x\to 1^-} f(x) = -\infty$ $\quad x = 1$ is V.A

(9) $\lim\limits_{x\to -1^+} f(x) = -\infty$

(10) $\lim\limits_{x\to -1^-} f(x) = \infty$ $\quad x = -1$ is V.A.

(11) Critical points: $x = 0$ → $f'(x)=0 \Rightarrow x^2=0 \Rightarrow x=0.$

(12) Increasing intervals: $(-\infty,-1)\cup(-1,0]$ → $f'(x)$ + + − − ; -1, 0, 1

(13) Decreasing intervals: $[0,1)\cup(1,\infty)$

(14) Local maximum values: $f(0) = 0$

(15) Local minimum values: None

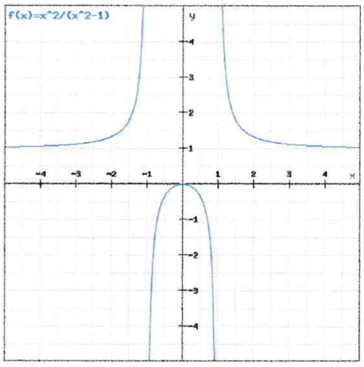
(16) Absolute maximum values: None

(17) Absolute minimum values: None

(18) Concave up intervals: $(-\infty,-1)\cup(1,\infty)$ $\quad f''(x)=0$ + − + ; -1, 1

(19) Concave down intervals: $(-1,1)$

(20) Inflection points: None



(Example 3) $f(x) = \dfrac{x}{x^2+1}$ $\quad f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$ $\quad f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}$

(1) Domain: $\mathbb{R}$

(2) Horizontal asymptotes: $y = 0$

(3) Vertical asymptotes: None

(4) Oblique asymptotes: None

(5) $\lim_{x\to\infty} f(x) = 0$

(6) $\lim_{x\to-\infty} f(x) = 0$

(7) Critical points: $x = 1$ and $x = -1$

(8) Increasing intervals: $[-1, 1]$

(9) Decreasing intervals: $(-\infty, -1] \cup [1, \infty)$

(10) Local maximum values: $f(1) = \frac{1}{2}$

(11) Local minimum values: $f(-1) = -\frac{1}{2}$

(12) Absolute maximum values: $f(1) = \frac{1}{2}$

(13) Absolute minimum values: $f(-1) = -\frac{1}{2}$

(14) Concave up intervals: $[-\sqrt{3}, 0] \cup [\sqrt{3}, \infty)$

(15) Concave down intervals: $(-\infty, -\sqrt{3}] \cup [0, \sqrt{3}]$

(16) Inflection points: $(-\sqrt{3}, -\sqrt{3}/4)$, $(0, 0)$, $(\sqrt{3}, \sqrt{3}/4)$