Chapter 7

Kinetic Energy and Work

7-1 Kinetic Energy

- Energy is required for any sort of motion.
- Energy:
  - Is a scalar quantity assigned to an object or a system of objects
  - Can be changed from one form to another
  - Is **Conserved** in a closed system, that is the total amount of energy of all types is always the same
- In this chapter we discuss one type of energy (**Kinetic Energy**)
- We also discuss one method of transferring energy (**Work**)

The **kinetic energy** *K* associated with the motion of a particle of mass *m* and speed *v*, where *v* is well below the speed of light, is

$$K = \frac{1}{2}mv^2$$

Notes:

1. Kinetic energy is zero for a stationary object
2. Kinetic energy is always positive ($K \geq 0$)
3. The unit of kinetic energy is a **joule** (**J**)

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$$

••5 A father racing his son has half the kinetic energy of the son, who has half the mass of the father. The father speeds up by 1.0 m/s and then has the same kinetic energy as the son. What are the original speeds of (a) the father and (b) the son?

At first, $K_{father} = \frac{1}{2} K_{son}$

$$\frac{1}{2} m_{father}\, v_{father}^2 = \frac{1}{2}\left(\frac{1}{2} m_{son}\, v_{son}^2\right) \dots (1)$$

$$m_{father} = 2m_{son} \dots (2)$$

The father speeds up by 1.0 m/s →→

$$v_{father,new} = v_{father} + 1.0\ m/s$$

$$K_{father,new} = K_{son}$$

$$\frac{1}{2} m_{father} \left(v_{father} + 1.0\ m/s\right)^2 = \frac{1}{2} m_{son}\, v_{son}^2$$

$$\frac{1}{2} m_{father} \left(v_{father}^2 + 2v_{father} + 1\right) = \frac{1}{2} m_{son}\, v_{son}^2$$

Use $m_{father}\, v_{father}^2 = \frac{1}{2} m_{son}\, v_{son}^2$

$$\frac{1}{2} m_{father} \left(v_{father}^2 + 2v_{father} + 1\right) = m_{father}\, v_{father}^2$$

$$v_{father}^2 - 2v_{father} - 1 = 0$$

$$v_{father} = \frac{+2 \pm \sqrt{4+4}}{2} = 2.4 \frac{m}{s} \quad or \quad -0.4\ m/s$$

$$v_{father} = 2.4\ m/s$$

$$v_{father,new} = 3.4\ m/s$$

Plug (2) in (1)

$$\frac{1}{2}(2m_{son})\, v_{father}^2 = \frac{1}{2}\left(\frac{1}{2} m_{son}\, v_{son}^2\right)$$

$$v_{son} = 2v_{father} = 2(2.4\ m/s) = 4.8\ m/s$$

7-2 Work and Kinetic Energy

- Account for changes in kinetic energy by saying energy has been transferred *to* or *from* the object
- In a transfer of energy via a force, **Work**

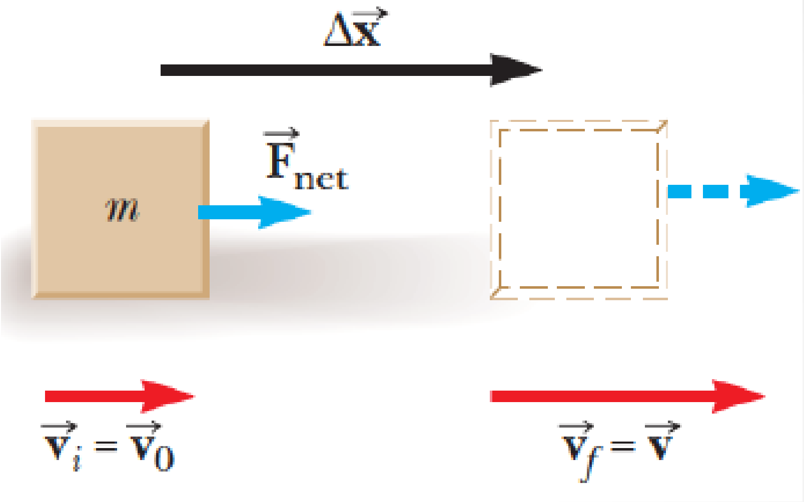
> Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

- This is not the common meaning of the word "work"
  - To do work on an object, energy must be transferred
  - Throwing a baseball does work
  - Pushing an immovable wall does not do work

An object undergoes a displacement and a change in velocity under the action of a constant net force F_{net}

$$v^2 = {v_0}^2 + 2\,a_x\,\Delta x$$

$$F_{net,x} = ma_x = m\left(\frac{v^2 - {v_0}^2}{2\Delta x}\right)$$



$$F_{net,x}\Delta x = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2$$

The right side tell us the kinetic energy has been changed by the force, and the left side tell us the change is equal to $F_{net,x}\Delta x$. Therefore, the work W done on the object by the force (the energy transfer due to the force) is:

$$\boldsymbol{F_{net,x}\Delta x = W}$$

- The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

$$W = Fd\cos\emptyset = \vec{F}\cdot\vec{d} \quad \text{(work, constant force)}$$

In which $\emptyset$ is the constant angle between the direction of $\vec{F}$ and $\vec{d}$

Note:

1. Only the component of $\vec{F}$ that is along the displacement $\vec{d}$ can do work on the object (Parallel component). The force component perpendicular to the displacement do zero work.

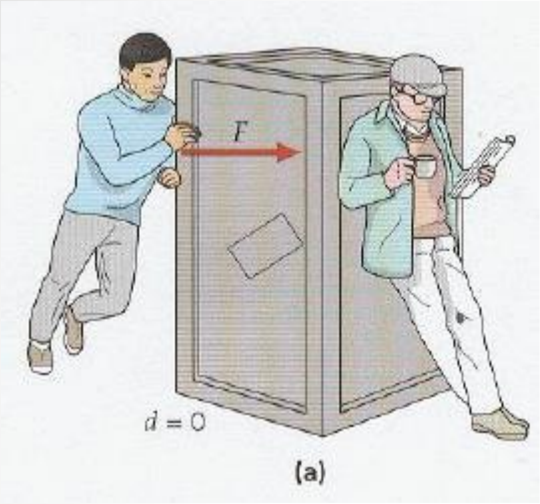
2. Work can be zero or positive or negative

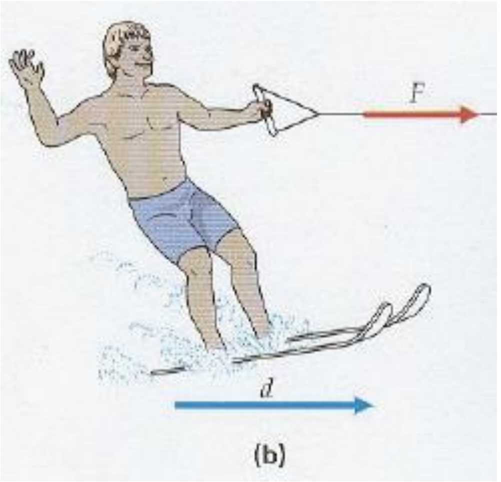
3. Work is scalar quantity.

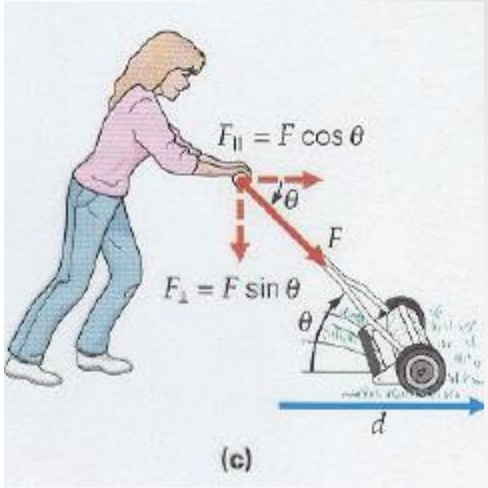
4. Work has the SI unit of joule

$$1\text{ joule} = 1\text{ J} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$$

Work Done by a Constant Force







If there is no displacement, no work is done:

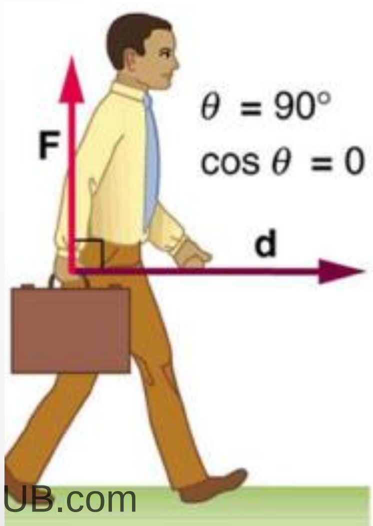
$$W = 0$$

For a constant force in the same direction as the displacement,

$$W = Fd$$

For a constant force at an angle to the displacement,

$$W = \vec{F}.\vec{d} = (F\cos\theta)d$$



For a constant force at an angle $\theta = 90^{\circ}$ to the displacement, $W = 0$

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

- For two or more forces, the **net work** is the sum of the works done by all the individual forces
- Two methods to calculate net work:
  - We can find all the works and sum the individual work terms.
  - We can take the vector sum of forces (F_{net}) and calculate the net work once

Work-Kinetic energy theorem states:

$$\Delta K = K_f - K_i = W$$

(*Change in kinetic energy*) = (*The net work done on the particle*)

we can write it as:

$$K_f = K_i + W$$

Example If the kinetic energy of a particle is initially 5 J:

- A net transfer of 2 J to the particle (positive work)
  - $K_f = 7$ J
- A net transfer of 2 J from the particle (negative work)
  - $K_f = 3$ J

Checkpoint 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from −3 m/s to −2 m/s and (b) from −2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

The **work-kinetic energy theorem** states:

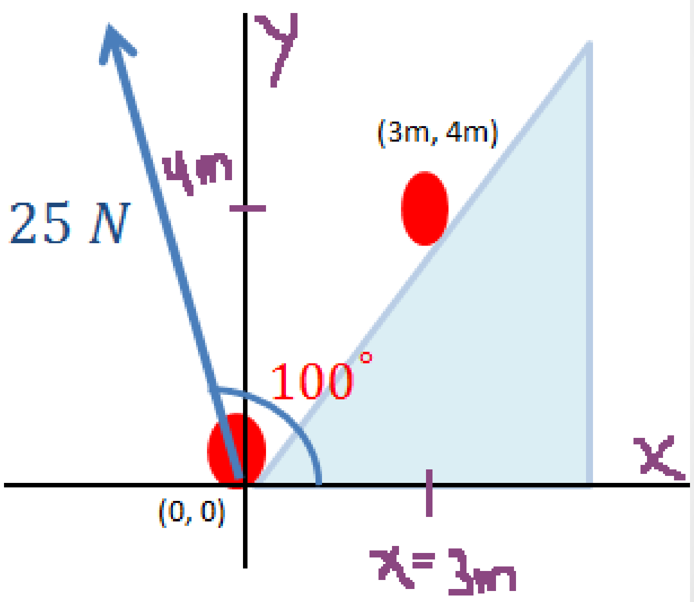
$$\Delta K = K_f - K_i = W$$

Answer: (a) Energy decreases

(b) Energy remains the same

(c) work is negative for (a) and work is zero for (b)

•10 A coin slides over a frictionless plane and across an xy coordinate system from the origin to a point with xy coordinates (3.0 m, 4.0 m) while a constant force acts on it. The force has magnitude 2.0 N and is directed at a counterclockwise angle of 100° from the positive direction of the x axis. How much work is done by the force on the coin during the displacement?



$$W = \vec{F}.\vec{d} = F_x \Delta x + F_y \Delta y$$

$$\vec{F} = 2\,N(\cos 100°\,\hat{\imath} + \sin 100°\,\hat{\jmath})$$

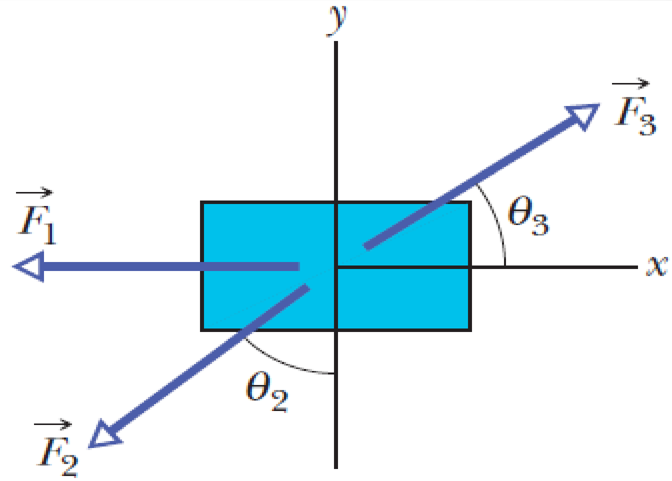
$$\vec{F} = (-0.347\,N)\hat{\imath} + (1.97N)\hat{\jmath}$$

$$\vec{d} = (3\,m)\hat{\imath} + (4\,m)\hat{\jmath}$$

$$W = \vec{F}.\vec{d}$$

$$W = (-0.347\,N)(3\,m) + (1.97N)(4\,m) = +6.84\,J$$

••14 GO Figure 7-27 shows an overhead view of three horizontal forces acting on a cargo canister that was initially stationary but now moves across a frictionless floor. The force magnitudes are $F_1 = 3.00$ N, $F_2 = 4.00$ N, and $F_3 = 10.0$ N, and the indicated angles are $\theta_2 = 50.0°$ and $\theta_3 = 35.0°$. What is the net work done on the canister by the three forces during the first 4.00 m of displacement?



$$F_{\text{net}\,x} = -F_1 - F_2 \sin 50.0° + F_3 \cos 35.0° = -3.00\,\text{N} - (4.00\,\text{N})\sin 35.0° + (10.0\,\text{N})\cos 35.0° = 2.13\,\text{N}$$

$$F_{\text{net}\,y} = -F_2 \cos 50.0° + F_3 \sin 35.0° = -(4.00\,\text{N})\cos 50.0° + (10.0\,\text{N})\sin 35.0° = 3.17\,\text{N}.$$

$$F_{\text{net}} = \sqrt{F^2_{\text{net}\,x} + F^2_{\text{net}\,y}} = \sqrt{(2.13\,\text{N})^2 + (3.17\,\text{N})^2} = 3.82\,\text{N}$$

$$\theta = 56.1^{\circ}$$

$$W = F_{\text{net}} d = (3.82\,\text{N})(4.00\,\text{m}) = 15.3\ \text{J}$$

The canister started from rest and moved 4.0 m in the same direction of F_{net}.

7-3 Work Done by the Gravitational Force

The work W_g done by the gravitational force $\vec{F}_g$ on a particle-like object of mass m as the objects moves through a displacement $\vec{d}$ is given by

$$W_g = mgd\cos\emptyset$$

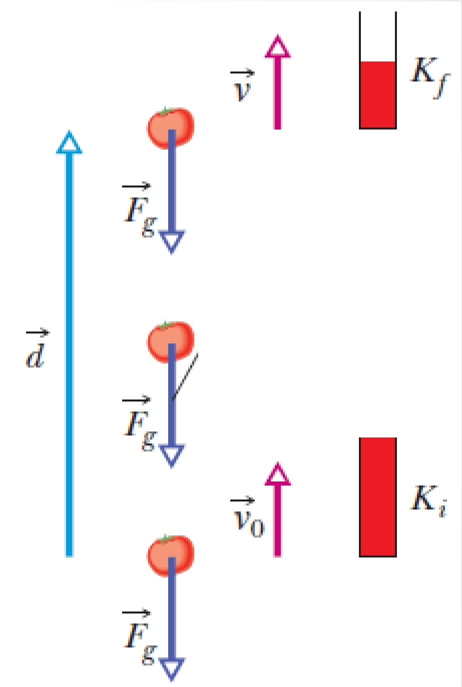
In which $\emptyset$ is the angle between $\vec{F}_g$ and $\vec{d}$

- For a rising object:

$$W_g = mgd\cos 180° = mgd(-1) = -mgd$$

- For a falling object:

$$W_g = mgd\cos 0° = mgd(+1) = +mgd$$



The force does *negative* work, decreasing speed and kinetic energy.

- Work done in lifting or lowering an object, applying an upwards force:

$$\Delta K = K_f - K_i = W_a + W_g$$

- For a stationary object:
  - Kinetic energies are zero
  - We find:

$$W_a + W_g = 0$$

$$W_a = -W_g.$$

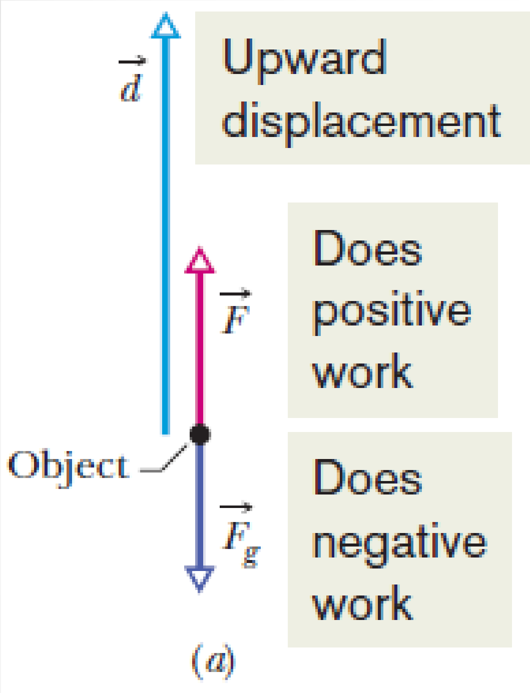
which tells us that the applied force transfers as much energy to the object as the gravitational force transfers from it.

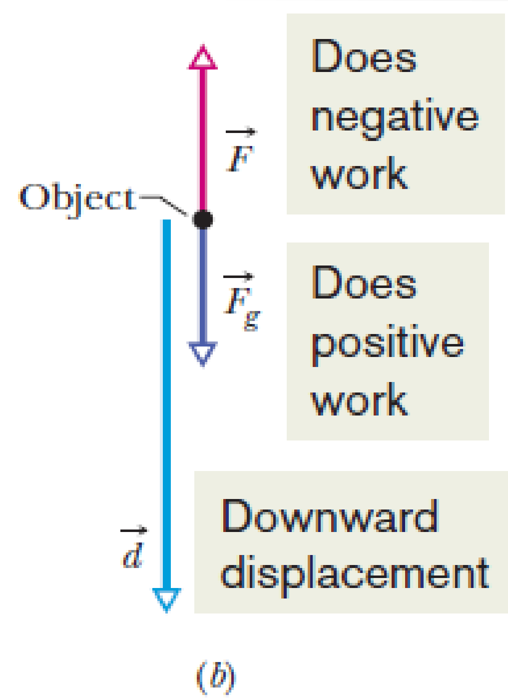
- In other words, for an applied lifting force:

$$W_a = -mgd\cos\phi \qquad \text{(work done in lifting and lowering; } K_f = K_i\text{)}$$

- Applies regardless of path

Work Done in Lifting and Lowering an Object



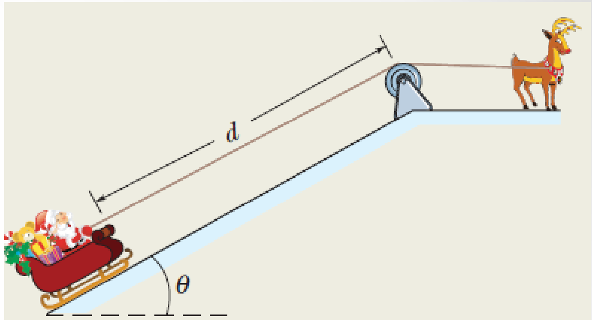


(*a*) An applied force $\vec{F}$ lifts an object. The object's displacement $\vec{d}$ makes an angle $\phi = 180°$ with the gravitational force $\vec{F}_g$ on the object. The applied force does positive work on the object.

(*b*) An applied force $\vec{F}$ lowers an object. The displacement $\vec{d}$ of the object makes an angle $\phi = 0°$ with the gravitational force $\vec{F}_g$. The applied force does negative work on the object.

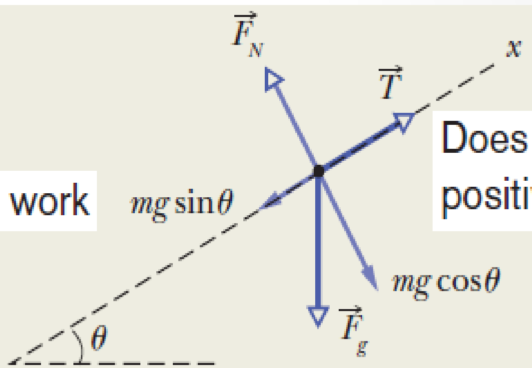
Sample Problem 7.04 Work in pulling a sleigh up a snowy slope

In this problem an object is pulled along a ramp but the object starts and ends at rest and thus has no overall change in its kinetic energy (that is important). Figure 7-8*a* shows the situation. A rope pulls a 200 kg sleigh (which you may know) up a slope at incline angle $\theta = 30°$, through distance $d = 20$ m. The sleigh and its contents have a total mass of 200 kg. The snowy slope is so slippery that we take it to be frictionless. How much work is done by each force acting on the sleigh?



Work W_N by the normal force.

$$W_N = F_N d \cos 90° = 0$$



Does negative work

Does positive work

Work W_g by the gravitational force.

$$W_g = F_g d \cos 120° = mgd \cos 120°$$
$$= (200\text{ kg})(9.8\text{ m/s}^2)(20\text{ m}) \cos 120°$$
$$= -1.96 \times 10^4\text{ J}.$$

$$F_{net,x} = ma_x$$
$$F_T - mg \sin 30° = m(0)$$
$$F_T = mg \sin 30°$$

Work W_T by the rope's force.

$$\Delta K = W = \text{zero}$$

$$0 = W_N + W_g + W_T = 0 - 1.96 \times 10^4\text{ J} + W_T$$
$$W_T = 1.96 \times 10^4\text{ J}.$$

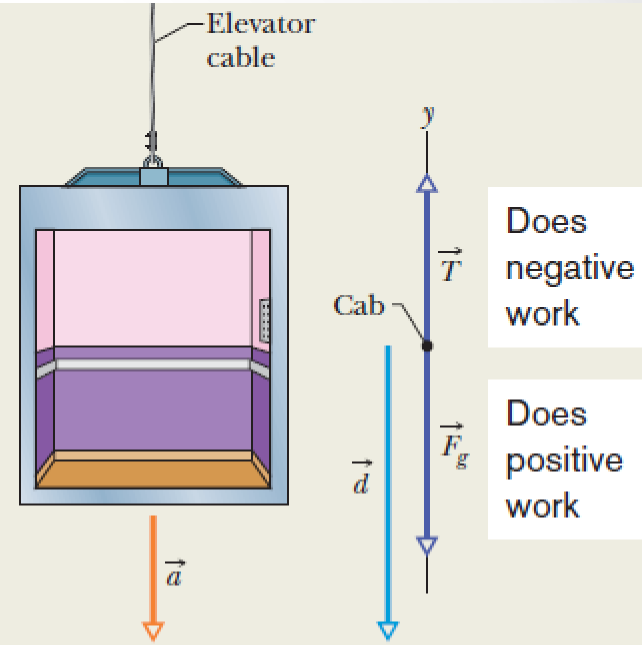
$$W_T = F_T d \cos 0° = (mg \sin 30°) d \cos 0°$$
$$= (200\text{ kg})(9.8\text{ m/s}^2)(\sin 30°)(20\text{ m}) \cos 0°$$
$$= 1.96 \times 10^4\text{ J}.$$

Sample Problem 7.05 Work done on an accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

$$W_g = mgd\cos 0° = (500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(1) = 5.88 \times 10^4\text{ J} \approx 59\text{ kJ}.$$



(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

$$F_g - T = ma.$$

$$W_T = Td\cos\phi = m(g - a)d\cos\phi.$$

$$W_T = m\left(-\frac{g}{5} + g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$
$$= \frac{4}{5}(500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\cos 180°$$
$$= -4.70 \times 10^4\text{ J} \approx -47\text{ kJ}.$$

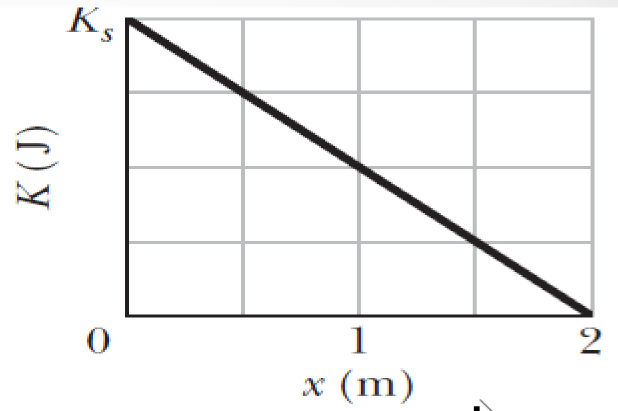
(c) What is the net work W done on the cab during the fall?

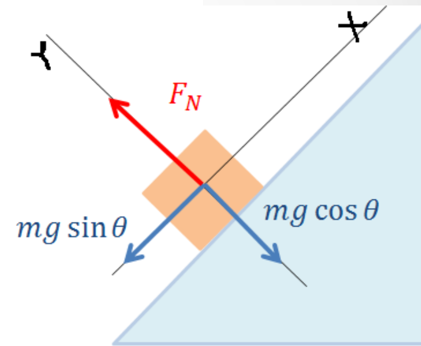
$$W = W_g + W_T = 5.88 \times 10^4\text{ J} - 4.70 \times 10^4\text{ J} = 1.18 \times 10^4\text{ J} \approx 12\text{ kJ}.$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500\text{ kg})(4.0\text{ m/s})^2 + 1.18 \times 10^4\text{ J}$$
$$= 1.58 \times 10^4\text{ J} \approx 16\text{ kJ}.$$

••20 A block is sent up a frictionless ramp along which an x axis extends upward. Figure 7-31 gives the kinetic energy of the block as a function of position x; the scale of the figure's vertical axis is set by $K_s = 40.0$ J. If the block's initial speed is 4.00 m/s, what is the normal force on the block?





By Work-Kinetic Energy Theorem:

$$W = \Delta K = \vec{F}.\vec{d} = F_x \Delta x$$

Slope of K versus x graph:

(0m, 40J), (2m, 0J)

$$slope = \frac{-40J}{2m} = -20\ J/m$$

$$F_x = -20\ N = -mg\sin\theta$$

No motion in y- direction:

$$F_N = mg\cos\theta$$

The block initial speed is $v_i = 4\ m/s$,

$$K_i = 40\ J = \frac{1}{2} m\, {v_i}^2$$

$$m = \frac{2\,K_i}{{v_i}^2} = \frac{2(40\ J)}{(4m/s)^2} = 5kg$$

Use

$$F_x = -20\ N = -mg\sin\theta = -(5kg)(9.8\,m/s^2)sin\theta$$

$$\theta = 24.09^{\circ}$$

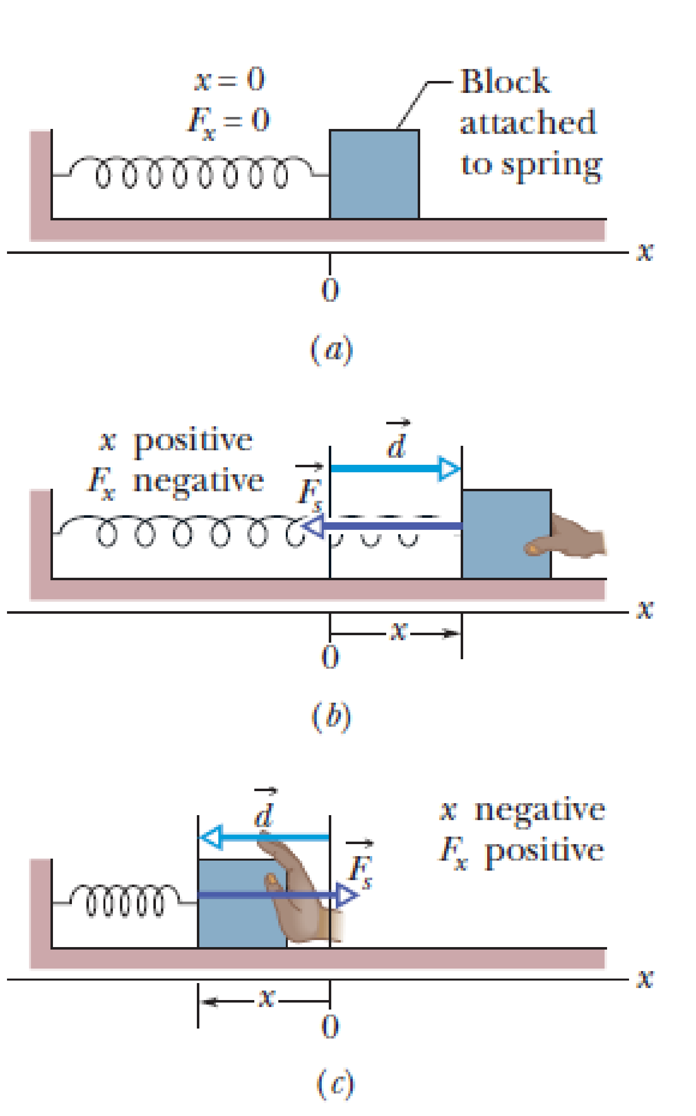
$$F_N = mg\cos\theta$$

$$F_N = (5kg)(9.8\,m/s^2)\cos(24.09^{\circ})$$

$$F_N = 44.73N$$

7-4 Work Done by a Spring Force

- A **spring force** is the *variable force* from a spring
  - A spring force has a particular mathematical form
  - Many forces in nature have this form
- Figure (a) shows the spring in its **relaxed state**: since it is neither compressed nor extended, no force is applied
- If we stretch or extend the spring it resists, and exerts a *restoring force* that attempts to return the spring to its relaxed state



- The spring force is given by **Hooke's law**:

$$\vec{F}_s = -k\vec{d}$$

- The negative sign represents that the force always opposes the displacement
- The **spring constant** *k* is a is a measure of the stiffness of the spring
- This is a variable force (function of position) and it exhibits a linear relationship between *F* and *d*
- For a spring along the *x*-axis we can write:

$$F_x = -kx$$

The Work Done by a Spring Force

- We can find the work by integrating: $$W_s = \int_{x_i}^{x_f} F_x \, dx$$

$$W_s = \int_{x_i}^{x_f} -kx \, dx = -k \int_{x_i}^{x_f} x \, dx$$

$$= (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2)$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \quad \text{(work by a spring force)}$$

  - Can be positive or negative
  - Depends on the *net* energy transfer

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

- For an initial position of $x = 0$:

$$W_s = -\frac{1}{2}kx^2$$

- For an applied force where the initial and final kinetic energies are zero (Stationary block):

$$W_a = -W_s$$

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

Checkpoint 2

For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7-10 are (a) −3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) −2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?

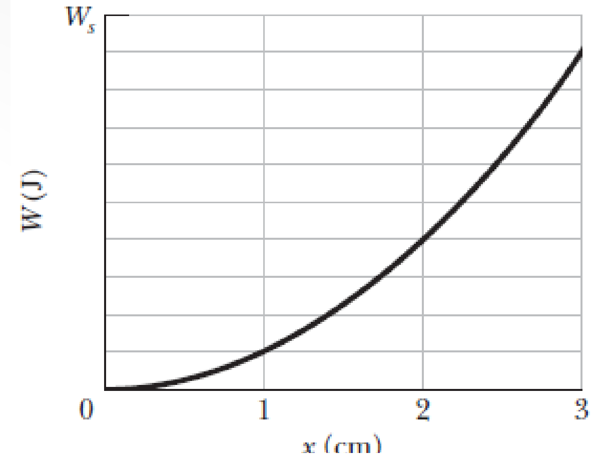
Answer: (a) positive

(b) negative

(c) zero

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \quad \text{(work by a spring force)}$$

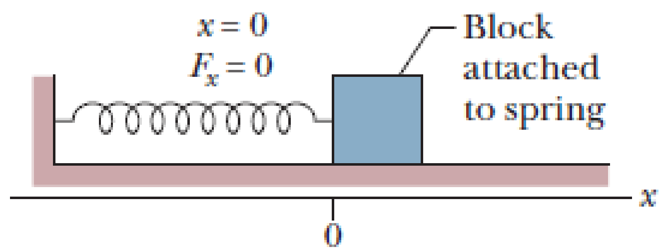
••29 In the arrangement of Fig. 7-10, we gradually pull the block from $x = 0$ to $x = +3.0$ cm, where it is stationary. Figure 7-35 gives the work that our force does on the block. The scale of the figure's vertical axis is set by $W_s = 1.0$ J. We then pull the block out to $x = +5.0$ cm and release it from rest. How much work does the spring do on the block when the block moves from $x_i = +5.0$ cm to (a) $x = +4.0$ cm, (b) $x = -2.0$ cm, and (c) $x = -5.0$ cm?



$$W_a = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$$x_i = 0 \rightarrow x_f = +3\ cm$$

$$0.9\ \text{J} = \frac{1}{2}k(0.03)^2$$



$$k = 2\ kN/m$$

a) $x_i = +5.0\ cm \rightarrow x_f = +4.0\ cn$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}\left(2000\ \frac{N}{m}\right)\left((0.05\ m)^2 - (0.04\ m)^2\right) = 0.9\ J$$

b) $x_i = +5.0\ cm \rightarrow x_f = -2.0\ cm$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}\left(2000 \times 10^3\ \frac{N}{m}\right)\left((0.05\ m)^2 - (-0.02\ m)^2\right) = 2.1\ J$$

c) $x_i = +5.0\ cm \rightarrow x_f = -5.0\ cm$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}\left(2000 \times 10^3\ \frac{N}{m}\right)\left((0.05\ m)^2 - (-0.05\ m)^2\right) = zero$$

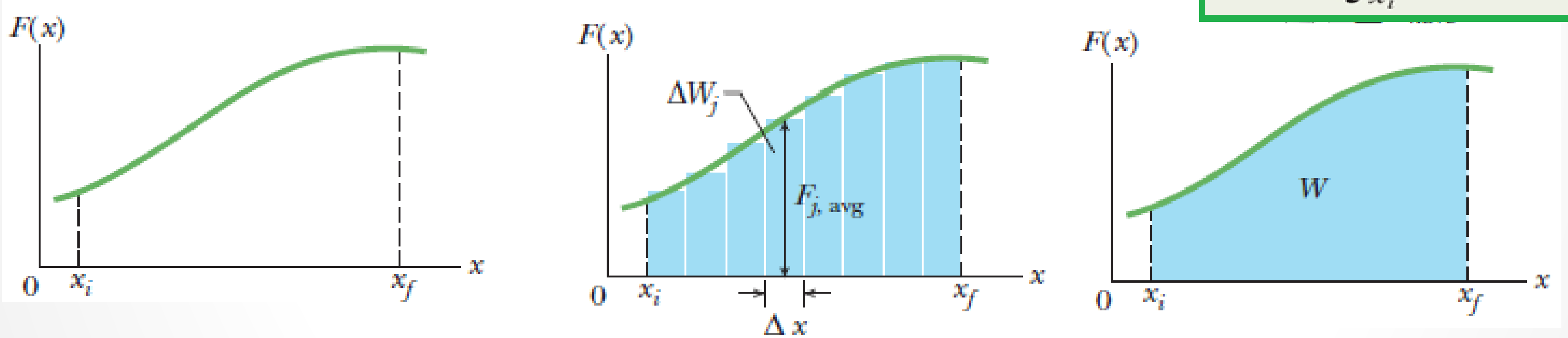
7-5 Work Done by a General Variable Force

- We take a one-dimensional example
- We need to integrate the work equation (which normally applies only for a constant force) over the change in position
- We can show this process by an approximation with rectangles under the curve

Work is equal to the area under the curve.

$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}}\, \Delta x$$

$$W = \int_{x_i}^{x_f} F(x)\, dx$$



In three dimensions, we integrate each separately

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy + \int_{z_i}^{z_f} F_z\, dz.$$

- The work-kinetic energy theorem still applies!

Work–Kinetic Energy Theorem with a Variable Force

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx$$

$$ma\,dx = m\frac{dv}{dt}\,dx$$

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$$

$$ma\,dx = m\frac{dv}{dx}v\,dx = mv\,dv$$

$$W = \int_{v_i}^{v_f} mv\,dv = m\int_{v_i}^{v_f} v\,dv$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$W = K_f - K_i = \Delta K$$

The work-kinetic energy theorem still applies!

Sample Problem 7.08 Work, two-dimensional integration

Force $\vec{F} = (3x^2\text{ N})\hat{\imath} + (4\text{ N})\hat{\jmath}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

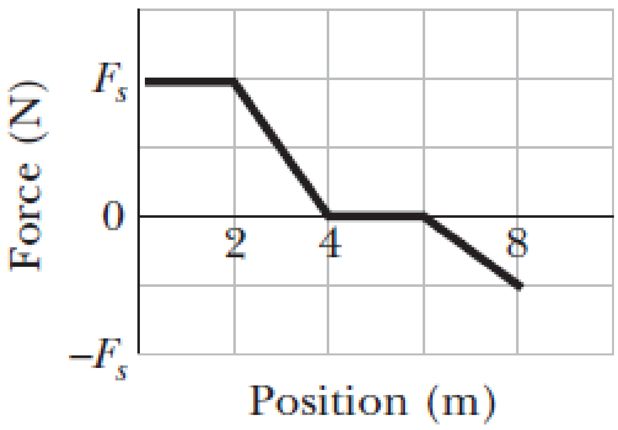
$$W = \int_2^3 3x^2\,dx + \int_3^0 4\,dy = 3\int_2^3 x^2\,dx + 4\int_3^0 dy$$

$$= 3[\tfrac{1}{3}x^3]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3]$$

$$= 7.0\text{ J.}$$

The positive result means that energy is transferred to the particle by force $\vec{F}$. Thus, the kinetic energy of the particle increases and, because $K = \frac{1}{2}mv^2$, its speed must also increase. If the work had come out negative, the kinetic energy and speed would have decreased.

•36 GO A 5.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in Fig. 7-39. The scale of the figure's vertical axis is set by $F_s = 10.0$ N. How much work is done by the force as the block moves from the origin to $x = 8.0$ m?



Work done by a variable force:

$$W = \int_{x_i}^{x_f} F_x \, dx = Area\ under\ the\ curve$$

$$W = \left(+\frac{1}{2}(4+2)(10)\right) + \left(-\frac{1}{2}(2)(5)\right)$$

$$W = +30 - 5 = +25\,J$$

Example: A particle of mass 0.02 kg moves along a curve with velocity $(5\hat{\imath} + 18\hat{k})m/s$. After some time, the velocity changes to $(9\hat{\imath} + 22\hat{\jmath})m/s$ due to action of a single force. Find the work done on the particle during this interval of time?

$$\vec{v}_i = (5\hat{\imath} + 18\hat{k})\, m/s \rightarrow \vec{v}_f = (9\hat{\imath} + 22\hat{\jmath})\, m/s$$

$$v_i^2 = (5)^2 + (18)^2 = 349(m/s)^2 \qquad v_f^2 = (9)^2 + (22)^2 = 565(m/s)^2$$

$$\text{Work done on the particle} = \Delta K = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

$$W_{done} = \Delta K = \frac{1}{2}(0.02)(565 - 349) = 2.16\, J$$

7-6 Power

- **Power** is the time rate at which a force does work
- A force does *W* work in a time Δt, the **average power** due to the force is:

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

- The **instantaneous power** at a particular time is:

$$P = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right) = Fv\cos\phi$$

$$P = \vec{F}\cdot\vec{v} = Fv\cos\phi$$

- The SI unit for power is the watt (W): $1\ \text{W} = 1\ \text{J/s}$

$$1\ \text{horsepower} = 1\ \text{hp} = 746\ \text{W}$$

- Work can be expressed as power multiplied by time, as in common unit kilowatt-hour:

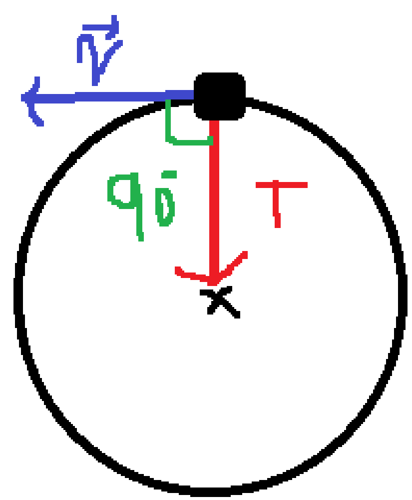
$$1 \text{ kilowatt-hour} = 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s})$$
$$= 3.60 \times 10^6 \text{ J} = 3.60 \text{ MJ}$$

Checkpoint 3

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Answer: Zero

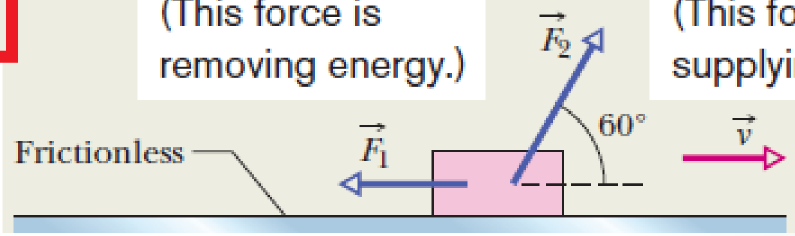
$P = Fv \cos \phi = 0 \quad (\phi = 90°)$



Sample Problem 7.09 Power, force, and velocity

$\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

$$P_1 = F_1 v \cos\phi_1 = (2.0\text{ N})(3.0\text{ m/s})\cos 180^\circ = -6.0\text{ W}.$$



The negative result tell us that force $\vec{F}_1$ is transferring energy from the box at the rate of 6 J/s.

$$P_2 = F_2 v \cos\phi_2 = (4.0\text{ N})(3.0\text{ m/s})\cos 60^\circ = 6.0\text{ W}.$$

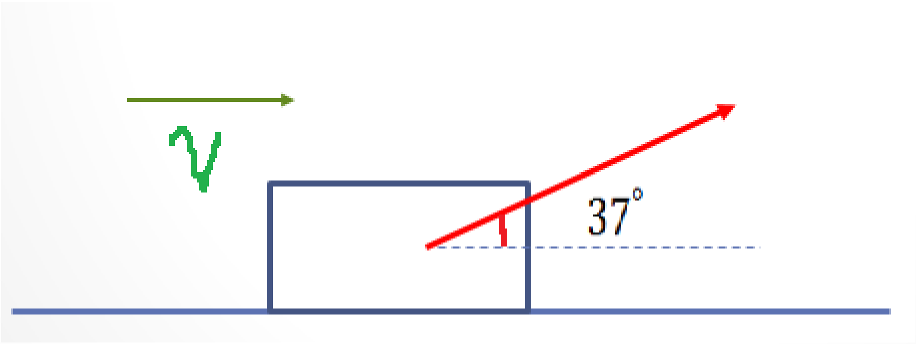
The positive result tell us that force $\vec{F}_2$ is transferring energy to the box at the rate of 6 J/s.

$$P_{net} = P_1 + P_2 = -6.0\text{ W} + 6.0\text{ W} = 0$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s.

•45 SSM ILW A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block?

$$P = \vec{F} \cdot \vec{v} = Fv\,cos\emptyset = (122N)\left(\frac{5m}{s}\right)\cos 37^{\circ} = 487\ watt$$



•46 The loaded cab of an elevator has a mass of 3.0×10^3 kg and moves 210 m up the shaft in 23 s at constant speed. At what average rate does the force from the cable do work on the cab?

The Average Rate of the Work Done by the cable on the Cab

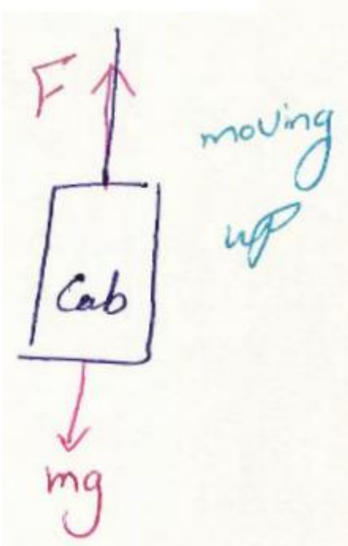
$\Rightarrow$ Power: $P = \vec{F} \cdot \vec{v} = F v \cos\theta$

- moving up with a constant speed

$$a = \text{zero} \longrightarrow F_{net} = \text{zero}$$

$$F = mg$$

$\theta = 0°$ $\Rightarrow$ Elevator moves up ward and the force of the cable is upward.



$$v = \frac{\Delta x}{\Delta t} = \frac{210\ m}{23\ s} = 9.13\ m/s$$

$$P = F v \cos\theta$$

$$P = mg \frac{\Delta x}{\Delta t} \cos(0°)$$

$$= 3.0 \times 10^3 (9.8)(9.13) = 268434.78 \text{ Watt}$$

$$P = 2.7 \times 10^5 \text{ watt}$$