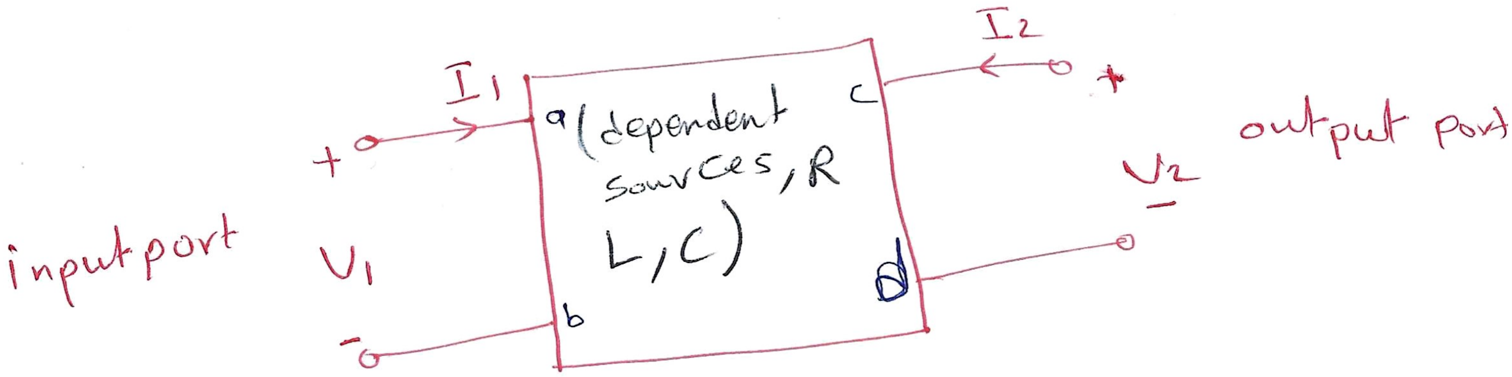
Chapter 18: Two-port Circuits



Two-port circuits can be modeled by 2X2 matrix to relate V/i variables. The matrix elements can be obtained by performing open, and short circuit tests.

There are 6 possible ways to form sets of two equations.

1- Z-parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

2- Y- Parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

3- A - Parameters

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

4- B - Parameters

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \begin{bmatrix} b_{11} & -b_{12} \\ b_{21} & -b_{22} \end{bmatrix} = \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

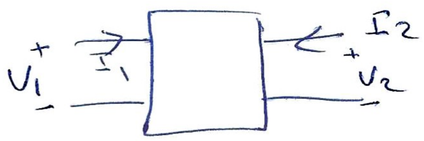
5 - h - parameters

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

6 - g - parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Calculating of z - parameters



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

If $I_2 = 0 \Rightarrow$ open Circuit at port II

$V_1 = z_{11} I_1 \Rightarrow z_{11} = \frac{V_1}{I_1}$

$V_2 = z_{21} I_1 \Rightarrow z_{21} = \frac{V_2}{I_1}$

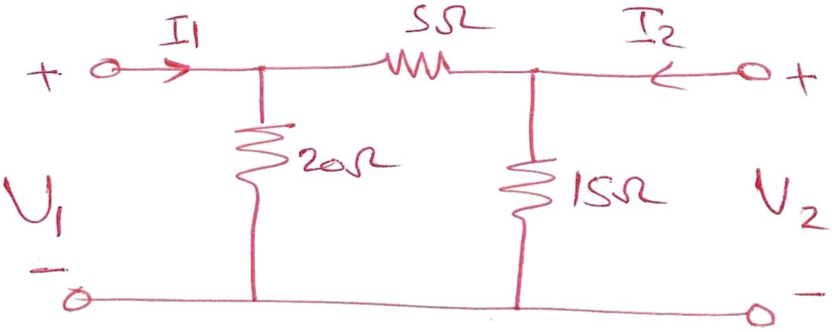
If $I_1 = 0 \Rightarrow$ open Circuit at port I

$V_1 = z_{12} I_2 \Rightarrow z_{12} = \frac{V_1}{I_2}$

$V_2 = z_{22} I_2 \Rightarrow z_{22} = \frac{V_2}{I_2}$

We can use Table 18.1 to convert to other parameters

Example: Determine the z-parameters using the z-parameter definitions for the Network shown below



If $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = 20 \,//\, (5+15) = \boxed{10\,\Omega} \Rightarrow I_1 = \frac{V_1}{10}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$V_2 = \frac{V_1(15)}{5+15} = 0.75\,V_1$$

$$I_1 = \frac{V_1}{10}$$

$$\Rightarrow Z_{21} = \frac{0.75\,V_1}{\frac{V_1}{10}} = 7.5\,\Omega$$

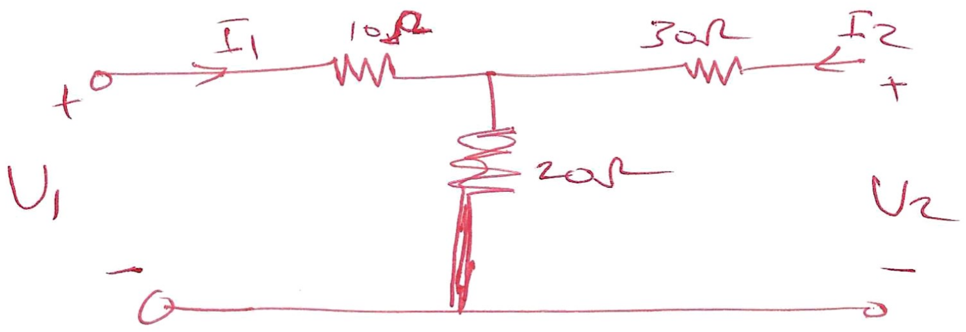
If $I_1 = 0$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = (5+20) \,//\, 15 = 9.375\,\Omega \Rightarrow I_2 = \frac{V_2}{9.375}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \Rightarrow Z_{12} = \frac{0.8\,V_2}{\frac{V_2}{9.375}} = 7.5\,\Omega$$

$$V_1 = \frac{V_2(20)}{5+20} = 0.8\,V_2$$

Examples- Determine the z-parameters using the z-parameter definitions for the network shown below



for $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}\Bigg|_{I_2=0} = 10 + 20 = 30\,\Omega \Rightarrow I_1 = \frac{V_1}{30}$$

$$Z_{21} = \frac{V_2}{I_1}\Bigg|_{I_2=0} \qquad V_2 = 20 I_1$$

$$Z_{21} = \frac{20\,I_1}{I_1} = 20\,\Omega$$

for $I_1 = 0$

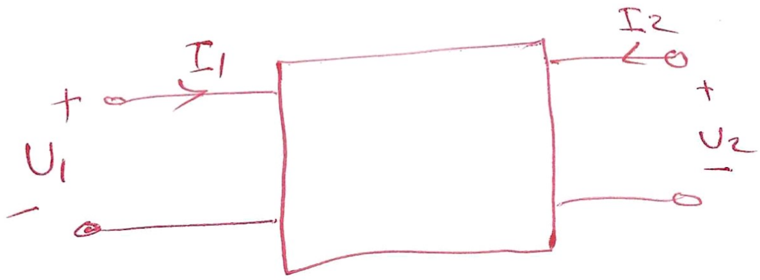
$$Z_{22} = \frac{V_2}{I_2}\Bigg|_{I_1=0} = 30 + 20 = 50\,\Omega$$

$$Z_{12} = \frac{V_1}{I_2} \qquad V_1 = 20\,I_2$$

$$Z_{12} = \frac{20\,I_2}{I_2} = 20\,\Omega$$

Example 8- The following measurements pertain to a two-port circuit operating in the sinusoidal steady state.

with port two open ($I_2 = 0$), a voltage equal to $150\cos(4000t)$ V is applied to port 1. The current into port 1 is $25\cos(4000t - 45°)$ A, and the port 2 voltage is $100\cos(4000t + 15°)$ V. With port 2 short circuited ($V_2 = 0$) a voltage equals to $30\cos(4000t)$ V is applied to port 1. The current into port 1 is $1.5\cos(4000t + 30°)$ A, and the current into port 2 is $0.25\cos(4000t + 150°)$ A. Find the a parameters that can describe the sinusoidal steady state behavior of the circuit



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$V_1 = a_{11} V_2 - a_{12} I_2$

$I_1 = a_{21} V_2 - a_{22} I_2$

for $I_2 = 0$ $V_1 = 150\angle 0$, $V_2 = 100\angle 15°$, $I_1 = 25\angle -45°$

for $V_2 = 0$ $V_1 = 30\angle 0$, $I_1 = 1.5\angle 30°$ A, $I_2 = 0.25\angle 150°$

for $I_2 = 0$

$$a_{11} = \frac{V_1}{V_2} = \frac{150\angle 0}{100\angle 15} = 1.5\angle -15°$$

$$a_{21} = \frac{I_1}{V_2} = \frac{25\angle -45}{100\angle 15} = 0.25\angle -60° \text{ S.}$$

for $V_2 = 0$

$$a_{12} = \frac{V_1}{I_2} = \frac{-30\angle 0}{0.25\angle 150} = 120\angle 30° \ \Omega$$

$$a_{22} = \frac{I_1}{I_2} = \frac{1.5\angle 30}{0.25\angle 150} = 6\angle 60°$$