Th (Fundemental Th of Calculus)

- $f(x)$ cont. on $[a,b]$
- $F(x)$ is any antiderivative of $f(x)$

$f(x) = 2x$
$F_1(x) = x^2 + 1$
$F_2(x) = x^2 - \sqrt{5}$
$\vdots$

Then

① $\displaystyle\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

② If $\displaystyle F(x) = \int_a^x f(t)\,dt$ cont. on $[a,b]$ and diff on (a,b)

then $F'(x) = f(x)\,1 = f(x)$

Exp Find the following derivatives

① $\displaystyle\frac{d}{dx}\int_0^x \cot(t)\,dt = \cot x \cdot 1 = \cot x$

② $\displaystyle\frac{d}{dx}\boxed{\int_{\frac{\pi}{2}}^{\pi}\left(\frac{\csc\theta}{1-\cos\theta}\right)d\theta}$ → تكامل محدود جوابه رقم $= 0$

③ $\displaystyle\left(\int_{\tan x}^{\frac{\pi}{4}} \frac{dt}{1+t^2}\right)' = \left(-\int_{\frac{\pi}{4}}^{\tan x} \frac{dt}{1+t^2}\right)'$

$\tan x$

$$= -\frac{1}{1+(\tan x)^2} \cdot \sec^2 x$$

$$= -\frac{\sec^2 x}{1+\tan^2 x} = -\frac{\sec^2 x}{\sec^2 x}$$

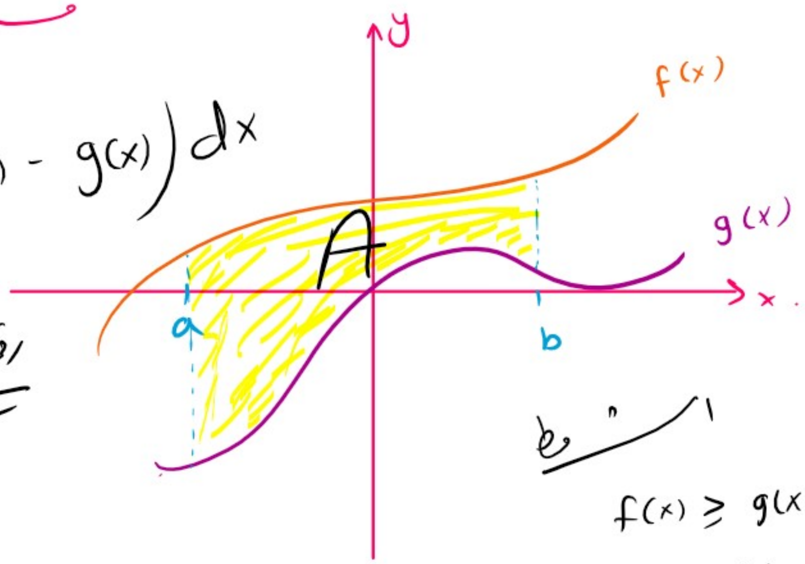
$$= -1$$

$$F(x) = \int_a^{u(x)} f(t)\,dt$$

$$F'(x) = f(u(x))\; u'(x)$$

Area مساحة

$$A = \int_a^b \big(f(x) - g(x)\big)\,dx$$



شروط

$f(x) \geq g(x)$ on $[a,b]$

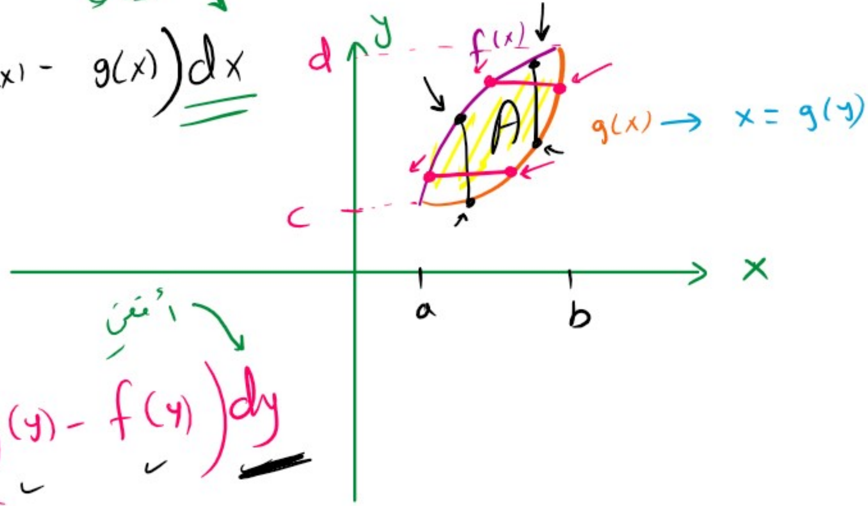
f, g integrable

(b)

$\int (f(x) - g(x))\,dx$ d ↑ y - - f(x)

عمودي

$$A = \int_a^b \big(f(x) - g(x)\big)\,dx$$



أفقي

$$= \int_c^d \big(g(y) - f(y)\big)\,dy$$

Exp $y = 5 - x^2$, $y = 1$

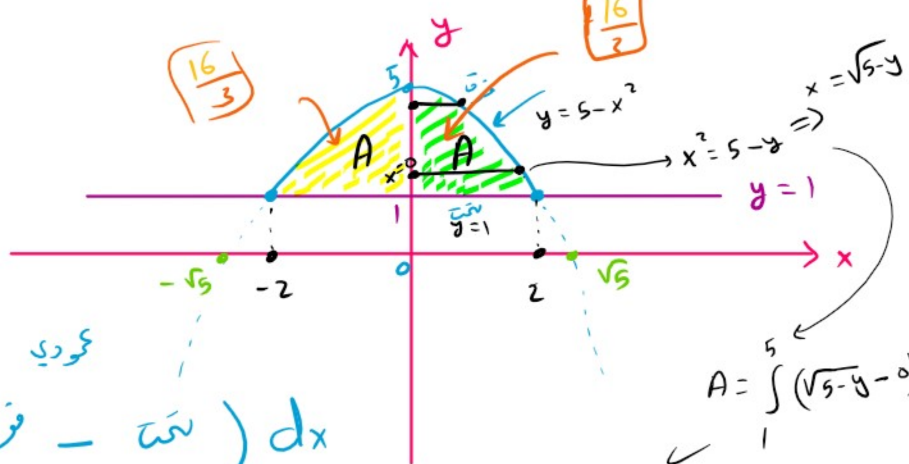
Find area eclosed by these functions

$y = y$

$5 - x^2 = 1$

$x^2 = 4$

$x = \pm 2$



$$A = \int_1^5 \left(\sqrt{5-y} - 0\right) dy$$

$$= \frac{16}{3}$$

عمودي

$$A = \int_0^2 (\text{فوق} - \text{تحت})\,dx$$

$$= \int_0^2 (5 - x^2 - 1)\,dx$$

$$= \int_0^2 (4 - x^2)\,dx = 4x - \frac{x^3}{3}\Big|_0^2 = 8 - \frac{8}{3} = \frac{16}{3}$$

$$2A = 2\left(\frac{16}{3}\right) = \frac{32}{3}$$

$$= \int^2 (5 - x^2 - 1)\,dx$$

$$= \int_{-2}^{2} (5 - x^2 - 1)\, dx$$

Exp Find area eclosed by $f(x) = 3x\sqrt{x^2+1}$ and x-axis on $[0,1]$

موجب على الفترة $[0,1]$

$$A = \int_0^1 3x\sqrt{x^2+1}\, dx$$

$u = x^2 + 1$

$du = 2x\, dx$

$\frac{du}{2} = x\, dx$

$x = 0 \Rightarrow u = 0^2 + 1 = 1$

$x = 1 \Rightarrow u = 1^2 + 1 = 2$

$$= \int_1^2 3\sqrt{u}\, \frac{du}{2}$$

$$= \frac{3}{2} \int_1^2 u^{\frac{1}{2}}\, du$$

$$= \frac{3}{2} \left. \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right|_1^2$$

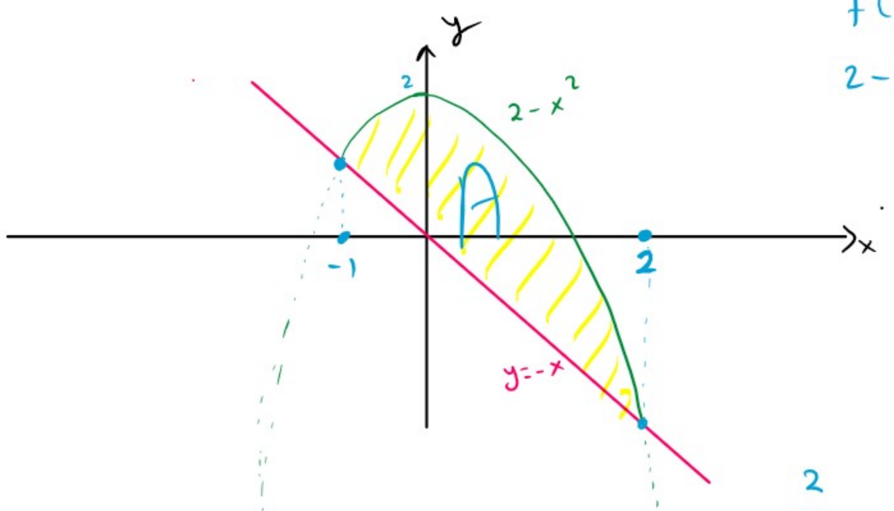
$$= \cancel{\frac{3}{2}} \left. \frac{u^{\frac{3}{2}}}{\cancel{\frac{3}{2}}} \right|_1^2 = \left. \sqrt{u^3} \right|_1^2$$

$$= \sqrt{8} - \sqrt{1}$$

$$= 2\sqrt{2} - 1$$ موجب

$= 2\sqrt{2} - 1$ مرضية

Exp Find area eclosed by $f(x) = 2 - x^2$ and $y = -x$



$f(x) = y$

$2 - x^2 = -x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad , \quad x = -1$

$$A = \int_{-1}^{2} \left(2 - x^2 - -x\right) dx = \int_{-1}^{2} \left(2 - x^2 + x\right) dx$$

$$= 2x - \frac{x^3}{3} + \frac{x^2}{2} \Big|_{-1}^{2}$$

$$= \left(4 - \frac{8}{3} + \frac{4}{2}\right) - \left(-2 - \frac{-1}{3} + \frac{1}{2}\right)$$

$$= 4 - \frac{8}{3} + 2 + 2 - \frac{1}{3} - \frac{1}{2}$$

$$= 8 - \frac{8}{3} - \frac{1}{3} - \frac{1}{2} = \boxed{\frac{9}{2}}$$

مساحة
مرضية

Exp $y = \sqrt{x}$, x-axis, $y = x - 2$

Find area eclosed by these curves and lines

Exp $y = \sqrt{x}$, ...

Find area eclosed by these curves and lines

1. with respect to x-axis
2. ,, ,, ,, y-axis

$y = y$

$\sqrt{x} = x - 2$

$\boxed{x = 4}$

$\sqrt{x} = x - 2$

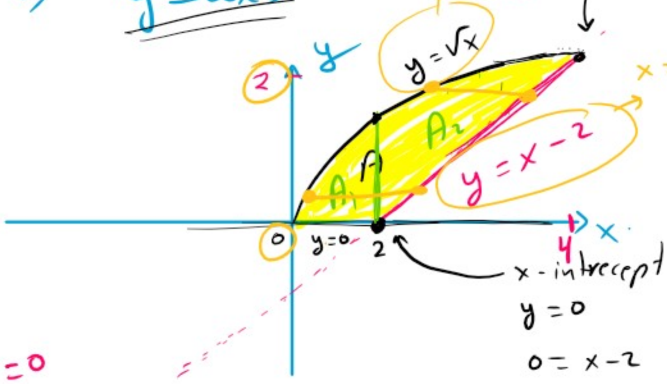
$x = (x-2)^2$

$x = x^2 - 4x + 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4$, ~~$x = 1$~~



x-intercept

$y = 0$

$0 = x - 2$

$x = 2$

$$A = \int_0^2 \left(y + 2 - y^2 \right) dy$$

$$= \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_0^2$$

$$= 2 + 4 - \frac{8}{3}$$

$$= 6 - \frac{8}{3}$$

$$= \frac{10}{3}$$

1. x-axis

$$A = A_1 + A_2$$

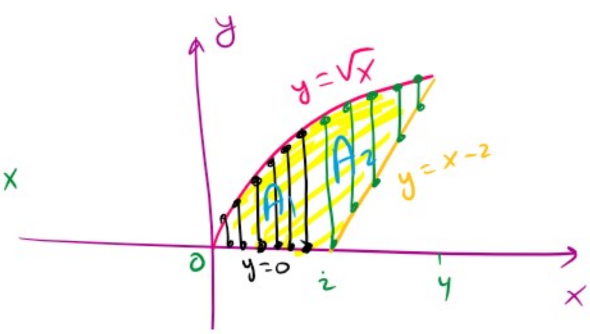
$$= \int_0^2 (\sqrt{x} - 0)\, dx + \int_2^4 \left(\sqrt{x} - (x-2)\right) dx$$

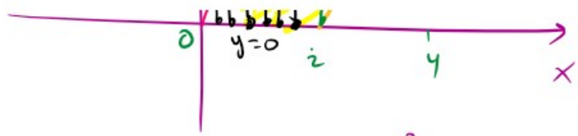
$$= \int_0^2 x^{\frac{1}{2}} dx + \int_2^4 (x^{\frac{1}{2}} - x + 2) dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} \Big|_0^2 + \frac{2}{3} x^{\frac{3}{2}} - \frac{x^2}{2} + 2x \Big|_2^4$$

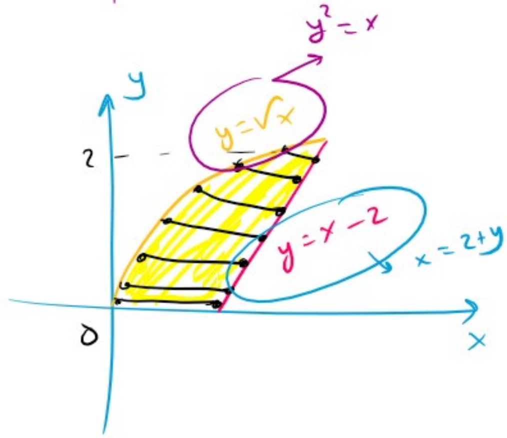
$$= \dots = \frac{10}{3}$$

$$A = \int \left(\text{فوق} - \text{تحت} \right) dx$$





$$A = A_1 + A_2$$



$$A = \int_0^2 \left(\overset{2+y}{\text{اليمين}} - \overset{y^2}{\text{اليسار}} \right) dy$$

Exp

$$\int \sqrt{\frac{x^4}{x^3-1}}\, dx$$

$$\sqrt{\frac{x^4}{x^3\left(1-\frac{1}{x^3}\right)}}$$

$$\left(\frac{x^4}{x^3-1}\right)^{\frac{1}{2}}$$

$$u = x^3 - 1$$

$$\int \frac{\sqrt{x^4}}{\sqrt{x^3-1}}\, dx$$

$$\int \frac{|x^2|}{\sqrt{x^3-1}}\, dx$$

$$u = x^3 - 1$$

$$du = 3x^2\, dx$$

$$\frac{du}{3} = x^2\, dx$$

$$\int \frac{x^2\, dx}{\sqrt{x^3-1}}$$

$$\int \frac{\frac{du}{3}}{\sqrt{u}} = \frac{1}{3} \int u^{-\frac{1}{2}}\, du$$

$\sqrt{u}$

$$= \frac{1}{3} \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{1}{3} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{1}{3} \cdot \frac{2}{1} \sqrt{u} + C$$

$$= \frac{2}{3} \sqrt{x^3 - 1} + C$$

اشتق

$\Downarrow$

$\sqrt{\frac{x^4}{x^3-1}}$