Math 1351

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Good luck for all

❤️

Chapter 0

(Section 1)

Sets :- مجموعات

عناصر ← معرفة بشكل واضح ← مجموعة

Definition Sets :- is a collection of well-defined objects.

مجموعة من العناصر معرفة بشكل واضح.

Examples :-

إناث

A = The set of all female students in this lecture

من الأعداد فردية

B = The set of all odd numbers between 2 and 8. ⇒ {3, 5, 7}

الأعداد الطبيعية

C = The set of Natural numbers. الأعداد طبيعية :- هي الأعداد الصحيحة الموجبة
وهي {1، 2، 3، ...}

رقم (الرمز) عدد

D = The set of all the digits of the number 2620.

في حال [illegible] (25) [illegible] digits [illegible] الأعداد
[illegible] رمز [illegible]

digits ← [illegible] ← 2620

رموز ← {2, 6, 0}. [illegible] 5 ∉ المجموعة

ملاحظة
∈ ← تنتمي
∉ ← لا تنتمي

* There are ~~tow~~ two ways to write a set :- يوجد طريقتين لتعبير عن المجموعات

1- By listing its elements :- طريقة الأولى سرد العناصر

A = {Nour, Abear, ---, Aya}
B = {3, 5, 7}
C = {1, 2, 3, 4, 5, ..., ∞}
D = {2, 6, 0} = {0, 6, 2} → غير مهم ترتيب في العناصر
المهم ذكر العناصر الموجودة وعدم تكرار العناصر أكثر من مرة.

⇒

✳ There are two ways to write a set :-

2- By describing its elements:- طريقة الصفة المميزة أو وصف العناصر

$A = \{X : X \text{ is a F student in this Lec}\}$ يتم تعبير عن المجموعة بأي رمز وبعد ذلك يتم سرد للتوضيح الرمز .

$B = \{X : X \text{ is odd and } 2 < X < 8\}$

$= \{3, 5, 7\}$.

أكبر . أصغر

استخدام الوصف بأي لغة المهم توصيل للمعلومة .

$C = \{X : X \text{ is a natural number}\}$.

$D = \{X : X \text{ is adigits of } 2620\}$ يمكن كتابة هذه الرقم اعمم أنو يكونوا موجودين الارقام

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✳ To indicate that an element belong (or dose not belong) to a set we use the symbols, ∈ and ∉ respectively.

$5 \in B$

$5 \notin D$

Definitions:- تعريف

1 → A set is finite (محدود) if all its elements can be listed.

Ex:- $A = \{1, 3, 5\}$ Finite. $C = \{1, 2, 3, \ldots, 9999\}$ Finite

Finite :- مجموعة محدودة .

Definitions :-

غير محدود

2 → A set is Infinite if all of it's elements can not be listed.

Ex:- $A = \{x : x \text{ is odd number}\}$ Infinite ← لأنه يوجد أعداد لا نهائية من الأعداد الفردية.

إسم

$B = \{x : x \text{ is } 1 < x < 3\}$ Infinite ← لأنه يوجد أعداد طبيعية كثيرة احتمال أن X ← يكون 1.5 أو 1.3 أو 2.5 ← مش شرط يكون العدد 2 لذلك غير محدود

فارغ

3 → A set is empty (Null) if it has no elements.

Ex:- $A = \{\ \} = \phi$ → فاي ← يعني مجموعة فارغة

$B = \{x : x \text{ is a chinese student in this Lectures}\} = \{\ \} = \phi$

← لأنه حسب الطلاب داخل المحاضرة لأنه لا يوجد أي طالب صيني لذلك المجموعة فارغة.

Question

Indicate whether the following sets are Finite or inFinite and write them in a second way :- اكتبها بطريقة أخرى

الأعداد الطبيعية أقل من 8

1- $\{x : x \text{ is a natural number less than } 8\}$ Finite محدودة

الطريقة الأخرى هي :- $\{1, 2, 3, 4, 5, 6, 7\}$

2- $\{10, 12, 14, \dots\}$ inFinite :- غير محدودة

الطريقة الأخرى هي $\{x : x \geq 10 \ \& \ x \text{ even}\}$,

$\{x : x \ \ x > 8 \ \& \ x \text{ is even}\}$

ملاحظة ← حيث أن even ← هو عدد زوجي.

Relations between sets :- العلاقات بين المجموعات

المساواة ← يعني يساوي

1 → ~~Tow~~ Two sets are equal if they contain the same elements. ←

Ex:- If $A = \{1, 2, a, b\}$ and $B = \{a, b, 1, 2\}$ then $A = B$

* امكان كتابة وحدة بطريقة السرد ووحدة بطريقة الوصف .

مشتركة [illegible] متقاطعين (غير منفصل)

2 → ~~Twos~~ Two sets are disjont if they have no common elements.

Ex:- If $A = \{1, 2, 3, 9\}$ and $B = \{1/2, 4, 8\}$

→ A and B are disjont

أما إذا كان يوجد عدد واحد فقط مشترك نضع jont بدل disjont .

مجموعة جزئية

3 → The set A is a Subset of the set B (denoted $A \subseteq B$) if every element of A is an element of B.

أعداد فردية — الأعداد الفردية

Ex:- $A = \{5, 9, 71\}$, $B = \{X : X \text{ is an odd number}\}$, $C = \{1, 2, 3\}$

$A \subseteq B \rightarrow$ B كل عناصر A موجودة في / $C \not\subseteq B$ / $A \not\subseteq C$

1- ملاحظة : $\boxed{\subseteq}$ ← مجموعة جزئية / $\boxed{\not\subseteq}$ ← مجموعة غير جزئية .

كل مجموعة جزئية من نفسها جزئية جزئية جزئية

Notes:- Every set is a subset of its self ($A \subseteq A$, $B \subseteq B$, $C \subseteq C$)

المجموعة الفارغة هي مجموعة جزئية من

The empty (Null) set is a subset of any set ($\emptyset \subseteq A$, $\emptyset \subseteq B$, $\emptyset \subseteq C$)

$\not\subseteq$

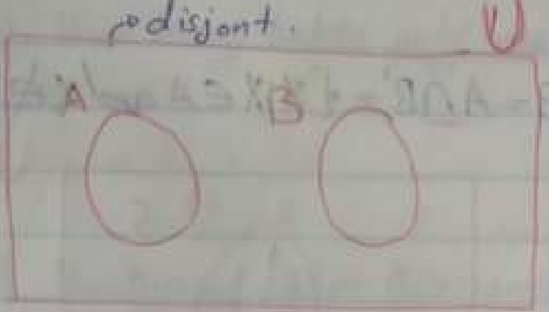
جمع وطرح الاعداد الحقيقية

Sets Operations and Venn diagrams أشكال فن

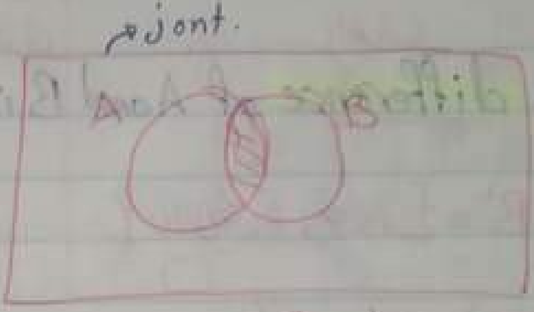
نفرض دائماً المجموعة جزئية من مجموعة أعظم نسميها U

By:- assumption, each set is subset of a largset called the Universal set. This set is represented by a rectangle in Venndiagrams. While the subsets are represented by circles. المجموعات الجزئية

Ex:- If A and B are sets, than the Venn disgram that represents them is



غير متقاطعة



متقاطعة

* Now do the following example to explain the set operations.

Ex:- If $U=\{X:X\in N \text{ and } x<11\}$, $A=\{2,4,7,9\}$

$\hookrightarrow =\{1,2,3,4,5,6,7,8,9,10\}$

$B=\{X:X$ is a Natural number less than $6\}$. Then.

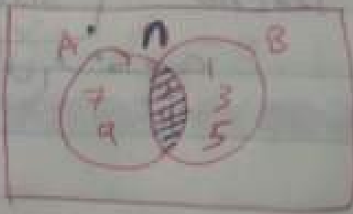
$\hookrightarrow =\{1,2,3,4,5\}$

تقاطع بينهما

* The intersection of A and B is $A\cap B=\{X:X\in A \text{ and } X\in B\}$.

$\cap$

$A\cap B=\{2,4\}$



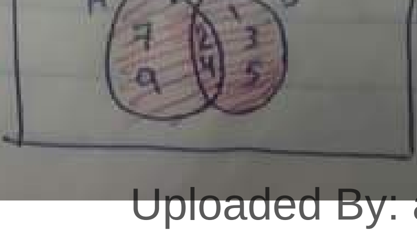
ملاحظة:-

اتحاد

* The union of A and B is $A\cup B=\{X:X\in A \text{ or } X\in B \text{ (or both)}\}$ ⇒ A اتحاد B = عندما يكون X عنصر من عناصر A او عنصر من عناصر B او كليهما

$A\cup B=\{2,4,7,9,1,3,5\}$

* عدم تكرار الأرقام المكررة في المجموعة

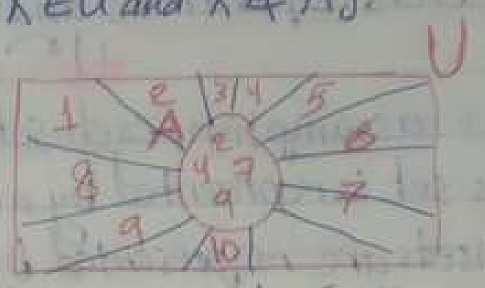


يعتمد على الحل
⟸

متممة

* The **complement** of A is $A' = \{x : x \in U \text{ and } x \notin A\}$.

$A' = \{1, 3, 5, 6, 8, 10\}$

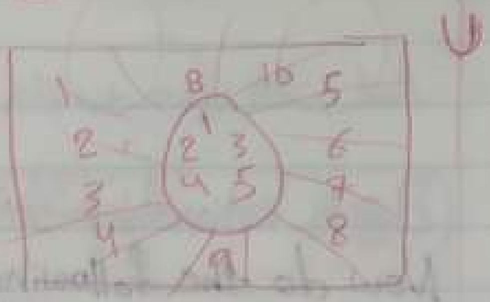


ملاحظة: كل اشي خارج A بس
يكون داخل U يكون هو complement
A يعني هو متممة A ← A'

الفرق   يعني الفرق

* The **difference** of A and B is $A - B = A \cap B' = \{x : x \in A \text{ and } x \notin B\}$.

$B' = \{6, 7, 8, 9, 10\}$



$= \begin{cases} A \cap B' = \{7, 9\} \\ A - B = \{7, 9\} \end{cases}$

* $B - A = \{1, 3, 5\} = B \cap A' = \{1, 3, 5\}$

عبارة عن العناصر الموجودة في B غير موجودة في A .

لغز لفهم السؤال :-
يوجد عنا قاضي وزوجته
ويوجد عنا استاذ واخته .
ذهبوا الى مطعم وطلبوا
٣ وجبات من الطعام و
كل واحد طلعله وجبة كاملة .
الحل الزوجة القاضي هي نفسها
اخت الاستاذ .

Judge	wife
Teacher	sister

$\cap$ نفس الشخصية

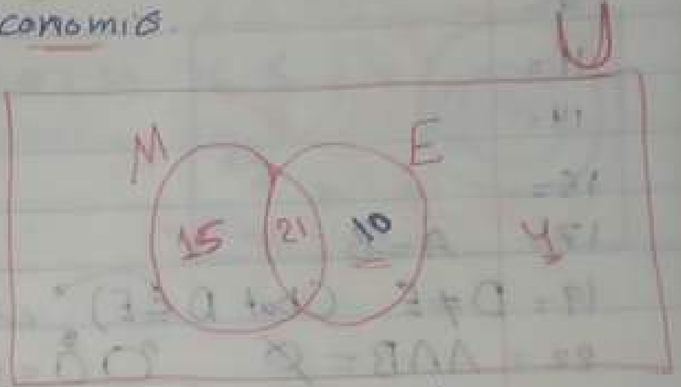
Application example :- مثال تطبيقي

* Records at a small college show the following about the enrollments of 50 first-years students in math and Economics.

→ 36 take math.

→ 21 take math and economics.

→ 4 do not take neither math nor economics.



a)- How many takes only math ?

↳ 36 − 21 = 15

15 + 21 = 36

b)- How many takes economicse ?

10 + 21 = 31

c)- How many takes only economicse ?

↳ 10

لأنه 15 ← بأخذو math فقط

و 21 ← بأخذو economice and math

و 4 ← طلاب ما أخذو اشي

40 = 4 + 21 + 15

50 − 40 = 10

↳ هو عدد الطلاب الي ما أخذو economice فقط

فعدد الطلاب الي بأخذو economice = 21 + 10 = 31 .

ملاحظات مهمة جداً جداً جداً :-

→ $A \cup \emptyset = A$

→ $A \cap \emptyset = \emptyset$

→ $A \cap A' = \emptyset$

→ $A + A' =$ Universal set.

→ $A - B = A \cap B'$

→ $(A')' = A$

→ $(A \cup B)' = A' \cap B'$

→ $(A \cap B)' = A' \cup B'$

→ $U' = \emptyset$

مهم

Outline

*حل ال

1. $12 \in \{1, 2, 3, 4\}$.

$3 = 6 \notin \{x : x$ is natural number greater than $5\}$.

$4 = 3 \notin \emptyset$

$6 = \{7, 8, 9\}$.

$8 = \{x : x$ is nanatural number gerate than $6\}$.

9 =

11 =

14 =

15 =

$17 = \quad A = B$

$19 = D \neq E \quad (\text{but } D \subseteq E)$ مجموعة جزئية من

$22 = A \cap B = \emptyset \qquad \overset{A}{\bigcirc}\ \overset{B}{\bigcirc} = \{\} = \emptyset$

$25 = A \cap B = \emptyset \xrightarrow{\text{الرسم}} \quad A = \emptyset \quad B = \{x, y, a, b\}$ $\emptyset$ لا يوجد عناصر مشتركة

$26 = A \cap B = \{3\}$

$28 = A \cup B = \{a, e, i, o, u, b, c, d\}$.

$29 = A \cup B = \{1, 2, 3, 4\} = B$

$31 = A' = U - A = \{4, 6, 9, 10\}$.

$35 = (A \cup B)' = A \cup B = \{1, 3, 5, 8, 7, 2, 4, 10\}$

$(A \cup B)' = A \cup B - U = \{6, 9\}$

$40 = A \cap (B' \cup C') = \quad B' = \{1, 2, 5, 6, 7, 9\} \quad C' = \{1, 3, 5, 7, 9\}$.

$B' \cup C' = \{1, 2, 5, 6, 7, 9, 3\}$

$A \cap (B' \cup C') = \{1, 2, 3, 5, 7\}$

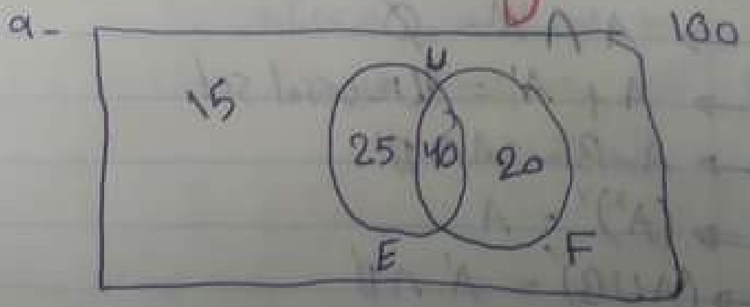
[illegible]

$43 = A - B = \{1, 7\}$

$46 = A - B = \{1, 2, 3, 4, 5\}$.

51 =

a-



b = 40

C = 85

d = 25

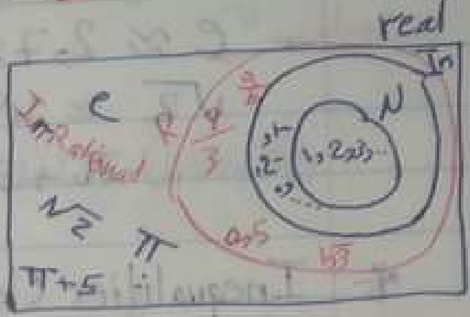
Section 0.2

The set of the real numbers can be represented by the **real number line** where each point on the line represents a real number.

مجموعة جزئية

* Subsets of the set of real numbers :

رمزها ⊆



1. Natural = الاعداد الطبيعية
   = {1, 2, 3, 4, ... ∞}.
   Natural ⊆ Real

2. Integers = الاعداد الصحيحة (هي الاعداد الطبيعية والصفر والسالبة وصفر)
   ↳ {..., 3-, 2-, 1-, 0, 1, 2, 3, ...}
   Integers ⊆ realnumber
   Natural ⊆ Integers

3. Rational number = الاعداد النسبية
   ↳ = { x : x = $\frac{a}{b}$, a, b are Integer and b ≠ 0 }.

Ex:- { $\frac{8}{3}$, $\frac{-2}{7}$, $4 = \frac{4}{1}$, $-19 = \frac{-19}{1}$, $0,5 = \frac{1}{2}$, $1,\overline{3} = \frac{4}{3}$ دوري

ملاحظة:- Any number decimal representation that either ends or repeates is rational number :- أي عدد مكتوب بالاعشار أو متكرار تعتبر من الاعداد النسبية مثل:

13,5678 → هو عدد كسري ٤ ٨ الاعشار منتهية
↳ $\frac{135678}{10000}$ هو عدد نسبي .

$2,\overline{67}$ → هو عدد نسبي .

4- Irrational = هي الاعداد الغير نسبية ، يعني ما بنقدر نكتبها على صورة كسر

$= \{x : x \in \mathbb{R} \text{ and } x \notin \text{ration}\}$

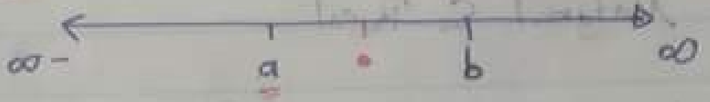
Ex :- $\pi \approx \frac{22}{7}$, 3,14 ----- هو ليس عدد نسبي

عدد نابيري $e \approx 2,7$ ------

$\sqrt{2} \approx 1,4$ -------

$\pi + 5 \rightarrow$ Irrational

* Inequalities ($<$, $\geq$)



$a \underset{\text{أصغر من}}{<} b$ / $b \underset{\text{أكبر من}}{>} a$

Ex :- $3 > 5$, $3 < 5$, $-2 < 0$, $[-2 > -4,5$ / $2,3 > 2,2222$

$\frac{9}{5} \boxed{>} \frac{11}{8}$

$1,8 \boxed{>} 1.375$

* Intervals :- الفترات

1- open Intervals :- الفترة المفتوحة (الاطراف غير داخلة ،)
↳ = $\{x : a<x<b\}$ a, b → نعرف انهم اي عددين حقيقيين (Real numbers)

Ex:- $\{x : x \ 2^- < x < 0\}$

]2-, 0[

2- closed Interval :- الفترة المغلقة (الاطراف داخلة)
↳ = $\{x : x \ a \leq x \leq b\}$.

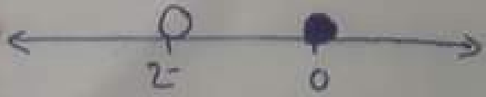
Ex:- $\{x : x \ 1 \leq x \leq 13\}$.

[1, 13]

3. half open interval هي الفترة الي يوجد فيها فترة مغلقة ومفتوحة (فيها طرف داخل وطرف ليس داخل)
↳ $\{x : x \ a \leq x < b\}$ / $\{x : x \ a < x \leq b\}$

Ex:- $[x : x \ 1 \leq x < 1,5]$
$\{x : x \ -2 < x \leq 0\}$

[1, 1,5[



]2-, 0]

Q :- write the types (نوعها) of the following intervals and express it in a secound and third way.

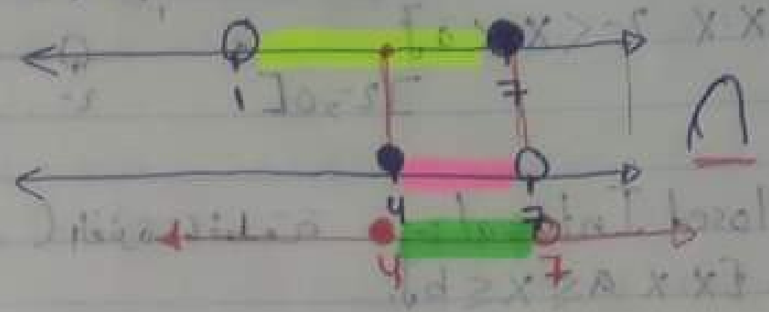
$+\infty > x \geq 3$

types :- half open.

[3, ∞[

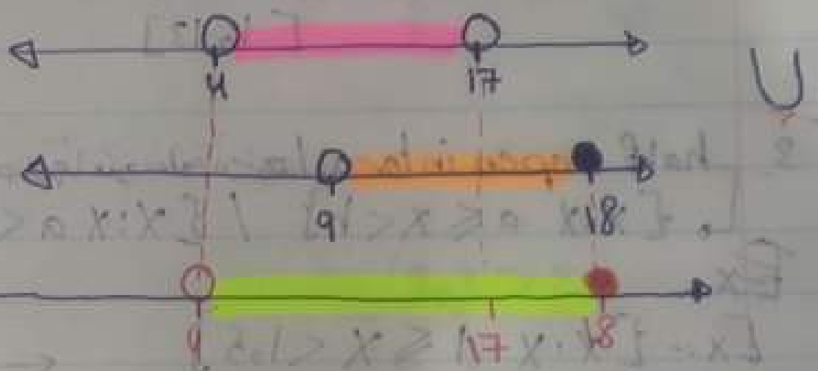
Q:- perform the following operations, and write your answer in interval notation :-

1). $1 < X \leq 7 \cap [4,7)$



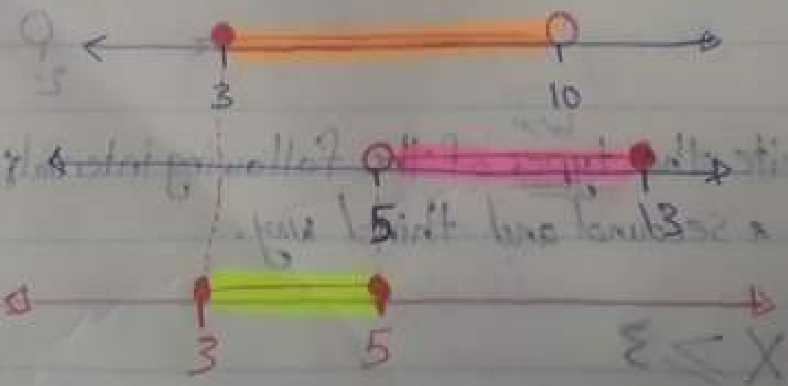
[(4,7)

2). $(4,17) \cup (9,18]$



.(4,18] =

Q:- $[3,10) - (5,13]$



= [3,5]

[redacted]

Absolute value and operations with real number

$$|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$$ القيمة المطلقة

$|-4| = 4$

$|16| = 16$

Ex:- $3+5 = 8$

$3-5 = -2$

$1 \overset{+}{\ominus} 2 = 3$

$(4) \times -2 = -8$

$-4 \times -2 = 8$

Q:- $2+(8 \div 2) =$ الاولوية دائماً للقسمة والضرب

$2+4 = 6$

الأولوية في العمليات :-

* ( ) parentheses للأقواس

* powers للقوة

* × or ÷ from left to right الضرب أو القسمة من اليسار لليمين

* − or + from left to right الطرح أو الجمع من اليسار لليمين

علي

homework:-

Q:- $\frac{(-3)^2 + |2-5|}{2(1-2) + 6 \div 3(2)}$

$= \frac{9 + |-3|}{2\times(-1) + 6 \div 3(2)} = \frac{9+3}{2(-1) + \underline{6 \div 3}(2)} =$

بنمشي من اليسار لليمين

$= \frac{12}{-2 + 2\times(2)}$

$= \frac{12}{-2+4} = \frac{12}{2} = \boxed{6}$

Section 0.3
Integral Exponents.

= Consider the expression $(1,5).(1,5).(1,5) \cdot (1,5) = (1,5)^4$

* In general, If a is a real number and n is positive integer, than $a^n = a.a.a.a \ldots a$ (n = factors), where n is the exponent الأس and a is the base.

Ex- $2^4 = 16$ $\qquad (-2)^4 = 16$

$-2^4 = -16$ $\qquad (-3)^3 = -27$

2 [illegible]

* Rules of exponents

For any real numbers a and b and any integers m and n.

1- $a^m . a^n = a^{m+n}$ $\quad$ Ex = $3^4 \times 3^{-2} = 3^{4+-2} = 3^2$ $\quad x^1 \times x^2 = x^{1+2} =$

2- $\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$ $\quad$ Ex = $\frac{4^7}{4^4} = 4^{7-4} = 4^3$ $\quad \frac{y^2}{y^{-1}} = y^{2\ominus 1} = y^3$

3- $(ab)^m = a^m . b^m$ $\quad$ Ex = $(3.2)^2 = 3^2 \times 2^2 = 9 \times 4 = 36$ $\quad (xy)^5 = x^5 . y^5$

4- $(a^m)^n = a^{m \cdot n}$ $\quad$ Ex = $(2^3)^2 = 2^{3 \times 2} = 2^6 = 64$ $\quad (x^2)^2 = x^{2 \times 2} = x^4$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $b \neq 0$ $\quad$ Ex = $\left(\frac{-2}{5}\right)^3 = \frac{(-2^3)}{(5)^3} = \frac{-8}{125}$ $\quad \left(\frac{x^2}{z^{-1}}\right)^3 = \frac{(x^2)^3}{(z^{-1})^3}$

6. $a^0 = 1$ $a \neq 0$ $\quad$ Ex = $45^0 = 1$ $\quad (-,0126)^0 = 1$ $\quad$ Nal = $0^0 = \varnothing$

*

7- $a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$. Ex $= 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $(-4)^{-3} = \frac{1}{-4^3} = \frac{1}{-64}$

8- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $(a, b \neq 0)$ Ex $= \left(\frac{x^2}{3}\right)^{-2} = \left(\frac{3}{x^2}\right)^2 = \frac{3^2}{x^4} = \frac{9}{x^4}$

عند تحويل السالب إلى موجب في الأس نقلب المقام على البسط .

Section 0.4

جذور Radicals and Rational Exponents

$V = 2 \times 2 \times 2 = 2^3 = 8$.

Now suppare we know that $v = 8$ and we went the root.
(we want b such that $b^3 = 8$)

This can be expressed as: $b^3 = 8$: $b = 2$

In general, ~~The~~ The n^{th} root of is $\sqrt[n]{a} = b$ only if $b^n = a$ subject to conditions page 22 where n is called the index and a is called the radicand.

Ex:- 1)- $\sqrt[2]{4} = 2$ $\sqrt[3]{64} = 4$ $\sqrt[3]{-27} = -3$

$\sqrt[2]{-9}$ = Not real number

* Definition :- تعريف
for a positive integer n = $a^{\frac{1}{n}} = \sqrt[n]{a^1}$

Ex:- $9^{\frac{1}{2}} = \sqrt[2]{9} = 3$ / $16^{\frac{1}{4}} = \sqrt[4]{18} = 2$ / $(-25)^{\frac{1}{2}} = \sqrt[2]{-25}$ = not real numbe

Definition:- تعريف هذا أشمل من التعريف الأول (more general).
for a positive integer n = $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ / $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$

جذر م   باسط مرفوع م

Q:- write the following in radical from and Simplify :-

$16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 8$ or $\sqrt[4]{16^3} = \sqrt[4]{4096} = 8$

$y^{\frac{-3}{2}} = (\sqrt[2]{y})^{-3}$ & $\frac{1}{(\sqrt[2]{y})^3}$ or $\sqrt[2]{y^{-3}} = \sqrt[2]{\frac{1}{y^3}}$ / $(6m)^{\frac{2}{3}} = \sqrt[3]{(6m)^2}$ ← أرفع القوة
$\sqrt[3]{6^2 m^2} = \sqrt[3]{36 m^2}$

اكتبهن بدون

Q:- write the following without radical sings:-

$\sqrt[2]{x^3} = x^{\frac{3}{2}}$ $\qquad \frac{1}{\sqrt[3]{b^2}} = \frac{1}{b^{\frac{2}{3}}}$ $\qquad \sqrt[3]{(ab)^3} = ab^{\frac{3}{3}} = ab^{1}$

* Rules for radicals :- if $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real than so

1- $\sqrt[n]{a^n} = a \Rightarrow (a^n)^{\frac{1}{n}} = a^{\frac{n}{n}} = a^1 = a$

Ex:- 1). $\sqrt[4]{7^4} = 7$ $\qquad$ 2). $\sqrt[2]{(-2)^2} = -2$

2- $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a.b}$

Ex:- $\sqrt[3]{2}\sqrt[3]{4} = \sqrt[3]{2.4} = \sqrt[3]{8} = 2$

3- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, $b \neq 0$

Ex:- $\frac{\sqrt[3]{54}}{\sqrt[3]{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27} = 3$

$\frac{\sqrt[4]{16}}{\sqrt[3]{2}} \neq \sqrt[4]{8}$

Simplifying Radicals :-

→ A radical is considered simplified if the power (القوة) of its radicand's (تحت الجذر) factors (العوامل الأولية) are each less than the index.

Ex:- $\sqrt[3]{5^2} = \sqrt[3]{\underline{25}}$ حللو للعوامل الأولية

$25 = 5 \cdot 5$ هسة بدي أعرف إذا مبسط أو لا؟ بنصب إلى القوة.

↗ factor

إذا كانت الأس أصغر من قوة الجذر بكون مبسط مثل :-

Simplifgied

$3 > 2$

أكبر ∴ هو مبسط في أبسط صورة.

$\sqrt[2]{14} = 14 = 7 \cdot 2 = \sqrt[2]{7^1 \cdot 2^1}$:- simplefied.

Ex:- $\sqrt[3]{xy^2} \rightarrow x \cdot y \cdot y = \sqrt[3]{x^1 \cdot y^2}$ simplified.

Ex:- $\sqrt[3]{16} = \sqrt[3]{2^4} = (2^4)^{\frac{1}{3}}$ simplify the folloing.

$16 = 2 \cdot 8$

$2 \cdot 4$

$2 \cdot 2$

$(2^4)^{\frac{1}{3}} = 2^{\frac{4}{3}} = 2^{1+\frac{1}{3}} = 2^1 \times 2^{\frac{1}{3}} = 2 \cdot \sqrt[3]{2}$

$\sqrt[2]{10x^3} = \sqrt[2]{2 \cdot 5x^3} = (2 \cdot 5 \cdot x^3)^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot (x^3)^{\frac{1}{2}}$

$10 = 2 \cdot 5$

= تبسيطها

$x^{\frac{3}{2}}$

$x^{\frac{2}{2}+\frac{1}{2}}$

$x^{\frac{2}{2}} \cdot x^{\frac{1}{2}}$

$\sqrt{2} \cdot \sqrt{5} \cdot x \cdot \sqrt{x} = x\sqrt{2 \cdot 5 \cdot x}$

Ex:- $\sqrt[2]{8x^5y^3z^2} = \sqrt[2]{2^3x^5y^3z^2} = 2x^2yz\sqrt{2^1x^1y^1z^0}$

$= 2x^2yz\sqrt{2^1 \cdot x^1 \cdot y}$.

outline 0.4.

⑲ $\sqrt[4]{m^2 n^5} = \sqrt[4]{m^2} \cdot \sqrt[4]{n^5}$

$= m^{\frac{2}{4}} \cdot n^{\frac{5}{4}}$

$= m^{\frac{1}{2}} \cdot n^{\frac{5}{4}}$

⑳ $-x^{\frac{-5}{3}} = -\sqrt[3]{x^{-5}} = -\sqrt[3]{\frac{1}{x^5}}$

㉑ $y^{\frac{1}{4}} \cdot y^{\frac{1}{2}} = y^{\frac{1}{4}+\frac{1\times2}{2\times2}} = y^{\frac{1}{4}+\frac{2}{4}} = y^{\frac{3}{4}} = \sqrt[4]{y^3} \Rightarrow (\sqrt[4]{y})^3$

㉙ $\frac{y^{\frac{-5}{2}}}{y^{\frac{-2}{5}}} = y^{\frac{-5}{2} - \frac{-2}{5}} = y^{\frac{5\times-5}{5\times2}+\frac{2\times2}{5\times2}} = y^{\frac{-25}{10}+\frac{4}{10}}$

$= y^{\frac{-21}{10}} = \frac{1}{y^{\frac{21}{10}}}$

㉞ $\left(x^{\frac{-2}{3}}\right)^{-\frac{2}{5}} \quad x^{\frac{-2}{3}\cdot\frac{-2}{5}} = x^{\frac{4}{15}}$.

㊱ Simplify

$\sqrt[3]{-64 \cdot x^6 \cdot y^3} = \sqrt[3]{-64 \cdot x^6 \cdot y^3}$ ——→ حول كل اس الى تقسيم $\frac{3}{3}$ تقريبا

$\sqrt[3]{-64} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^3} =$

$-4 \cdot x \cdot x \cdot y = -4x^2y$

㊵ $\sqrt{32x^5y} = \sqrt{16 \cdot 2 \cdot x^2 \cdot x^2 \cdot x \cdot y} = \sqrt[2]{16} \cdot \sqrt[2]{2} \cdot \sqrt[2]{x^2} \cdot \sqrt[2]{x^2} \cdot \sqrt{x} \cdot \sqrt{y}$

وسيط وسيط x وسيط x

$= 4x^2\sqrt{2xy}$

Section 0.5

Operation with Algebraic Expressions

العبارات الجبرية

1) Algebraic expressions are additions, subtractions, multiplications, divisions, or roots of one or more real numbers or letters called variables, that represents real numbers.

جمع طرح ضرب قسمة الجذور عدد بالحروف متغير

Ex :- 1) $3x^2$ 2) $2y-6xz$ 3) $3x-2\sqrt{y}$ → عبارات جبرية

→ term :- is a product of real number (coefficient) and one or more variables to powers.

معامل

الحد :- هو ضرب عدد حقيقي مع أكثر متغير مرفوعة إلى قوة

Ex :- 1) $3x^2$ 2) $5xyz$ 3) x^2 4) $3\sqrt[2]{x}$ 5) $6x^0$ (constant)

coefficient → معامل coefficient = 1 $6*1=6$ ثابت

→ The degree of a term is the sum of its variable's powers.

درجة :- درجة الحد هي مجموع القوى الموجودة على الأحرف

Ex :- 1) $3x^2=2$ 2) $5x^1y^1z^1=3$ 3) $x^2=2$ 4) $3\sqrt[2]{x}=3x^{\frac{1}{2}}=\frac{1}{2}$ 5) $6x^0=0$

وممكن تكون كسرية

→ A polynomial :- is the sum of a finite number of terms with nonnegative integer powers on the variables.

كثير حدود :- مجموع من الحدود المنتهية بأس غير سالب وصحيح على المتغيرات

Ex :- 1) $x^2-3y+\frac{1}{2}x+x^3y^5$ = polynomail 2) $x^4+\frac{2}{y^{-3}}+2 \Rightarrow x^4+2y^3+2$ = polynomail

3) $x^3+y^3-x^{-1}$ = not polynomail 4) $y^2-2\sqrt{x}=y^2-2x^{\frac{1}{2}}$ = not polynomail

مش صحيح

→ The degree of a polynomail is the degree of the term with the highest degree

درجة كثير الحدود :- هي أعلى درجة في الحد

→ Leading term :- الحد الي معاه أعلى درجة

→ Leading coeffisient :- المعامل الي بكون مع الحد الي أعلى درجة

Ex :- 1). $x^2 - 3y + \frac{1}{2}x^1 + 1\overset{⑦}{x^2y^5}$

degree = 7 ← أعلى درجة في الحد

Leading coeffisient = 1 ← الرقم الي بأعلى درجة للحد

Leading term = x^2y^5 ← الحد الي فيه أعلى درجة (الثابت)

2). $1x^4 + 2y^3 + 2$

→ degree = 4   Leading ceffisient = 1   Leading term = x^4

عشان كده تتكون ٣ حدود تسمى

* polynomial with Three term is called trinomial = مثل

$2x^5 + x^3 - 2$ $yr - y^2 + x$

تتكون ٢ تسمى

* polynomail with two term is called binomial = مثل

$x^5 + x^2$ $2xy + 3$ $-1x^3 - y^2$

تتكون ١ تسمى

* polynomial with one term is called monomial = مثل

$3x^2$ xy^9 $4y^2zx$

Ⓠ :- if a polynomial contains only one variable x, than it is a polynomial in x.

Ex :- 1)- $x^3 - x^1 + 6$ $3x^7 - x^2 + x - 1$

degree :-	3	7
Leading term :-	x^3	$3x^7$
Leading Coeffisint :-	1	3

Operations :- العمليات الحسابية

Combining polynomials :- (الجمع والطرح والقسمة والضرب) العمليات الحسابية

نقدر نجمع ونطرح الحدود المتشابهة :- هي الحدود التي متشابهين بالمتغيرات بنفس القوة.

Ex :- a). $(4xy + 3x) + (5xy - 2x)$ ← شرح الجمع

$= 4xy + 5xy + 3x - 2x$

$= 9xy + x$

b). $(3x^2 + 4xy + 5y^2 + 1) - (6x^2 - 2xy + 4)$ ← مثال على الطرح

$3x^2 + 4xy + 5y^2 + 1 - 6x^2 + 2xy - 4$.

$3x^2 - 6x^2 + 4xy + 2xy + 5y^2 + 1 - 4$

$= -3x^2 + 6xy + 5y^2 - 3$

Multiplication :- هي ضرب العبارات الجبرية كل طرف في العبارة الجبرية الاولى مع الطرف في العبارة الجبرية الثانية.

Ex.- $(4x^2 + 3xy + 4x) \times (2x - 3y)$.

$= (4x^2 \cdot 2x + 4x^2 \cdot -3y) + (3xy \cdot 2x + 3xy \cdot -3y) + (4x \cdot 2x + 4x \cdot -3y)$

$= 8x^3 - 12x^2y + 6x^2y - 9xy^2 + 8x^2 - 12xy$.

$= 8x^2 - 6x^2y - 9xy^2 + 8x^2 - 12xy$.

Ex :- $(x+4) \times (x+2)$.

$= x \cdot x + x \cdot 2 + 4x + 4 \cdot 2$

$= x^2 + 2x + 4x + 8$

$= x^2 + 6x + 8$

* Special products :- حالات خاصة

1- $(x+a)^2 = x^2 + 2xa + a^2.$

2. $(x-a)^2 = x^2 - 2xa + a^2.$

3. $(x+a)(x-a) = x^2 - a^2 \Rightarrow (x-a)^2$

4. $(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3 -$

5. $(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3.$

أمثلة :-

1- $(x+2)^2 = x^2 + 2\cdot2\cdot x + 2^2 =$

$= x^2 + 4x + 4.$

2. $(y-3)^3 = y^3 - 3\cdot3\cdot y^2 + 3\cdot3^2\cdot y - 3^3$

$= y^3 - 9y^2 + 27y - 27.$

$= y^3 - 9y^2 + 27y - 27.$

* Division :- القسمة

* بنستخدم القسمة الطويلة عندما يكون لدينا مقدارين بنفس المتغير

* أما إذا كان عندي أكثر من متغير ما بنستخدم القسمة الطويلة .

مثال :-

$$15x^{2}y^{3} \div 3xy^{5} = \frac{\overset{5}{\cancel{15}}\, x^{2} y^{3}}{\underset{1}{\cancel{3}}\, x\, y^{5}}$$

$$= 5x^{1}y^{-2} = \frac{5x}{y^{2}}$$

$\leftarrow$ * بنحلها على طريقة سكشن 0.3

* أما القسمة الطويلة :-

Ex :-

$$(4x^{3}-13x-22) \div (x-3) = 4x^{2}+12x+23+\frac{47}{(x-3)}$$

المقسوم منه dividend

divisor المقسوم عليه

الناتج $\leftarrow$

المقسوم عليه $\leftarrow$ divisor $\big)$ dividend المقسوم منه

الباقي : (remainder)

$\Rightarrow$

$4x^{2}+12x+23$ $\leftarrow$ الناتج

$(x-3)\,\big)\; 4x^{3}-13x-22$

$\overset{+}{\ominus}12x^{2}$

$-\;\cancel{4x^{2}}-12x^{2}$

$\cancel{12x^{2}}-13x-22$

$-\;12x^{2}-36x$

$\cancel{23x}-22$

$-\;23x-69$

$47 \leftarrow$ الباقي

$$\text{اذا} = 4x^{2}+12x+23+\frac{47}{(x-3)}$$

Ex :- $(3x^2-2x+4) \div (x+1) = 3x-5+\frac{9}{(x+1)}$

$$
\begin{array}{r|l}
 & 3x-5 \\
\hline
x+1 & 3x^2-2x+4 \\
- & \cancel{3x^2}+3x \\
\hline
- & \cancel{-5x}+4 \\
 & \cancel{-5x}-5 \\
\hline
 & 9
\end{array}
$$

4⊖5.
9

Ex :- $(4x^3+2x-1) \div (x^2+2) = 4x+\frac{-6-1}{(x^2+2)}$

$$
\Rightarrow \quad
\begin{array}{r|l}
 & 4x \\
\hline
x^2+2 & 4x^3+2x-1 \\
- & \cancel{4x^3}+8x \\
\hline
 & -6x-1
\end{array}
$$

outline:-

1). $10-3x-x^2$

a. dgree = 2
b. leading coefficient = -1
c. constant term = 10
d. number of variables = 1

④. $2x^5+7x^2y^3-5y^6+0$

$x=6$
$= -5$
$= 0$
= two variables (several variables).

⑨. $10xy-4(xy)^2$.

$10xy-4(x^2-2xy+y^2)$
$\underline{10xy}-4x^2+\underline{8xy}-4y^2$.

$18xy-4x^2-4y^2$

㉒. $y^3-[y^2-(y^3+y^2)]-[y^3+(1-y^2)]$

$y^3-[\cancel{y^2}-y^3-\cancel{y^2}]-[y^3+1-y^2]$.

$y^3-[0y^3]-[y^3+1-y^2)$.

$\rightarrow y^3+\cancel{y^3}-\cancel{y^3}-1+y^2$

y^3-1+y^2

㉝. $(4x+3)^2$.

$=16x^2+24x+9$.

㉓. $(5x^3).(7x^2)$.

$5x^3 . 7x^2$
$\rightarrow 35x^{3+2}$
$\rightarrow 35x^5$

outline:-

(40) $\left(\frac{2}{3}+x\right)\left(\frac{2}{3}-x\right)$ $\overset{\text{قاعدة}}{\Longrightarrow}$ $(a+b)(a-b)=a^2-b^2$

$\left(\frac{2}{3}\right)^2(x)^2=\frac{4}{9}-x^2$

(49) $(18m^2n+6m^3n+12m^4n^2)\div 6m^2n$

$$\frac{18m^2n+6m^3n+12m^4n^2}{6m^2n}=$$

$$=\frac{\overset{3}{\cancel{18m^2n}}}{\underset{1}{\cancel{6m^2n}}}+\frac{6\overset{1}{m^3}n}{6m^2n}+\frac{\overset{2}{12}\,\overset{2}{m^4}\,\overset{1}{n^2}}{6m^2n^1}=3+m+2m^2n$$

(53) $(x+1)^3=(x+1)^2(x+1)$

$\Rightarrow=(x^2+2x+1)\cdot(x+1)$

$x^2\cdot x+x^2\cdot 1+2x\cdot x+2x\cdot 1+1\cdot x+1\cdot 1$

$x^3+x^2+2x^2+2x+x+1$

$x^3+x^2+2x^2+3x+1$

$\rightarrow x^3+3x^2+3x+1$

(59) $(x^4+3x^3-x+1)\div(x^2+1)$

$$\begin{array}{r|l} & x^2+3x-1 \\ \hline x^2+1 & x^4+3x^3-x+1 \\ & -\cancel{x^4}+x^2 \\ \hline & \cancel{3x^3}-x^2-x+1 \\ & -\cancel{3x^3}+3x \\ \hline & \cancel{x^2}-4x+1 \\ & -\cancel{x^2}-1 \\ \hline & -4x-2 \end{array}$$

الباقي [illegible] ← $\underline{-4x-2}$

$\Rightarrow=x^2+3x-1+\left(\frac{-4x-2}{x^2+1}\right)$

Section 0.6

العامل المشترك

* If all the terms of a polynomial has a common factor than it can be factored out اخراج العوامل برا

Ex:

$2x^5 + 2x^1 + 2 = 2(x^5 + x + 1)$ → * أقسم بأخراج 2 عامل مشترك من كل الحدود

$2x^5 + \underset{2\ 3}{6}x^1 + \underset{4\ 2}{8} = 2(x^5 + 3x + 4)$ → عامل 2 موجود في كل الحدود لذا نقوم بإخراجه

(x) اقل قوة مكررة

$y^1x^5 + 2x^1y^2 + 2y = y(x^5 + 2xy + 2)$

$-3x^7 + \underset{2\ 3}{6}x^2 = 3x^2(-x^5 + 2)$ ⇒ factored completely ⇐ هنا أخرجنا عاملين لذا لاحظ

* factoring trinomials: لا عوامل مشتركة بين الحدود

Since $(x+a)(x+b) = x^2 + bx + ax + ab = x^2 + (a+b)x + ab$ therefore certain trinomals in x can be factoral to $(x+a)(x+b)$

Ex: $x^2 + 5x + 6 = (x + 2)(x + 3)$ نأخذ عددين ضربهم 6 و مجموعهم يساوي 5

تأكد: $x^2 + 5x + 6$ (✓)

$2+3=5$ / $2\times3=6$

$x^2 - 7x + 6 = (x - 1)(x - 6)$

$-1\times-6=6$

$-1+-6=-7$

$y^2 - 10y + 25 = (y - 5)(y - 5)$

$-5\times-5=25$

$-5+-5=-10$

Ex :-

أعد كتابتها بشكل صحيح

$x^2 - 8 - 2x =$ $\qquad$ $-4 \times 2 = -8$

$x^2 - 2x - 8 = (x-4)(x+2)$ $\qquad$ $-4 + 2 = -2$

بنكتبها على شكل حد مربع

$9x^2 + 6x - 3 =$

$3^2x^2 + 6x - 3 =$

$(3x)^2 + 2 \times 3x - 3 =$ $\qquad$ $z = 3x$

بنفرض اسم z مربع $(3x^2)$

$z^2 + 2z - 3 = (z+3)(z-1)$

$(3x+3)(3x-1)$ $\qquad$ $3 \times -1 = -3$

$3 + -1 = 2.$

$x^2 - 16 = (x-4)(x+4)$ $\qquad$ $-4 \times 4 = -16$

لانه فرق بين مربعين $\qquad$ $-4 + 4 = 0$

* Special factorizations :- $\qquad$ الفرق بين

الفرق بين مربعين .

Difference of two squares :- $x^2 - a^2 = (x+a)(x-a)$

Ex:- $x^2 - 16 = (x-4)(x+4)$

$25x^2 - 36y^2 =$

$(5x)^2 - (6y)^2 = (5x - 6y)(5x+6y).$

الفرق بين مكعبين

Difference of ~~two~~ two cubes :- $a^3 - b^3 = (a-b)(a^2 + ab + b^2).$

Ex:- $x^3 - 64 = (x^3 - 4^3) = (x-4)(x^2 + 4x + 16).$

$27 - 8x^3 = (3^3 - (2x)^3) = (3 - 2x)(9 + 6x + 4x^2).$

* Sum of two cubes :- $a^3+b^3=(a+b)(a^2-ab+b^2)$.

Ex :- $x^3+64=(x^3+4^3)=(x+4)(x^2-4x+16)$

$27+8x^3=(3^3+(2x)^3)=(3+2x)(9-6x+4x^2)$.

outline:-

④. $12y^3z+4yz^2-8y^2z^3$ 12=④x3 | 4=④x1 | 8=④.2.

$4yz(3y^2+z-2yz^2)$.

⑨. $x^2+8x+12=(x+2)(x+6)$ → معادلة تربيعية

⑩. $x^2-2x-8=(x-4)(x+2)$

⑰. $49a^2-144b^2$

$7^2a^2-12^2b^2=(7a)^2-(12b)^2$

$(7a+12b)(7a-12b)$

* $x^2-y^2=(x+y)(x-y)$.

* $(ab)^n=a^n.b^n$.

a(19) $9x^2+21x-8$

$3^2x^2+7.3.x-8$.

$\rightarrow (3x)^2+7(3x)-8 \Rightarrow (3x-1)(3x+8)$

㉑. $4x^2-x=x(4x-1)$ Commonfactor.

Section 0.7

Algebraic Fractins :- الكسور الجبرية (كسور)

Algebraic Fractins are :- $\frac{\text{algebraic expression}}{\text{algebraic expresion}}$ → $\frac{\text{numerator}}{\text{denominator}}$ $\frac{\text{البسط}}{\text{المقام}}$

أي مقدار عددي وجبري يكون على شكل $\frac{A}{B}$

Ex: $\frac{6}{8}$ $\frac{3x^2-14x+8}{x^2-16}$ $\frac{ab}{c}$

→ In this section, we will [illegible] learn how to multiply, divide, add and subtract algebraic fractins. رح نتعلم طريقة عمليات على الكسور (الضرب والقسمة والجمع والطرح)

Simplifying fractions :- to simplify, factor the denominator and numberator and divide them by the common factors [illegible]

Ex: $\frac{6}{8} = \frac{3\cancel{2}}{4\cancel{2}} = \frac{3}{4}$

ضرب Products of fractins :- to multiply, multiply the numberator together and denominator together.

Ex: $\frac{4}{5} \times \frac{10}{12} = \frac{40 \div 10}{60 \div 10} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

[illegible]

$$\frac{-4x+8}{3x+6} \times \frac{2x+4}{4x+12} =$$

$$\frac{\cancel{4}(-x+2)}{3\cancel{(x+2)}} \cdot \frac{2\cancel{(x+2)}}{\cancel{4}(x+3)} = \frac{(-x+2)2}{3\cdot(x+3)} = \frac{-2x+4}{3x+9}$$

القسمة Quotients of fractions :- to divide , multiply the dividend by the reciprocal (مقلوب) of the divisor.

Ex :- $\frac{4}{7} \div \frac{5}{21} = \frac{4}{{}_1\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{5} = \frac{12}{5}$

Ex :- $\frac{a^2b}{c} \div \frac{ab}{c^2} = \frac{\cancel{a^2}\cancel{b}}{\cancel{c}} \times \frac{\overset{2}{\cancel{c}}}{\cancel{ab}} = \frac{ac}{1} = ac.$

Ex :- $\frac{6x^2-6}{x^2+3x+2} \div \frac{x-1}{x^2+4x+4} =$

$\frac{6x^2-6}{x^2+3x+2} \times \frac{x^2+4x+4}{x-1} =$

$\frac{6(x^2-1)}{(x+1)(x+2)} \cdot \frac{(x+2)(x+2)}{(x-1)} =$

$\frac{6\cancel{(x-1)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x+2)}} \cdot \frac{\cancel{(x+2)}(x+2)}{\cancel{(x-1)}} = 6(x+2) = 6x+12.$

* Combining Fraction :- to add or subtract fractions, find a common then add or subtract the numerator.

Ex :- $\frac{3\times8}{12\times8}+\frac{7\times12}{8\times12}=\frac{24}{96}+\frac{84}{96}=\frac{108}{96}=\frac{54}{48}=\frac{27\div3}{24\div3}$

$=\frac{9}{8}$.

* Least common Denominator :-(LCD) :- [illegible]

Ex :- $\frac{3(2)}{12(2)}\ \frac{7(3)}{8(3)}=$

$=\frac{6+21}{24}=\frac{27\div3}{24\div3}=\frac{9}{8}$

$12 :- 2 . 2 . 3$

$8 :- 2 . 2 . 2$

[illegible] LCD = $2^3 . 3$

distinct fact :-

2 , 3

$2\times2\times2\times3 = 24$

[illegible]

Ex :- $\frac{x . 3x}{x . a^2}+\frac{4 . a}{ax . a}$

$=\frac{3x^2+4a}{a^2x}$

$a^2 = a . a$

$ax = a . x$

distinct Factors

a , x

LDC :- $a^2 . x^1$

Ex :- $\frac{(y+1)\ y-3}{(y+1)(y-5)^2}-\frac{(y-2)(y-5)}{(y^2-4y-5)(y-5)}$

$(y-5)^2 = y^2-10y+25 = (y-5)(y-5)$

$(y^2-4y-5) = (y-5)(y+1)$

distinct factors :- $(y-5)(y+1)$

LCD :- $(y-5)^2(y+1)$

$=\frac{(y-3)(y+1)-(y-2)(y-5)}{(y-5)^2(y+1)}$

$=\frac{(y^2+y-3y-3)-(y^2-5y-2y+10)}{(y-5)^2(y+1)}$

$=\frac{(\cancel{y^2}-2y-3)-(\cancel{y^2}+7y+10)}{(y-5)^2(y+1)}=\frac{5y-13}{(y-5)^2(y+1)}$.

* find the least commen denomintor :- for :-

$\frac{1}{x^3-x}$, $\frac{x+1}{x^2}$, $\frac{x}{y+1}$

distinct factors:-

$x, (x-1), (x+1), (y+1)$

LCD:- $x^2, (x-1)^1, (x+1)^1, (y+1)^1$

$x^3 - x = x(x^2-1) \Rightarrow (x-1)(x+1)$

$x^2 = x \cdot x$

$y+1 = y+1$

Chapter 1

(Section 1.1) معادلة

Solutions of linear Equations and المتباينات Inequalities in One Variable.

معادلة · عبارة · متساوي

An: equation :- is a statement that two algebraic expressions are equal.

المعادلة :- عبارتين جبريتين بينهما إشارة مساواة.

Ex:- 1. $X+\cancel{1}=-6$ $\Rightarrow X=-7$
$\cancel{-1}$ -1

2. $X^2+X=2X+6 \rightarrow$ معادلات

3. $xy=6$

4. $6(3-x)=5+\frac{x-1}{2}$

تعريف The values of the variables that makes the statements trua are called solutions of the equations.

* Solving linear equations :-

Ex:- $X+\cancel{1}=-6 \Rightarrow X=-6+-1=\underline{-7} \rightarrow -7+1=-6.$
$\cancel{-1}$ -1
↳ solutions.

Ex.: $\cancel{4X}-7=8X+2 = -7=4X+\cancel{2} = \frac{-9}{4}=\frac{4X}{\cancel{4}} = \left\{ X=\frac{-9}{4} \right\}$ solution
$\cancel{-4X}$ $-4X$ -2 $\cancel{-2}$

$\cancel{4}\cdot\frac{-9}{\cancel{4}}-7 \overset{??}{=} \overset{2}{\cancel{8}}\cdot\frac{-9}{\cancel{4}_1}+2$

(✓) $-16 \doteq -16$. نعم متساويات

Ex:- $X+8=8(X+1)$

$X+\cancel{8}=8X+\cancel{8}$
$\cancel{-8}$ $\cancel{-8}$

$\cancel{X}=8X$
$\cancel{-X}$ $-X$

$\frac{0}{7}=\frac{7X}{\cancel{7}} = X=0$ (solution).

Ex:- $\frac{2x}{3} - 1 = \frac{x-2}{2}$

$6\cdot\left(\frac{2x}{3} - 1\right) = 6\cdot\left(\frac{x-2}{2}\right)$

$\overset{2}{\cancel{6}}\cdot\frac{2x}{\underset{1}{\cancel{3}}} - 6\cdot 1 = \overset{3}{\cancel{6}}\cdot\frac{x-2}{\underset{1}{\cancel{2}}}$

$4x - 6 = \cancel{3x} - 6$
$-3x \qquad \cancel{-3x}$

$x \cancel{-6} = -6$
$\cancel{+6} \qquad +6$

$x = 0$

Ex:- $\frac{2x-2}{x-3} = 4 + \frac{5}{x-3}$

$(x-3)\cdot\left(\frac{2x-2}{x-3}\right) = (x-3)\cdot\left(4 + \frac{5}{x-3}\right)$

$\cancel{x-3}\cdot\frac{2x-2}{\cancel{x-3}} = x-3\cdot 4 + \frac{5}{\cancel{x-3}}\cdot\cancel{(x-3)}$

$2x - 2 = 4x - 12 + 5$

$2x - 2 = 4x - 7$
$-2x \qquad -2x$

$-2 = 2x \cancel{-7}$
$+7 \qquad \cancel{+7}$

$\frac{2x}{2} = \frac{5}{2} = x = \frac{5}{2}$

Ex:- $\frac{2x-1}{x-3} = 4 + \frac{5}{x-3}$

$\cancel{(x-3)}\cdot\frac{2x-1}{\cancel{x-3}} = (x-3)\cdot 4 + \frac{5}{\cancel{x-3}}\cdot\cancel{(x-3)}$

$2x-1 = 4x-12+5$

$\cancel{2x}-1 = 4x-7$
$-\cancel{2x} \quad\quad -2x$

$-1 = 2x\cancel{-7}$
$+7 \quad\quad \cancel{+7}$

$\frac{6}{2} = \frac{\cancel{2}x}{\cancel{2}} \Rightarrow x = \underline{\underline{3}} =$ No solution.

Ex:- $\frac{3x}{2x+10} = 1 + \frac{1}{x+5}$

$(2(x+5))\cdot\frac{3x}{\cancel{2x+10}} = 2(x+5)\cdot 1 + \frac{1}{\cancel{x+5}}\overset{2}{\leftarrow}\cdot(2(x+5))$

$3x = 2x+10+2$

$3x = \cancel{2x}+12$
$-2x \quad\quad -\cancel{2x}$

$x = 12$

Ex:- $\cancel{2x}+5 = -3+\cancel{2x} \Rightarrow 5 \neq -3$ always false $\therefore$ No solution
$-\cancel{2x} \quad\quad -\cancel{2x}$

Ex:- $2x+6 = 2(3+x)$

$\cancel{2x}+6 = 6+\cancel{2x} \rightarrow 6 = 6$ always true $\therefore$ Solution or real number
$-\cancel{2x} \quad\quad -\cancel{2x}$

اكل كل الاعداد الحقيقية

* Solving linear equations of two variables for one variable :-

Ex :- 1). $4x + 3y = 12$ (Solve the following equations for y)

$$\underset{-4x}{4x} + 3y = \underset{-4x}{12}$$

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{12-4x}{3}$$

$$y = \frac{12-4x}{3}$$

Ex :- 2). $(9x + \frac{3}{2}y = 11)\times 2$

$$= \underset{-18x}{18x} + 3y = \underset{-18x}{22}$$

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{22-18x}{3}$$

$$y = \frac{22-18x}{3}$$

* Linear Inequalities :-

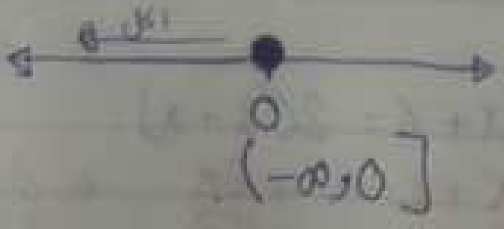
متباينة Inequality :- is a statement that one algebraic expression is greater or less than another algebraic expression.

المتباينة :- عبارتين علاقة (مهما طرفين) بـ طرف أكبر أو أصغر من الطرف الثاني (معادلة بس أكبر وأصغر).

Ex :-

1) $3x + \cancel{2} \leq 2$
$\quad\; -\cancel{2} \quad -2$

$$\frac{3x}{3} \leq \frac{0}{3} \Rightarrow x \leq 0$$



$(-\infty, 0]$

Ex:-

2). $\cancel{2x} - 1 > 3x + 5$

$-\cancel{2x} \qquad -2x$

$-1 > x \cancel{+5} \quad \rightarrow \quad -6 > x$

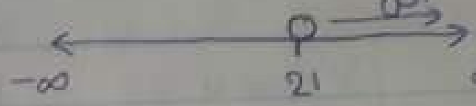
$-5 \qquad \cancel{-5}$

$(-\infty, -6)$

3). $\cancel{+17} - x < -4$

$\cancel{-17} \qquad -17$

$\frac{\cancel{-}x}{\cancel{-1}} < \frac{-21}{-1}$:



$(21, \infty)$

$x > 21$

قلبنا الاشارة لانه عندما نضرب بالسالب او ما

نقسم على السالب نقوم بتغير الاشارة مثل

$3 > 2$

$-1 \times 3 \; \bigcirc \; -1 \times 2$

$-3 \; \textcircled{<} \; -2$

4). $\frac{3x}{4} - \frac{1}{6} \geq x - \frac{2(x-1)}{3}$

$4 = 2 . 2$

$6 = 3 . 2$

$3 = 3$

$LCD = 2^2 . 3^1$

$\overset{3}{\cancel{12}} . \frac{3x}{\cancel{4}} - \overset{2}{\cancel{12}} . \frac{1}{\cancel{6}} \geq 12 . x - \overset{4}{\cancel{12}} . \frac{2(x-1)}{\cancel{3}}$

$9x - 2 \geq 12x - 8(x-1)$

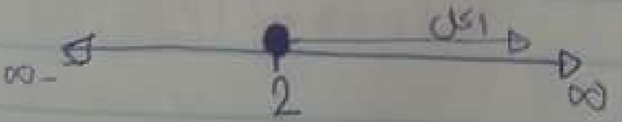
$9x - 2 \geq 12x - 8x + 8$

$9x - 2 \geq \cancel{4x} + 8$

$-4x \qquad \cancel{-4x}$

$5x \cancel{-2} \geq 8 \Rightarrow \frac{\cancel{5}x}{\cancel{5}} \geq \frac{10}{5} \Rightarrow x \geq 2$

$\cancel{+2} \qquad +2$



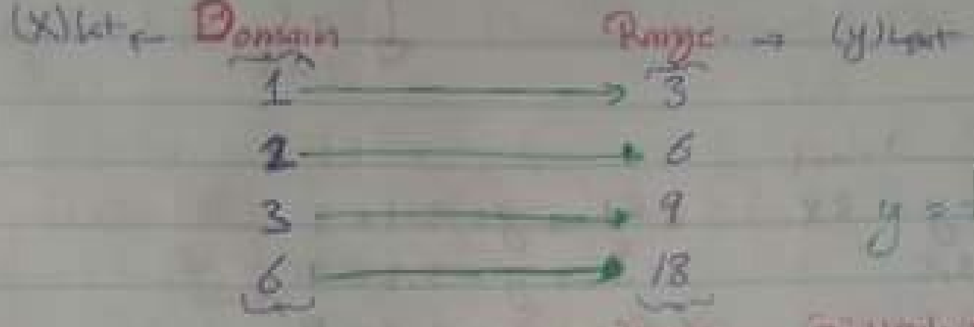
$= [2, \infty)$

(Section 1.2)
functions 80 (الاقترانات)

علاقة بين مجموعتين المجال المدى

function :- is relation between two sets (domain and range) such that for each that for each element in the domain there corresponds only one element of the range.

يوجد شرط حتى نسمي الاقتران ان كل عنصر المجال يرتبط بعنصر واحد من مجموعة المدى



$y = f(x)$

مجموعة الازواج المرتبة جدول رسم

A function may be defined by a set of ordered pairs, a table, a graph or an equation → معادلة

عرض

1- set of ordered pairs :- مجموعة الازواج المرتبة

$$\{\overset{(x,\ y)}{(1,3)}\ \overset{(x,\ y)}{(2,6)}\ \overset{(x,\ y)}{(3,9)}\ \overset{(x,y)}{(6,18)}\}$$

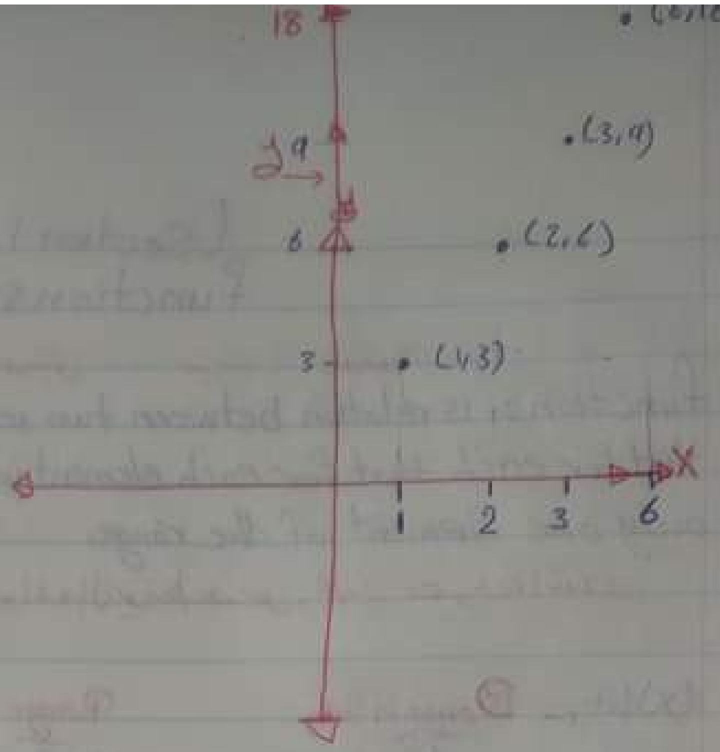
domain = $\{1,2,3,6\}$.
Range = $\{3,6,9,18\}$.

Ex :- $\{(1,\textcircled{4})\ (3,7)\ (1,\textcircled{2})\}$ هذا ليس اقتران لان العدد 1 مرتبط بعنصرين في المدى
(Not a function)

2- table :- الجدول

x	1	2	3	6
y	3	6	9	18

3_ Graph :- الرسم



4- equation :- المعادلة

$y = 3X$

$y=3X$ $X=1, 2, 3, 6$

$X=1 \rightarrow y = 3\cdot 1 = 3$

$X=3 \Rightarrow y = 3\cdot 3 = 9$

$X=6 \rightarrow y = 3\cdot 6 = 18$

$X=4 \rightarrow y = 3\cdot 4 = 12$ X غير معرف

Evaluating functions: finding the range (y, output, dependent variable) for a specified domain (X, input, independent variable).

Ex:- evalute the functionabov at $X=2$ and at $X=6$

at $X=2 \rightarrow y=6$ $y=3\cdot 2=6$

at $X=6 \rightarrow y=18$ $y=3\cdot 6=18$

Ex:- evaluate the function $y=\frac{2-x}{x+4}$ at $x=6$ and $\boxed{x \neq -4}$

not in the domain

at $X=6 \Rightarrow y=\frac{2-6}{6+4} = \frac{-4}{10} = \frac{-2}{5}$

at $X=-4 \rightarrow y=\frac{2--4}{-4+4} = \frac{6}{0} \rightarrow$ undefinated (غير معرف)

Domain and Range.

If the domain is not specified than it is the set of all real number inputs that results in real number outputs the range is ~~that~~ the set of all outputs obtaind from evaluating all the domain.

Ex. $y = f(x) = x^2$ Domain is all real number $(-\infty,\infty)$.

$y = 1 + \frac{1}{x-2}$ Domain $(-\infty,\infty) - \{2\}$ [illegible] $x - 2 = 0$, $+2$, $x = 2$

$f(x) = \frac{3}{2x - \frac{1}{3}}$ $3(2x - \frac{1}{3}) = 0 \times 3$

$6x - 1 = 0$, $+1$ $+1$ $\rightarrow \frac{6x}{6} = \frac{1}{6} = x = \frac{1}{6}$

Domain $(-\infty,\infty) - \{\frac{1}{6}\}$.

$f(x) = \sqrt{4-x}$ $\cancel{4} - x \geq 0$, $\cancel{-4}$ -4

Domain $(-\infty, 4]$ $-x$ $-x \geq -4$ $x-$

$x \leq 4$

opration with functions. $(f+g)(x) = f(x) + g(x)$

$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$ $g(x) \neq 0$.

Ex: if $f(x) = 3x+3$ and $g(x) = 8-2x$ $h(x) = x^2 - 1$ than.

1) $(f+g)(x) = f(x) + g(x)$

1.5) $(f+g)(2) = 3x+3+8-2x$

$2 + 11 = 13 \Leftarrow = x + 11$

2) $(h-g)(x) = h(x) - g(x)$

$x^2 - 1 - (8-2x)$

$x^2 - 1 - 8 + 2x$

$x^2 + 2x - 9$

3- $(g \cdot h)(x) = g(x) \cdot h(x)$.

$(g \cdot h)(1)$ $(8-2x) \cdot (x^2-1) = 8x^2 - 8 - 2x^3 + 2x$

$= -2x^3 + 8x^2 + 2x - 8 \Rightarrow -2 + 8 + 2 - 8 = 0$

4). $\left(\frac{g}{h}\right)(x) = \frac{g(x)}{h(x)} = \frac{8-2x}{x^2-1}$

5). $\left(\frac{f}{h}\right)(x) = \frac{f(x)}{h(x)} = \frac{3x+3}{x^2-1} = \frac{3\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{3}{(x-1)}$

$\left(\frac{f}{h}\right)(x) = \frac{3}{x-1}$ domain $(-\infty,\infty) - \{1\}$

5,5) $\left(\frac{f}{h}\right)(7) = \frac{3}{7-1} = \frac{3}{6} = \frac{1}{2}$

$f(x)$ evaluate $f(x)$ at $x = 3$

$f(3)$

evaluate $f(x)$ at $x = \frac{-1}{2}$

$f(\frac{-1}{2})$

evaluate $f(x)$ at $x = b$

$f(b)$

evelauate $f(x)$ at $x =$ [illegible]

$f($[illegible]$)$

* Composite functions:- f composed of g written as $(f \circ g)(x) = f(g(x))$

مركبة

Ex:- if $f(x) = 3x+3$ and $g(x) = 8-2x$ $h(x) = x^2-1$ than:-

1-$(f \circ g)(x) = f(g(x))$
$= 3(8-2x)+3$
$= 24-6x+3$
$= -6x+27.$

2). $(h \circ f)(x) = h(f(x))$
$= (3x+3)^2-1$
$= 9x^2+18x+9-1$
$9x^2+18x+8$

1.5) $(f \circ g)(g(3)) = -6x(3)+27 =$
$= 9$

2.5) $(h \circ f)(1) = h(f(1))$
$9(1)^2+18(1)+8$
$9+18+8 = 35$

6) $(f \circ g \circ h)(x) = f(g \circ h) = f(g(h(x))$
$= 3[8-2(x^2-1)]+3$
$= 3(8-2x^2+2)+3$
$= 24-6x^2+6+3$
$= -6x^2+33$

* $f(x) = \frac{3}{2x-7}$

$$2x \cancel{-7} = 0$$
$$\cancel{+7} \quad +7$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{7}{2} \Rightarrow x = \frac{7}{2}$$

Domian $(-\infty,\infty) - \{\frac{7}{2}\}$

* $g(x) = \sqrt[4]{x-3}$

$$x \cancel{-3} \geq 0$$
$$\cancel{+3} \quad +3$$

$$x \geq 3$$

Domian $[3, \infty)$

* $m(x) = \sqrt[4]{x+3}$

$$x \cancel{+3} \geq 0$$
$$\cancel{-3} \quad -3$$

$$x \geq -3$$

Domian $[-3, \infty)$

Section 1,3
Linear Functions

A Linear function is a function of the form $y = f(x) = ax + b$ where a and b are constants (not variables). slope intercept form.

Ex :- $y = f(x) = \circled{2}x + \circled{4}$ → $y = f(x) = 2x + 4$

a = 2 b = 4

$h(x) = 2x - 5$

a = 2 b = -5

$\frac{4x}{-2} = \frac{2}{-2} - y$ $= -y = 4x - 2$

$y = 2 - 4x$

a = -4 b = 2

$y = -x + 4$

a = -1 b = 4

$y = f(x) = 3 + 6x$

a = 6 b = 3

a ← عامل x يسمى

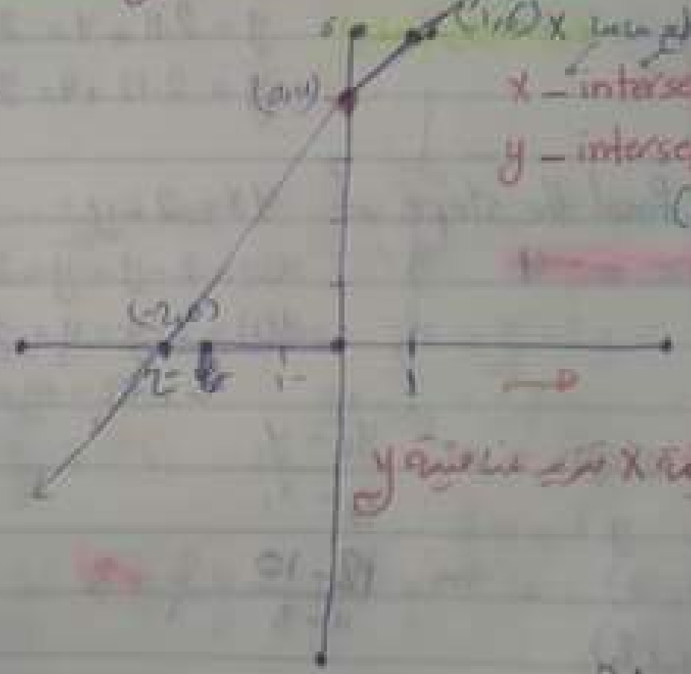
b ← هو الحد الثابت

* In this section we will learn how to graph linear function, find their slopes (rate of ~~ch~~ change), and find their equations.

1- graphing linear functions :-

$y = 2x + 4$

	x	y
y=2(0)+4=4	0	4
y=2(1)+4=6	1	6
y=2(-2)+4=0	-2	0



(1,0) x تقاطع مع (-2,0)

x – intersept point

y – intersept point

(0, 4)

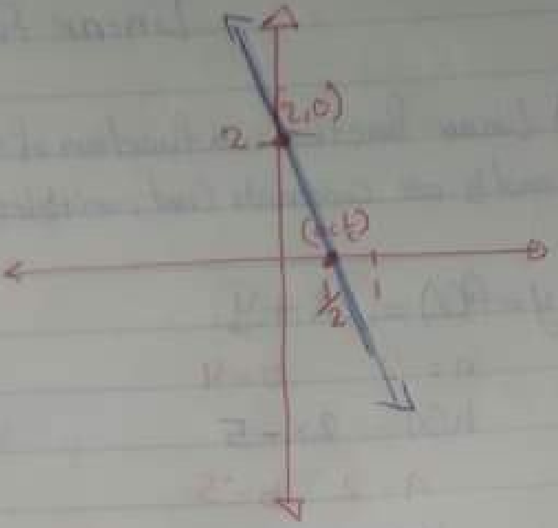
كلما زادت x تزيد معها y

Ex :- Graph $4x = 2-y$

$4 \cdot (0) = 2-y$
$0 = 2-y$
$y = 2$

$4x = 2-0$
$\frac{4x}{4} = \frac{2}{4}$
$x = \frac{1}{2}$

x	y
0	2
$\frac{1}{2}$	0



* Rate of change (slope) :- نسبة التغير

كم (y) تتغير عندما نزيد قيمة (X) قيمة واحدة ها

1) * find the slope of $y = 2x + 4$

Slope = 2

$y = 2 \cdot 3 + 4 = 10$
$y = 2 \cdot 4 + 4 = 12$
$y = 2 \cdot 11 + 4 = 26$
$y = 2 \cdot 12 + 4 = 28$

* عندما نزيد قيمة X قيمة واحدة تزيد قيمة y بمقدار وحدتين.

x	y
3	10
4	12
11	26
12	28

زيادة = 2

زيادة = 2

2) * find the slope of $4x = 2-y$

slope = -4

$4(0) = 2-y = y = 2$
$4(1) = 2-y = \frac{4}{-2} = \frac{-2-y}{-2}$
$-1+2 = +y \times + = -2 = y$

x	y
0	2
1	-2
2	-6

* or slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$

1) $\overset{x}{(3}, \overset{y}{10)}$ $(4, 12)$ $= m = \frac{12-10}{4-3} = \frac{2}{1} = 2$

2) $\overset{x}{(0}, \overset{y}{2)}$ $(2, -6)$ $= \frac{-6-2}{2-0} = \frac{-8}{4} = -4$

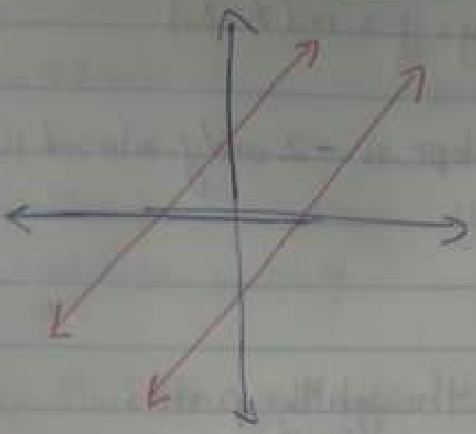
* or slop :- $(y = ax + b)$
↳ a = slope
b = is y-intersept

1). $y = 2x + 4$
$a = 2$
slope = 2

2). $4x = 2-y$
$y = -4x + 2$
$a = -4$
Slope = -4

متوازيين

* Tow Lines are parallel if their slope are equal

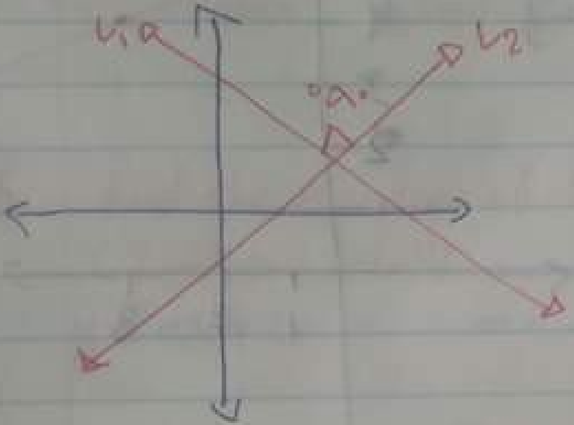


إذا كان نفس (slope) للخطين ما بيتقاطع حتى لو اشتواك النهاية

$m_1 = m_2$

متعامدين   ضرب

* Tow lines are perpendicular if ~~thier slope~~ product of thier slopes is nightinson



$m_1 \times m_2 = -1$

إذا كان حاصل ضرب (slop) = -1 يكون الخطين متعامدان .

Ex:- are the lines $\underbrace{3X = 2 - 2y}_{}$ and $\underbrace{y = \boxed{\frac{2}{3}}X + 107}_{}$

$L_1 \rightarrow$ $L_2 \rightarrow$

$m_1 \neq m_2$ - $\overset{x}{\text{Parallel}}$, $\boxed{\text{perpendicular}}$ or neither

متعامدين

slop = $3X = \cancel{2} - 2y$

$-2 \quad -2$

$\frac{3X}{-2} - \frac{2}{-2} = \frac{\cancel{-2}y}{\cancel{-2}}$

$y = \boxed{\frac{-3}{2}}X + 1$

$m_2 = \frac{2}{3}$ $m_1 = \frac{-3}{2}$

$m_2 \cdot m_1 = \frac{-\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} = \boxed{-1}$

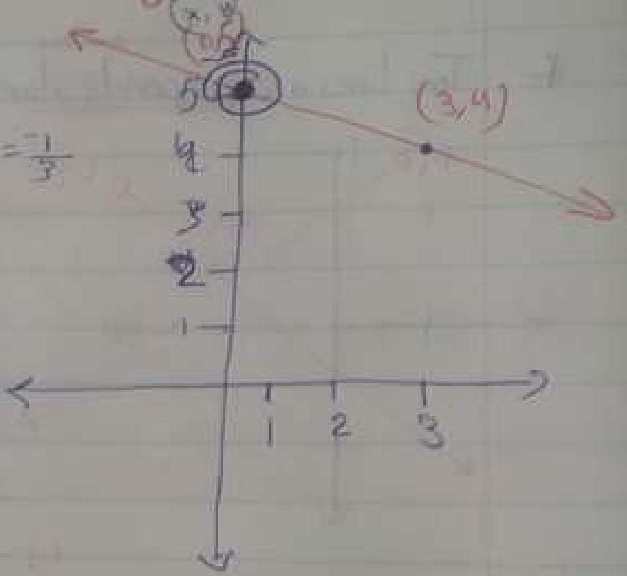
* writing equations of lines 80 $\overset{\text{slope}}{a}$ $y = ax + b$ → y-intersept

$y - y_1 = m(x - x_1)$

Ex:- find thequations of lines whose ~~slop~~ slope is −2 and y-intersek is 7.

equationse ⟹ $y = -2x + 7$

Ex- Find the equationse of the line that passe through the points $(3,4)$ and $(\overset{x}{0}, \overset{y}{5})$.

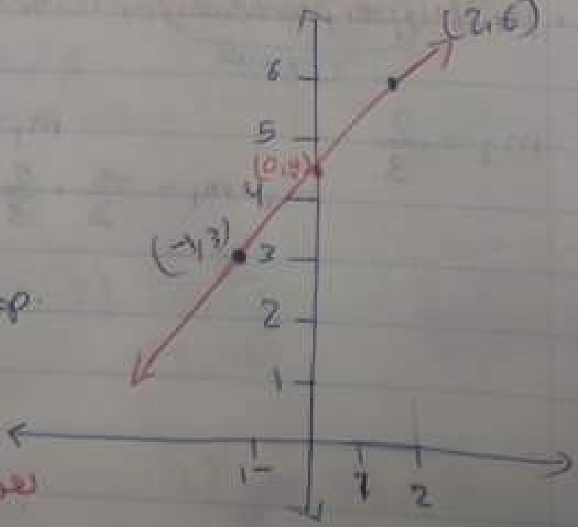


$y = ax + b$

$\underset{\text{slope}}{m} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-4}{0-3} = \dfrac{-1}{3}$

$y = \dfrac{-1}{3}x + 5$

Ex:- find the equation of the line that passes thereght the points $(-1,3)$ and $(2,6]$



$y = \underset{\text{slop}}{a}x + \underset{\text{y-intersect}}{b}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6-3}{2-(-1)} = \dfrac{3}{3} = \boxed{1}$ slop

$y = x + b$ نعوض وحدة من النقطتين الاولى او الثاني

$6 = 2 + b$

$-2 \quad -\cancel{2}$

$\boxed{b = 4}$

∴ the Equation $y = 1x + 4$

Ex:- find the equations of the line that passes throught (4,0) and (1,3)

$y = ax + b$ $m = \frac{3-0}{1-4} = \frac{3}{-3} = \boxed{-1}$

$y = -1x + b$ equations $y = -x + 4$

$0 = \cancel{-4} + b \rightarrow b = 4$

$+4 \quad \cancel{+4}$

Ex:- find the equations of the line that passer through the point (2,1) and

متوازيين parallel to $y = \underset{\rightarrow}{3}x + 117$.

نفس slope

$y = \overset{}{\underset{\text{slope}}{a}}x + \underset{\text{y-intecst}}{b}$

$y = 3x + b$ equations → $y = 3x + -5$

$1 = 3 \times 2 + b$

$1 = 6 + b \rightarrow b = -5$

$-6 \quad -6$

Ex:- find the equations of the line that passer through the points (2,4) and (-1,4)

$y = ax + b$ $m = \frac{4-4}{-1-2} = \frac{0}{-3} = 0$

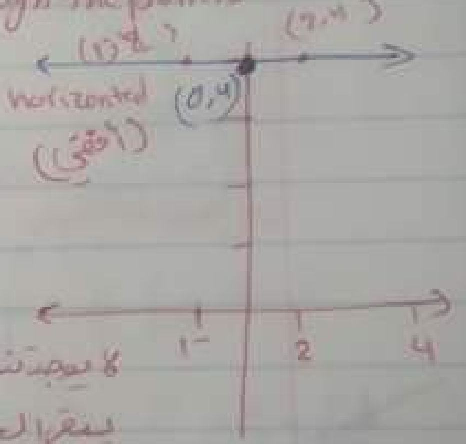
$y = 0(x) + b$ $y = 4$

$y = 2 \times 0 + b$

$\underline{4 = b}$

∴ the equations → $y = 4$

لا يوجد تزايد أو تناقص
يبقى العدد ثابت

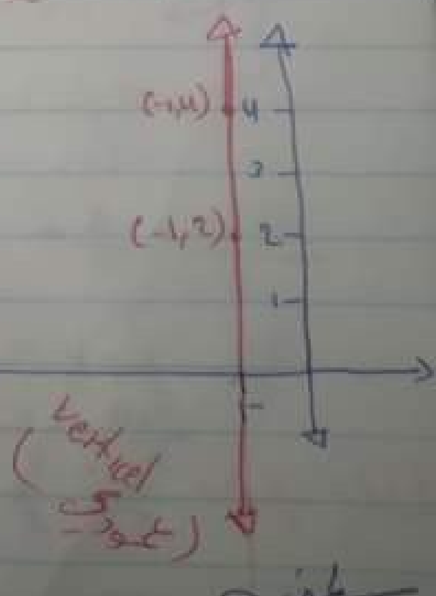


Ex:- find the equation of the line that passes through (-1,2) and (-1,4)

$y = ax + b$ $m = \frac{4-2}{\cancel{-1+1}} = \frac{2}{0}$ = undefined

ليست علاقة اقتران لأن
العنصر (-1) مرتبط بعنصرين

∴ equationes is $X = -1$



Sectione 1.5
Solutions of systems of linear Equations.

A slutions of the linear equation with two variables (x and y) is the pair (x, y) that will make the equation ture.

Ex:- for the equation $4x + 3y = 11$ $(0, \frac{11}{3})(\frac{11}{4}, 0)(\frac{2}{4}, 3)$ are all solutions.

$$4 \cdot (0) + \cancel{3} \cdot \frac{11}{\cancel{3}} = 11 \ (\checkmark)$$

Suppous we went a slution that satisfes ~~are~~ more than one equation (system of equations). This solution is called a simmultaneous solution and it could be found using three methods.

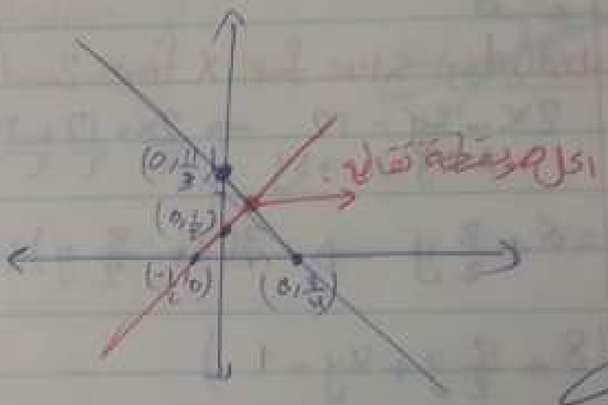
Ex:- Find a simultaneous solution for the following system of equations.

[illegible] $4x + 3y = 11$

$2x - 5y = -1$

x	y
0	$\frac{1}{5}$
$-\frac{1}{2}$	0

1- Graphical methods.



2- Substitution :- تعويض

$4x + 3y = 11 \rightarrow$ Solve for $\cancel{4x} + 3y = 11 \rightarrow \frac{\cancel{3}y}{\cancel{3}} = \frac{11 - 4x}{3}$
$2x - 5y = -1$ $\quad -\cancel{4x} \quad -4x$

$$y = \frac{11}{3} - \frac{4x}{3}$$

$$2x - 5\left(\frac{11}{3} - \frac{4x}{3}\right) = -1$$

$$y = \frac{11}{3} - \frac{4 \times 2}{3}$$

$$\cancel{3}\left(2x - \frac{55}{\cancel{3}} + \frac{20}{\cancel{3}}x\right) = -1 \cdot 3$$

$$y = \frac{11}{3} - \frac{8}{3} = \frac{3}{3} = \boxed{1}$$

$6x - 55 + 20x = -3 \rightarrow 26x - 55 = -3 \rightarrow \frac{\cancel{26}x}{\cancel{26}} = \frac{52}{26} \rightarrow \boxed{x = 2}$
$\quad +55 \quad +55$

(2, 1)

3- elemination :- (الحذف

$4x + 3y = 11$

$2 \cdot (2x - 5y) = -1 \cdot 2$

$\cancel{4x} + 3y = 11$

$- \quad \cancel{4x} - 10y = -2$

$\frac{\cancel{13}y}{\cancel{13}} = \frac{13}{13} \Rightarrow y = 1 \rightarrow 4x + 3 \cdot 1 = 11$

$4x + \cancel{3} = 11$

$\quad \cancel{-3} \quad -3$

~~$4x + 3y = 11$~~

~~[illegible] $+ 3y = 11 \rightarrow 3y = 8$~~

$\frac{\cancel{4}x}{\cancel{4}} = \frac{8}{4} \quad x = 2$

$(2, 1)$

Ex :-

تعويض.

1). substintion solue for x from 2and

$3x + 4y = 1 \rightarrow$

$2x - 3y = 12$

$2x \cancel{- 3y} = 12 \rightarrow \frac{2x}{2} = \frac{12}{2} + \frac{3y}{2}$

$\quad \cancel{+3y} \quad +3y$

$x = 6 + \frac{3}{2}y \rightarrow 3(6 + \frac{3}{2}y) + 4y = 1$

$2 \cdot (18 + \frac{9}{2}y + 4y = 1)$

$36 + 9y + 8y = 2$

$\cancel{36} + 17y = 2$

$\cancel{-36} \quad -36$

$\frac{\cancel{17}y}{\cancel{17}} = \frac{-34}{17} \rightarrow y = -2$

$(x, -2)$

$3x + 4 \cdot -2 = 1$

$3x + \cancel{-8} = 1$

$\quad \cancel{+8} \quad +8$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{9}{3}$

$\boxed{x = 3}$

Ex:- elemintion بالحذف

$3\cdot(3x+4y=1) \rightarrow 9x+\cancel{12y}=3$

$4\cdot(2x-3y=12) \quad 8x-\cancel{12y}=48$

$\frac{\cancel{17x}}{\cancel{17}}=\frac{51}{17} \rightarrow \boxed{x=3}$

(3, -2).

$3\cdot3+4y=1$

$\frac{\cancel{9}}{\cancel{-9}}+4y=\frac{1}{-9} \rightarrow \frac{\cancel{4}y}{\cancel{4}}=\frac{-8}{4} \rightarrow y=\boxed{-2}$

* Inconsistent system:- Not they have the same slope.

Ex:- $2\cdot(4x+3y)=(9)\cdot2$

$8x+6y=18$

$\cancel{8x}+\cancel{6y}=8$

$-\cancel{8x}+\cancel{6y}=18$

$\boxed{0=10}$ ∴ No solution

Chapter 2

Sactions 2.1

Quadriatic Equations 80 المعادلات التربيعية

A quadratic equation in one variable has a general form of

معادلة تربيعية ← $ax^2 + bx + c = 0 \quad (a \neq 0)$

Ex :- 1) $x^2 - 3x + 2 = 0$

$a = 1$, $b = -3$, $c = 2$

2) $3x^2 + 6x + 12 = 0$

$a = 3$, $b = 6$, $c = 12$.

3). $4x^2 - 9 = 0$

$a = 4$, $b = 0$, $c = 9$.

4). $\cancel{7x^2 - 4} = x^2 - 10x$

$\cancel{-7x^2 + 4}$ $-7x^2$ $+4$

$-1 \cdot (+6x^2 + 10x + 4) = (0) \cdot -1$

$a = +6$, $b = +10$, $c = 4$

Soluing quadratic equations :-

Using Quadratic Formula.

Once the equation is in the genral form $(ax^2 + bx + c = 0)$ than the Solution Is can be found using the quadratic formula.

$$x = -b \pm \frac{\sqrt{discriminant}}{2a} \quad \text{where } \overset{\text{مميز}}{discriminant} = b^2 - 4ac$$

Not 80

disicrimiant < 0 No real solution.

disicrimiant $= 0$ one solution $x = \frac{-b}{2a}$

disicrimiant > 0 Two solution

Solve Ex:-

1). $x^2 - 3x + 2 = 0$

$a = 1, b = -3, c = 2$

$x = \frac{-b \pm \sqrt{dis}}{2a}$ $x = \frac{\overset{+}{\ominus} 3 \pm \sqrt{1}}{2 \cdot 1} \Rightarrow x = \frac{3 \pm 1}{2}$

$disc = b^2 - 4ac.$

$(-3)^2 - 4 \times 1 \cdot 2$

$9 - 8 = 1$

$\rightarrow x = \frac{3+1}{2} = \frac{4}{2} = \underline{\underline{2}}$

or

$\rightarrow x = \frac{3-1}{2} = \frac{2}{2} = 1$

2). $3x^2 + 6x + 12 = 0$

$a = 3, b = 6, c = 12.$

$disc = (6)^2 - 4 \cdot 3 \cdot 12.$

$= 36 - 144 < 0$

$= -108 < 0$ ∴ No real solution.

3). $4x^2 - 9 = 0$

$a = 4, b = 0, c = -9.$

$disc = 0^2 - 4 \cdot 4 \cdot -9$

$disc = 0 \overset{+}{\ominus} 144 = 144$

$x = \frac{-0 \mp \sqrt{144}}{2 \cdot 4} = \frac{-0 \mp \sqrt{144}}{8} = -$

$\rightarrow x = \frac{\pm 12}{8} \Rightarrow \frac{\cancel{\pm 12}^{\cancel{6}^{3}}}{\cancel{8}_{\cancel{4}_{2}}} = \frac{3}{2}$

or

$\rightarrow \frac{-12}{8} = \frac{-3}{2.}$

Using Factoring :

If the quadratic equation can be factord, than we can use the Zero property.

if $ab=0$ the either $a=0$ or $b=0$ or both.

Ex : Solve the followins :-

1)- $x^2-3x+2=0$

$(x-1)(x+2)=0 \rightarrow x-1=0$, $+1$ $+1$, $x=1$ / $x-2=0$, $+2$ $+2$, $x=2$.

$x=1$ or $x=2$.

2). $4x^2-9=0$

$(2x)^2-(3)^2=0$

$(2x-3)(2x+3)=0 \rightarrow (2x-3=0$, $+3$ $+3$ $\rightarrow \frac{2x}{2}=\frac{3}{2} \rightarrow x=\frac{3}{2}$

$2x+3=0$, -3 -3, $\frac{2x}{2}=\frac{-3}{2} \rightarrow x=\frac{-3}{2}$

$x=\frac{3}{2}$ or $x=\frac{-3}{2}$.

Section 2.2.
Quadratic Functions

A quadratic function has a general form of $y = f(x) = ax^2 + bx + c$ where $a \neq 0$

Ex:-

1- $f(x) = x^2 + 2x - 3$

$y =$ $a = 1, b = 2, c = -3$

2) $2y + \cancel{24} = 2x + 2x^2$
   $\quad -24 \qquad -24$ $\rightarrow \frac{2y}{2} = \frac{2x}{2} + \frac{2x^2}{2} - \frac{24}{2}$

$y = x^2 + x - 12$

$a = 1, b = 1, c = -12.$

3). $y = (x+2)^2 - 3 \rightarrow y = x^2 + 4x + 4 - 3$

$y = x^2 + 4x + 1$

$a = 1, b = 4, c = 1$

The graph of the quadratic function is a parabola open upward U (if $a > 0$) or downward ∩ (if $a < 0$). :- الرسم القطع المكافئ .

1- $a > 0 \rightarrow$ U   2. $a < 0$ ∩ (x, y) vertex $= \left(\overset{x}{\frac{-b}{2a}}, \overset{y}{f\left(\frac{-b}{2a}\right)}\right)$

Ex:-

1- $f(x) = x^2 + 2x - 3 \rightarrow$ Graph

$y = x^2 + 2x - 3$

$a = 1, b = 2, c = -3$

Vertex $= \left(\overset{x}{\frac{-b}{2a}}, \overset{y}{f\left(\frac{-b}{2a}\right)}\right)$

$\frac{-b}{2a} = \frac{-2}{2.1} = \boxed{-1}$

$f(-1) = (-1)^2 + 2 \cdot -1 - 3.$

$f(-1) = 1 - 2 - 3$ $\Longrightarrow$

$f(-1) = \overset{(x, y)}{(-1, -4)}$

⇒ Ex:- مثال

$a>0 \rightarrow U \rightarrow$ [illegible]

x-intercept (Zero of the funtion)

$y = x^2 + 2x - 3$

$0 = x^2 + 2x - 3$

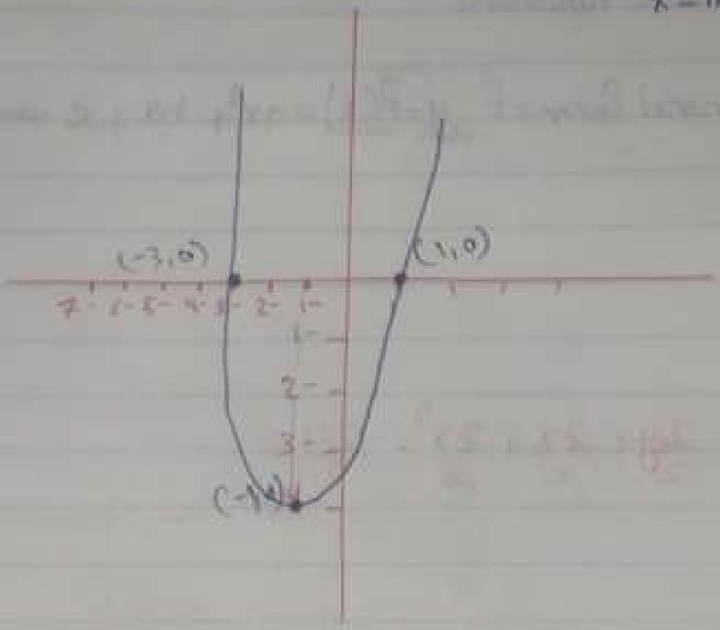
$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

y-intersat

$y = (0)^2 + 2(0) - 3$

$y = -3$



2). Ex:- $2y + 2y = 2x + 2x^2$

$y = x^2 + x - 12$

$a=1$, $b=1$, $c=-12$

vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \rightarrow \left(\frac{-1}{2}, -12,25\right)$

$\frac{-1}{2 \cdot 1} = \frac{-1}{2}$

$f\left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)^2 + \frac{-1}{2} - 12$

$f\left(\frac{-1}{2}\right) = \frac{1}{4} + \frac{^{2 \cdot}1}{2 \times 2} - \frac{^{4 \cdot}12}{1 \cdot 4}$

$\frac{1}{4} + \frac{-2}{4} - \frac{48}{4}$

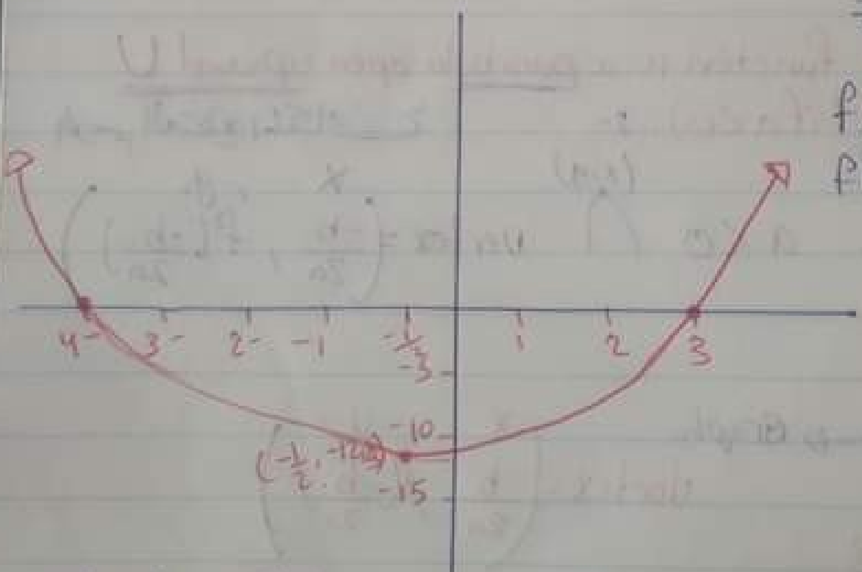
$= \frac{-49}{4} = -12,25$

Zero

$0 = x^2 + x - 12$

$0 = (x-3)(x+4)$

$x = +3$, $x = -4$



what is the pointed Value -12,25 (Mimimum)

optimal point $= \left(-\frac{1}{2}, -12,25\right)$.

is the optimal point a min or Max

= minimam point.

Vertax = optimal point.

$$\begin{tikzcd} & \text{optimal point} \arrow[dl] \arrow[dr] & \\ \text{Maximum point} & & \text{Minimum point} \end{tikzcd}$$

Maximum point

$a < 0$

Minimum point

$a > 0$

$$\left(\overset{x}{\frac{-b}{2a}}, \overset{y}{f\left(\frac{-b}{2a}\right)}\right)$$

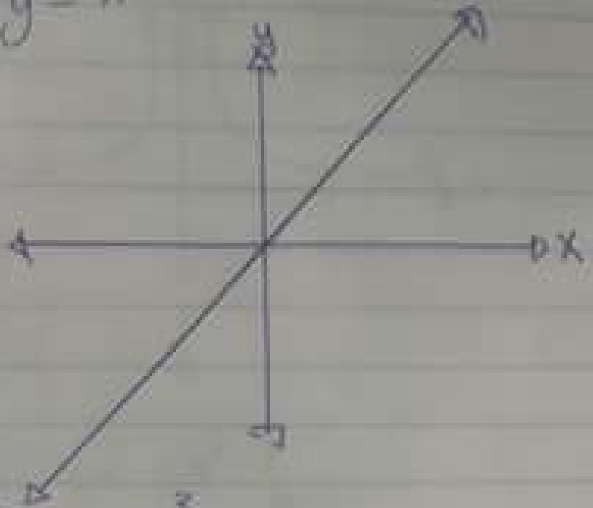
$f\left(\frac{-b}{2a}\right)$ optimal vertex.

Sactions 2.4

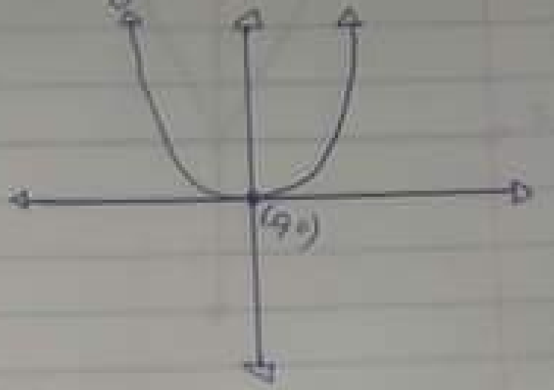
Spacial Functions and Their Graphs.

(هدول الشكلين عبارة عن رسمات بنحفظهم وتطبيق القواعد عليهم.

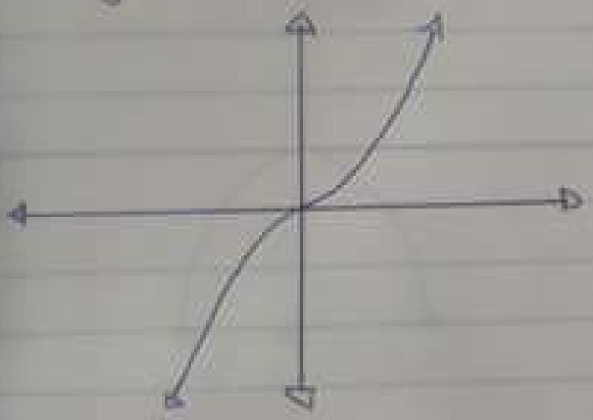
① $y = x$ ← $y = ax + b$



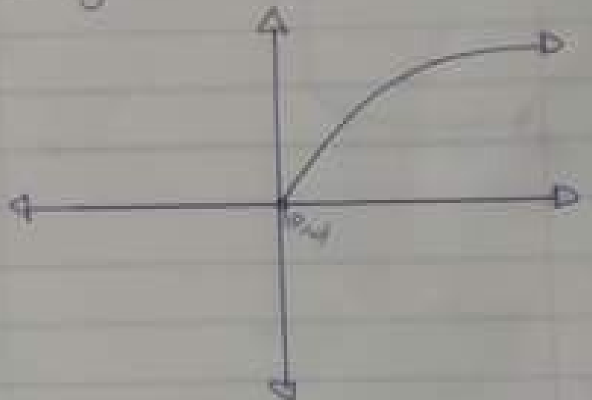
② $y = x^2$ ← $y = ax^2 + bx + c$ → $a \neq 0$.



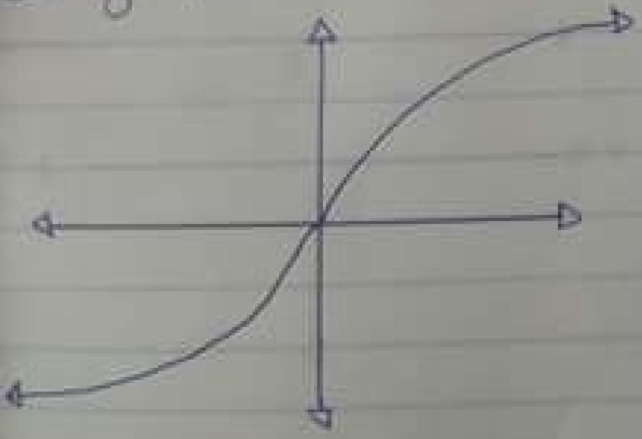
③ $y = x^3$



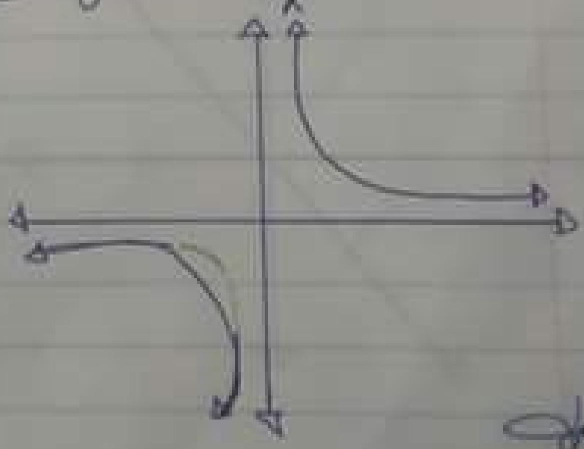
④ $y = x^{\frac{1}{2}} = \sqrt{x}$



⑤ $y = x^{\frac{1}{3}} = \sqrt[3]{x}$

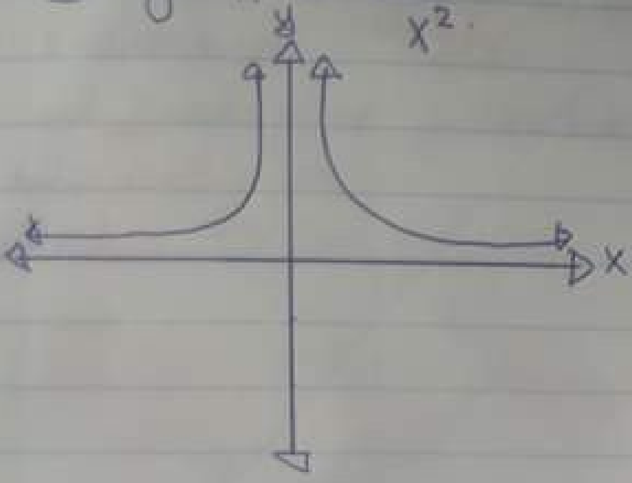


⑥ $y = x^{-1} = \frac{1}{x}$

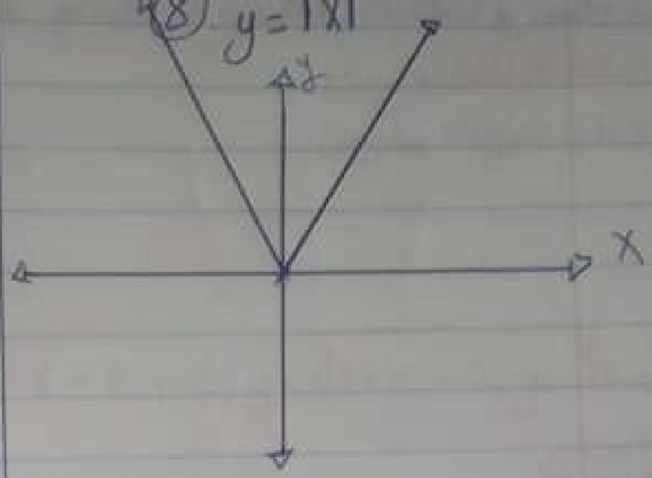


Sactions 2.4

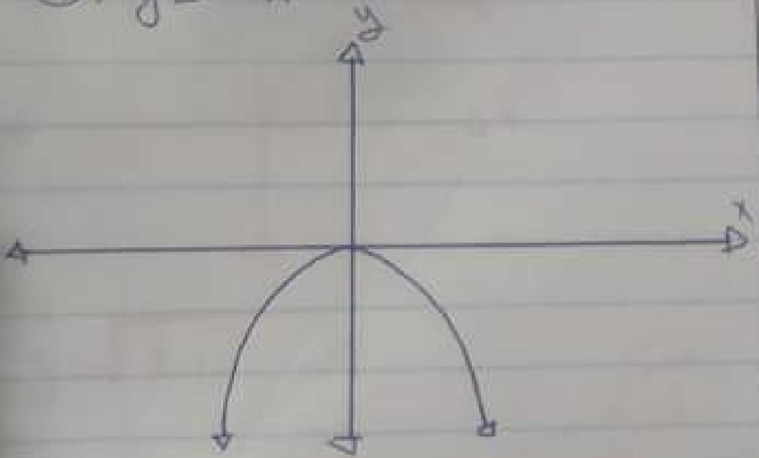
⑦. $y = x^{-2} = \frac{1}{x^2}$.



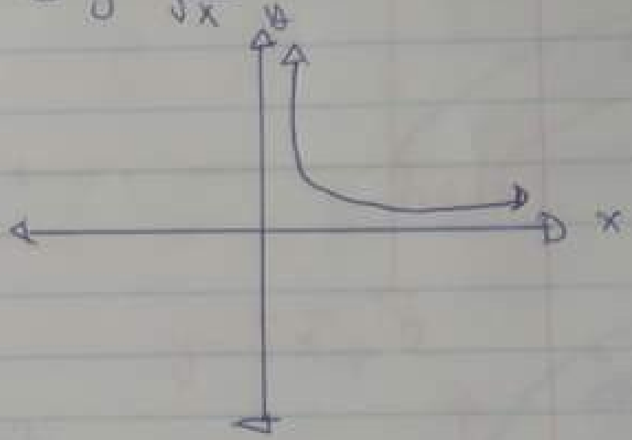
⑧. $y = |x|$



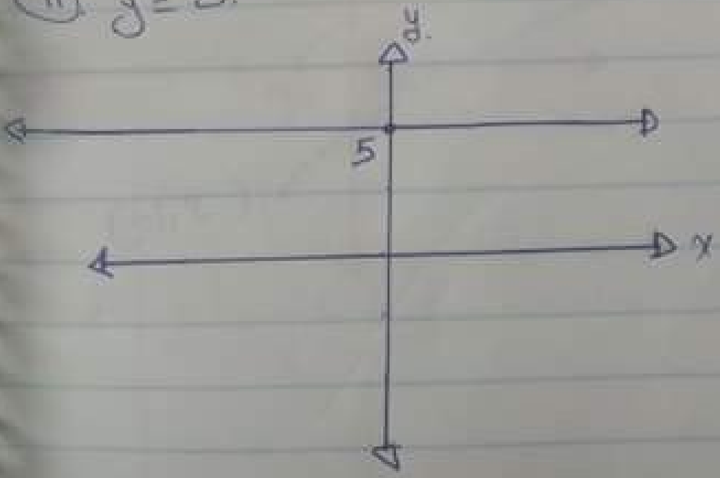
⑨. $y = -x^2$



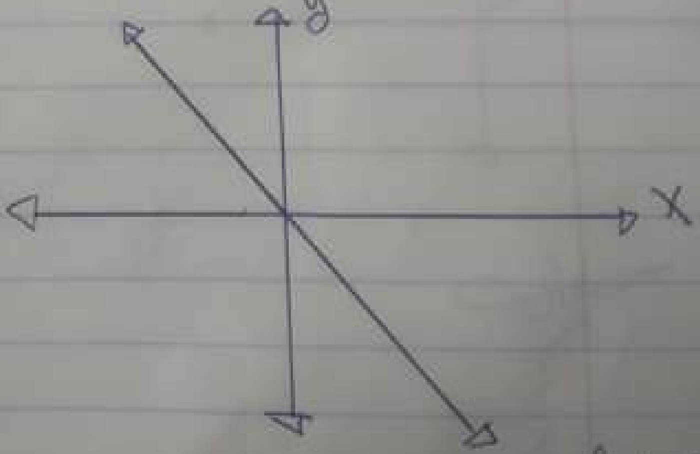
⑩. $y = \frac{1}{\sqrt{x}}$



⑪. $y = 5$.



⑫. $y = -x$



* Shifts of Graphs 80

The graph of $y = f(\overline{x-h}) + k$ is the graph of $y = f(x)$ shifted $\underline{h}$ units in the x-direction $f(x)$

لليسار ← Left if $h<0$

لليمين ← right is $h>0$

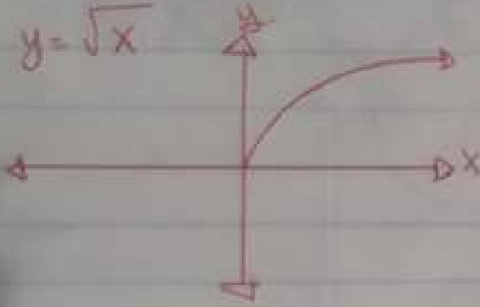
and $\underline{k}$ unitsinthe y-direction.

لفوق ← up if $k>0$

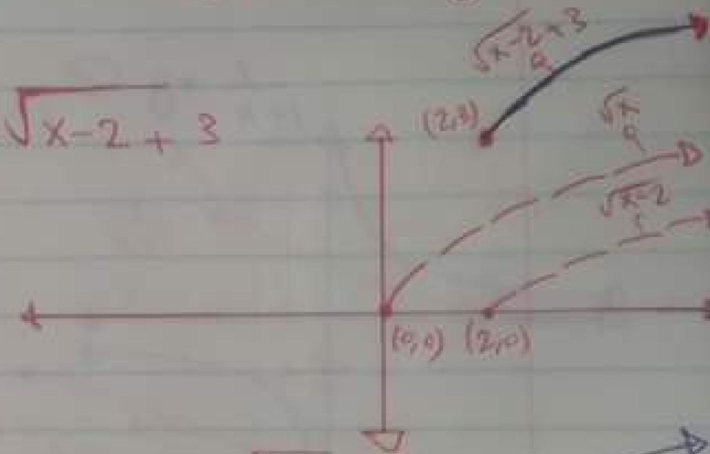
لتحت ← down if $k<0$

Ex:- Graph $y=\sqrt{x}$ than graph $y=\sqrt{x-2}+3$, $y=\sqrt{x+1}-2$, $y=\sqrt{x}+2$ and $y=\sqrt{x+1}$

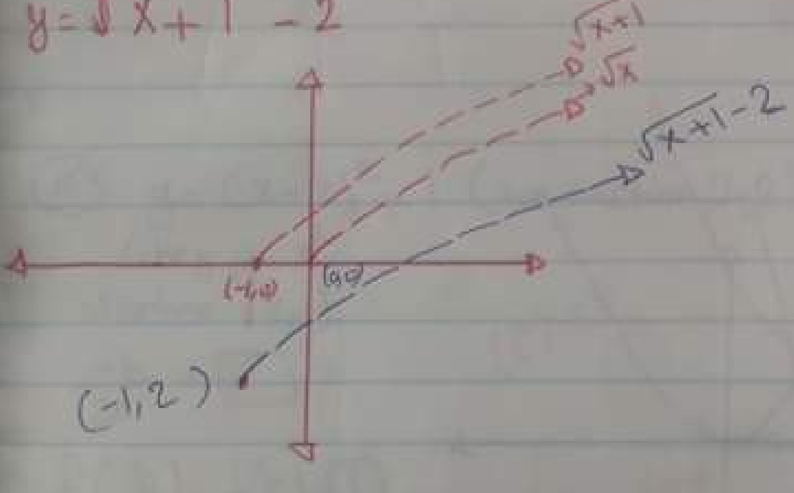
$y=\sqrt{x}$



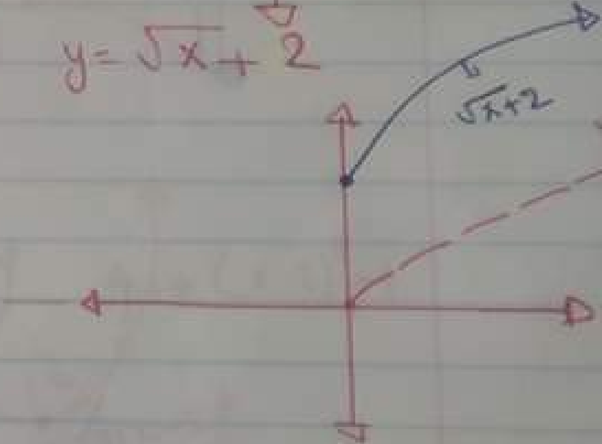
$y=\sqrt{x-2}+3$



$y=\sqrt{x+1}-2$

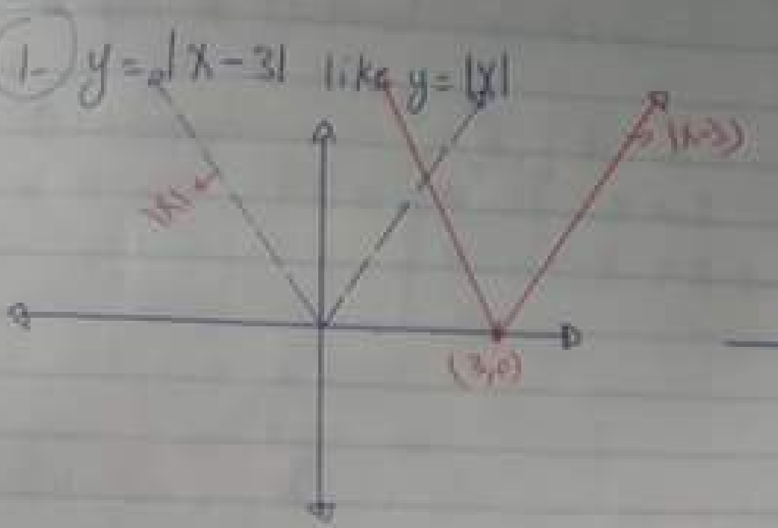


$y=\sqrt{x}+2$

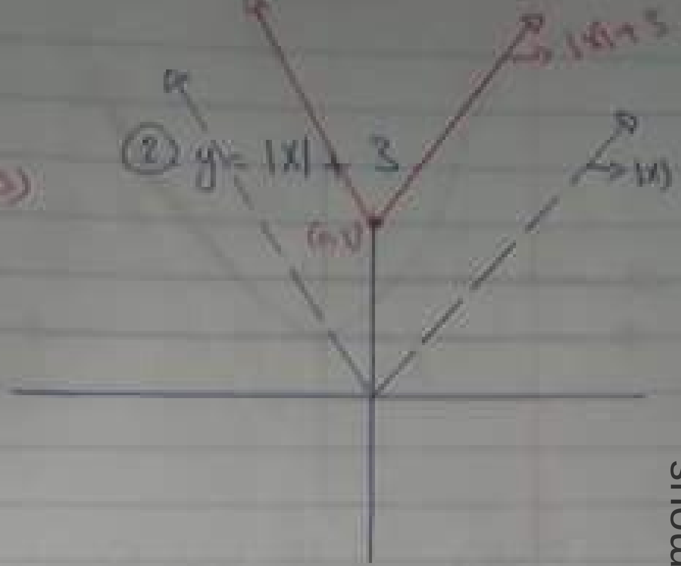


Ex:- graph:-

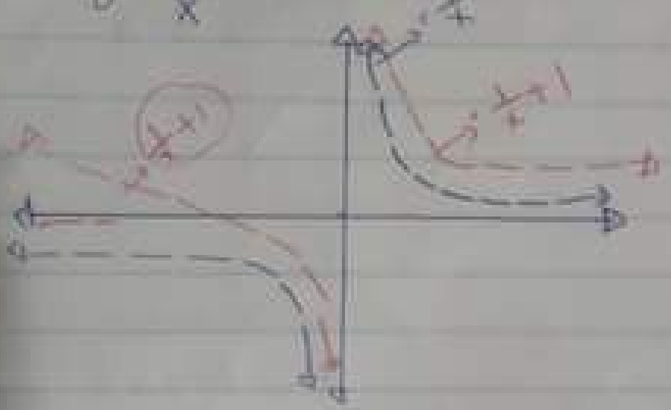
①1- $y = |x-3|$ like $y = |x|$



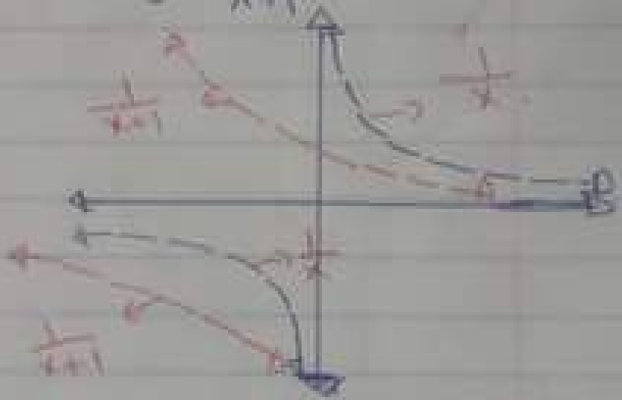
② $y = |x| + 3$



③. $y = \frac{1}{x} + 1$



④ $y = \frac{1}{x+1}$



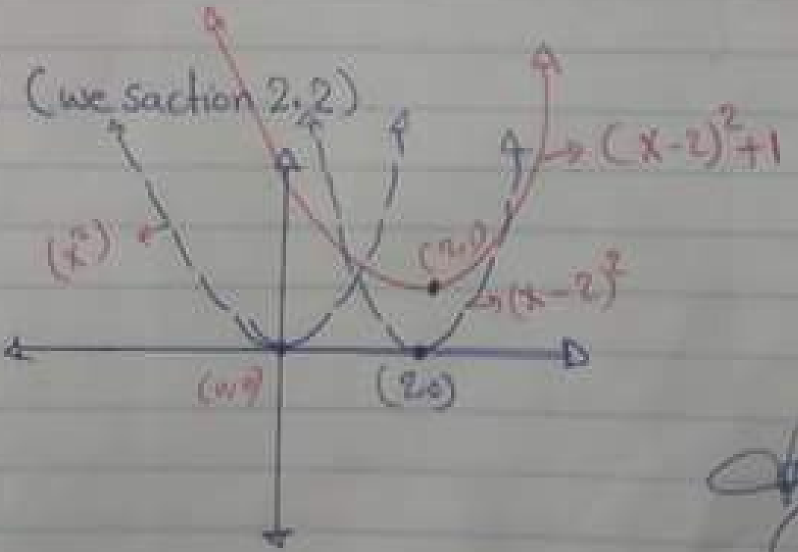
⑤. $y = (x-2)^2 + 1$ (we saction 2.2)

Likey $y = x^2$

vertex (2,1)

$\frac{-b}{2a} = \boxed{2}$

$f(\frac{-b}{2a}) = f(2) = \boxed{1}$

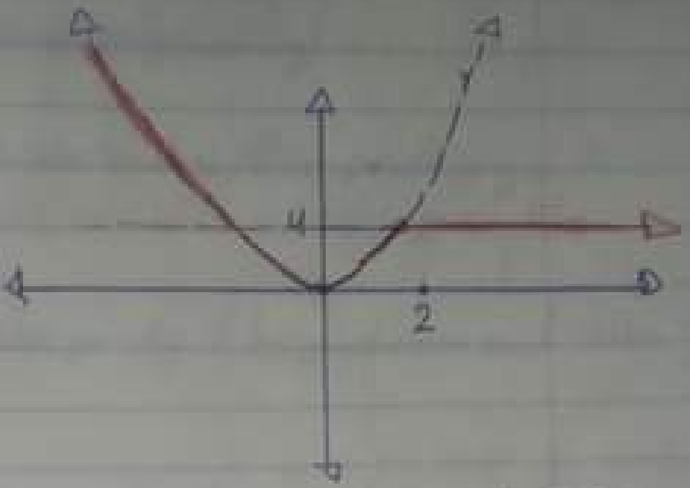


أقتران متعدد القواعد

Piecewise Defined Functions :-

Ex:- Graph

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 4 & \text{if } x > 2 \end{cases}$$



Chapter 4

Section 4.1
Inequalities and linear programming

* Suppose that the daily profit, a Carpenter makes, from producing and selling chairs and tables are 10$ per Chairs and 15$ per table

How much profit would the cepenter makes per day if he produceces and selles 4 chairs per day? $10 \times 4 = 40\$$

How much profit would the capenter makes per day if he produceces and selles 2 Chairre and 2 table per day? $10 \times 2 + 15 \times 2 = 50\$$.

In general, if we let ⇒ X = number of chairs produced and sold each day
y = number of table produced and sold each day

Than the total daily profit $= p = 10X + 15y$

* what value of X and y will maximize the daily profit?

* Suppose that each chairs requieres $1m^2$ of wood, each table requieres $2m^2$ of woods, and the total amount of wood availlable each day is $14m^2$. This will impose the fllowing constraint on X and y. ⇒ $1X + 2y \leqslant 14$

* Suppose that each Chair requieres 2 houres of labour, each tobal requieres 1 houres of labors, and the total dily work time 10 houres. This will impose the following contranits on X.

⇒ $2X + 1y \leqslant 10$

Now what value of X and y will maximize the daily profit? 4, 2

This kind of problem where you need to optimize a function called the Object Function $p = 10x +$

Subject to a system of linear inequalities called Constraints $\begin{cases} x + 2y \leqslant 14 \\ 2x + y \leqslant 10 \\ x \geqslant 0 \\ y \geqslant 0 \end{cases}$ is called linear programming prob

* To solve linear programming problems, we need to know how to solve linear inequalities in ~~two~~ two variables. (Section 4.1)

1.1 ⇒ radical → $\underset{-4}{x+4} \geq \underset{-4}{15} \Rightarrow x \geq 11$ ←

Examples:- Graph the solution of:- $4x - 2y \leq 6$

$4x - 2y = 6$

$x=0 \rightarrow 4\cdot0 - 2y = 6$

$0 \frac{-2y}{-2} = \frac{6}{-2}$

$y = -3 \rightarrow (0,-3)$

$y=0 \rightarrow \frac{4x}{4} - 2\cdot0 = \frac{6}{4}$

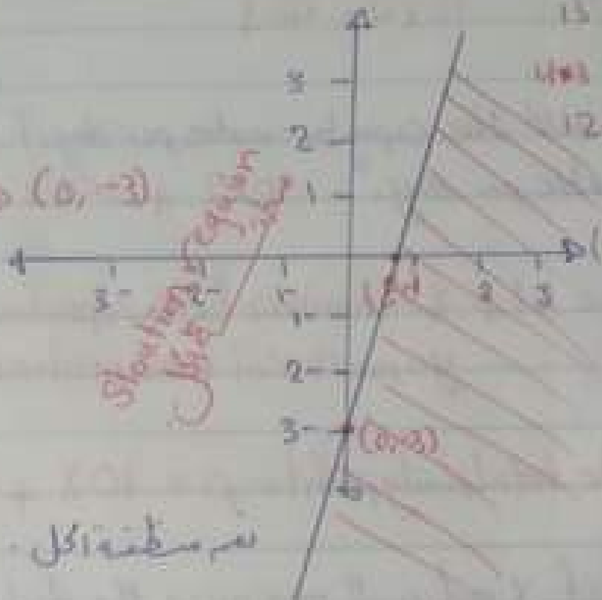
$x = \frac{6}{4}$

$(\frac{6}{4}, 0)$

text point (0,0)

$4\cdot0 - 2\cdot0 \leq 6$

$0 \leq 6$ (✓) نفس منطقة الحل



[illegible]

$\overset{x}{3}, \overset{y}{-4}$ ↳ (3, -4)

$4\cdot3 - 2\cdot-4 \leq 6$

$12 + 8 \leq 6$

$20 \leq 6$ $\boxed{X}$

(3,-4) Not solution

Ex:- Graph the solution of:- $2x + y > 4$ / [illegible]

[illegible]

$2x + y = 4$

x	y
0	4
2	0

$2\cdot0 + y = 4$

$\frac{2x}{2} + 0 = \frac{4}{2}$

$x = 2$

text point (0,0)

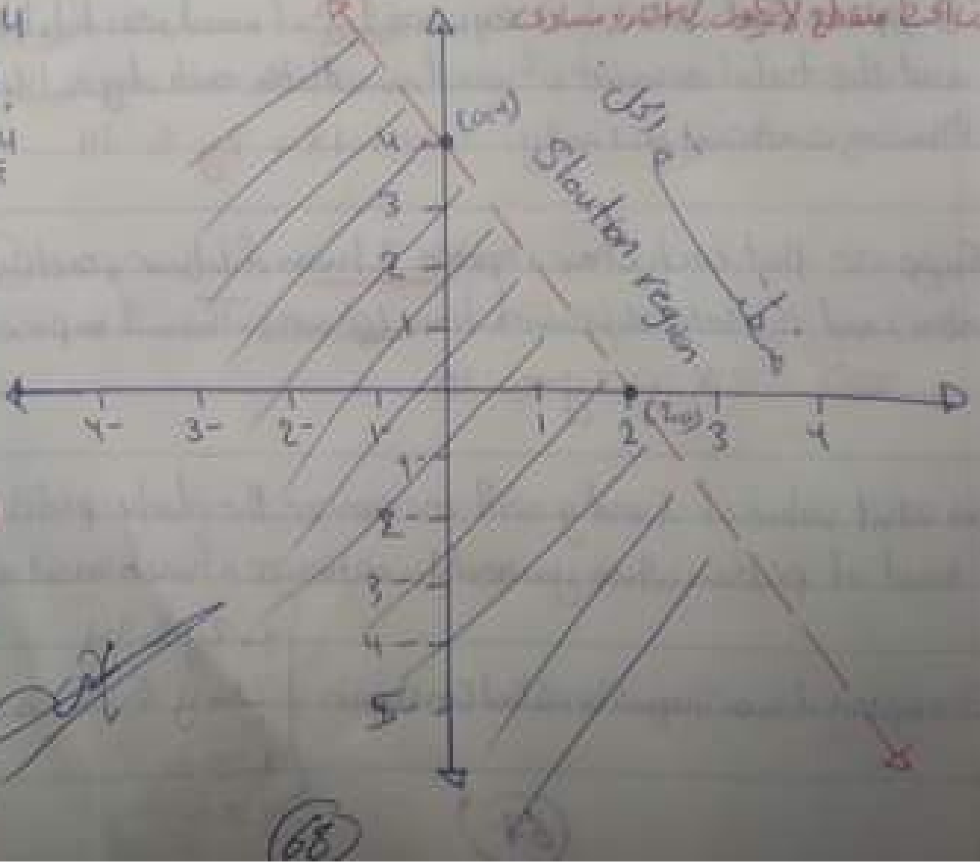
$2\cdot0 + 0 > 4$

$0 > 4$ X

text point (3,0)

$2\cdot3 + 0 > 4$

$6 > 4$ (✓)



Examples:- Graph the solution of $3x - 2y \geq 4$
$x + y > 3$

→1) $3x - 2y = 4$

$3 \cdot 0 - 2y = 4$

$\frac{-2y}{-2} = \frac{4}{-2}$

$y = -2 \rightarrow (0, -2)$

$3x - 2 \cdot 0 = 4$

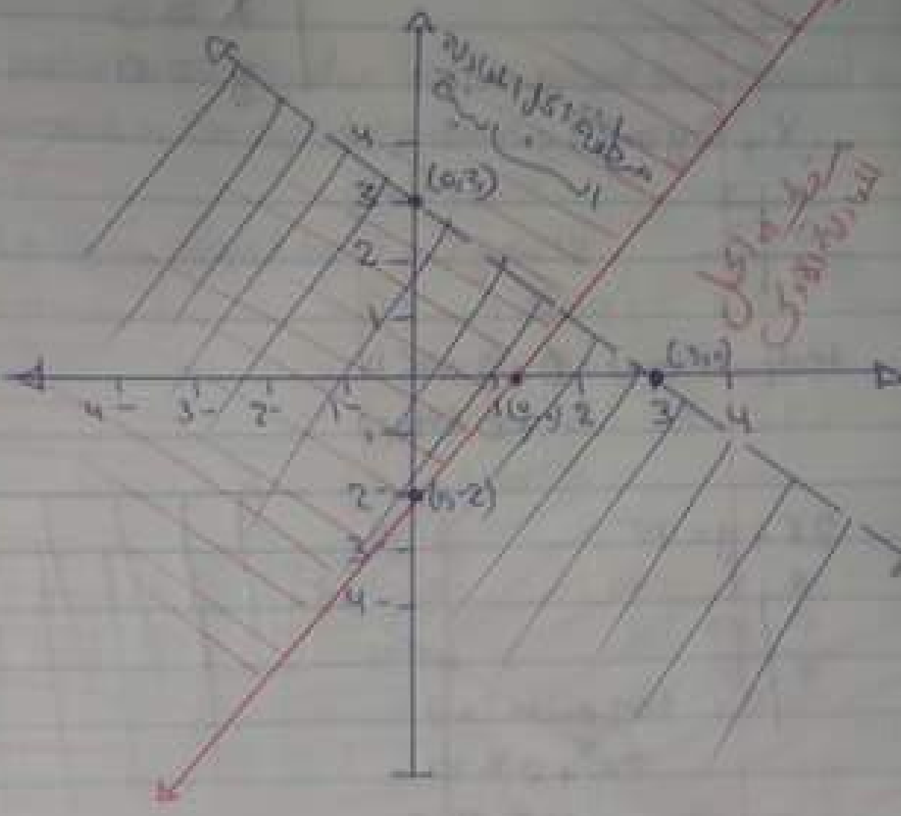
$\frac{3x}{3} = \frac{4}{3}$

$x = \frac{4}{3} \rightarrow (\frac{4}{3}, 0)$

test point (0,0)

$3 \cdot 0 - 2 \cdot 0 \geq 4$

$0 \geq 4 \rightarrow \boxed{X}$



2. $x + y = 3$

$0 + y = 3 \rightarrow (0,3)$

$x + 0 = 3 \rightarrow (3,0)$

test point (0,0)

$0 + 0 > 3$

$0 > 3 \boxed{X}$

Examples :- Graph the solution of

$$\begin{cases} x + 2y \leq 14 \\ 2x + y \leq 10 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$x + 2y = 14$

x	y
0	7
14	0

textpoint (0,0) = $0 + 2 \cdot 0 \leq 14$

$0 \leq 14$ (✓)

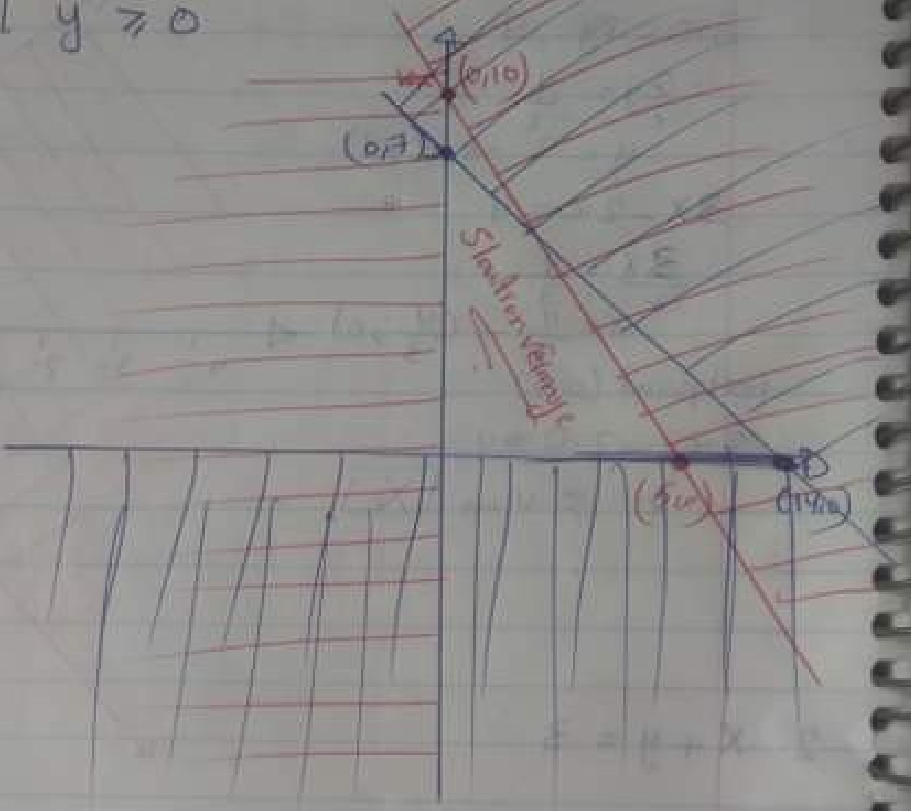
$2x + y = 10$

x	y
0	10
5	0

text point = (0,0)

$2 \cdot 0 + 0 \leq 10$

$0 \leq 10$ ✓



$x \geq 0$ → X الاعداد الموجبة

$y \geq 0$ → y الاعداد الموجبة

Section 4.2

Linearprogramming.. Graphical Methods

* A problem where we need to optimize an objective function subject to contramint is Call a linear programming problem such as the capender problem:-

Maximize P = 10X + 15y → subject to { X + 2y ≤ 14, 2X + y ≤ 10, X ≥ 0, y ≥ 0

X + 2y = 14

X	y
0	7
14	0

taxt point (0,0)

X + 2.0 ≤ 14

0 ≤ 14 (✓)

2X + y = 10

X	y
0	10
5	0

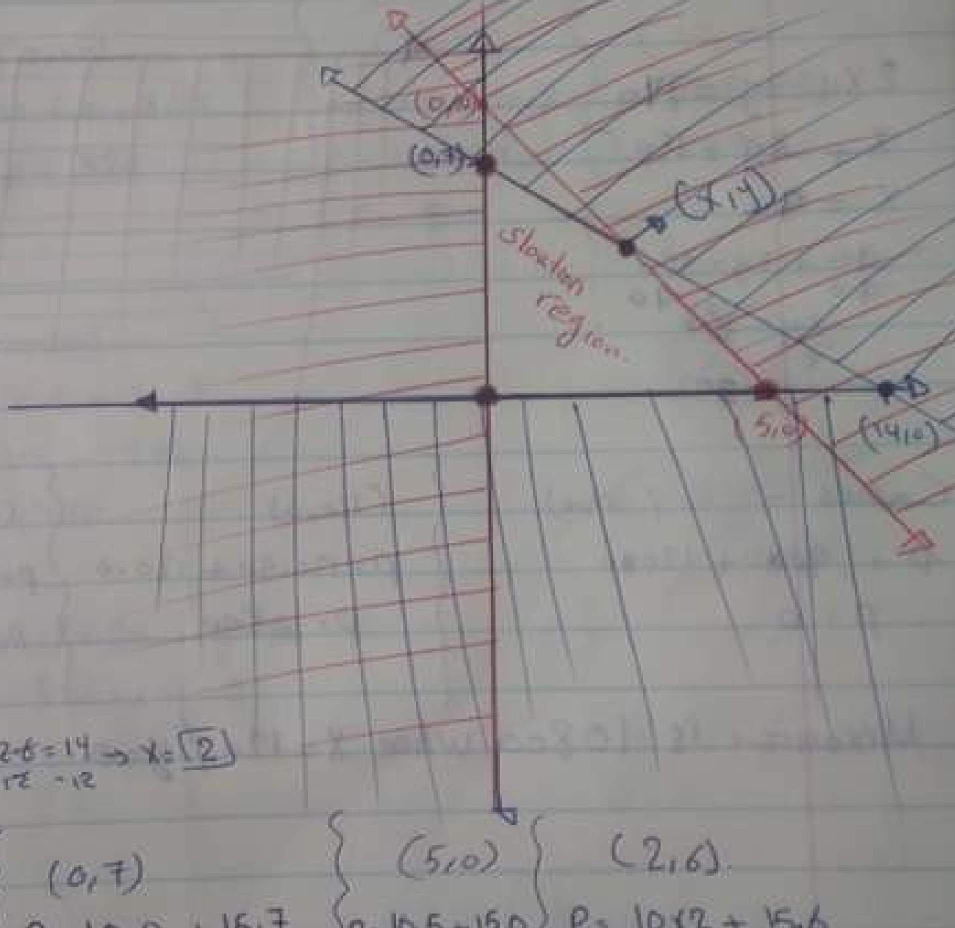
taxt point (0,0)

20 + 0 ≤ 10

0 ≤ 10 ✓

X = 0 → (1,0)

X - y (0,1)



نقاطة عين ⇐

(X + 2y = 14)·2

2X + y = 10

$\cancel{2X} + 4y = 28$

$- \cancel{2X} + y = 10$

$\frac{3y}{3} = \frac{18}{3} \Rightarrow y = 6$

$X + 2\cdot6 = 14 \Rightarrow X = \boxed{2}$
-12 -12

نقاط الشرك

Corners	(0,0)	(0,7)	(5,0)	(2,6)
	P= 10X + 15y P= 10.0 + 15.0 P=0	P= 10.0 + 15.7 P= 105	P= 10.5 + 15.0 = 50	P= 10×2 + 15.6 P = 20 + 90 P = 110

Maximize point is 110 $ when ⇒ X = 2, y = 6

Examples : Maximize 90X + 120y Subject to $\begin{cases} 2X + 3y \leq 240 \\ X + 3y \leq 150 \\ x \geq 0 \\ y \geq 0 \end{cases}$

① $2X + 3y = 240$

x	y
0	80
120	0

testpoint (0,0) $\Rightarrow 2.0 + 3.0 \leq 240$

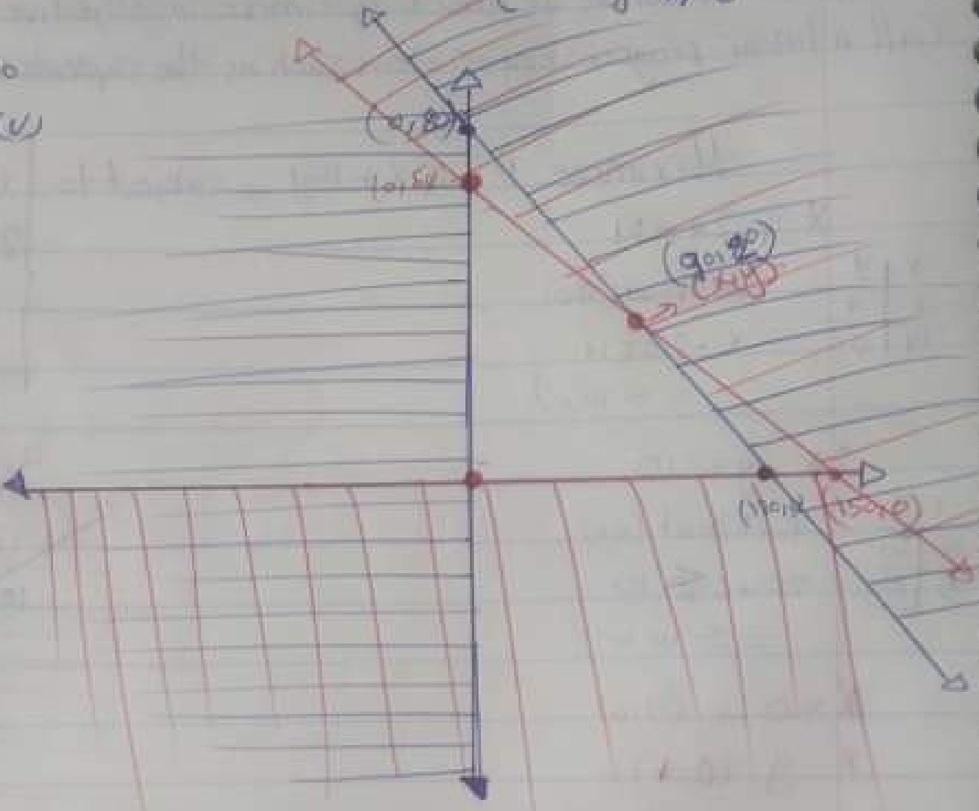
$0 \leq 240$ (✓)

② $X + 3y = 150$

x	y
0	50
150	0

totalpoint (0,0) $0 + 3.0 \leq 150$

$0 \leq 150$ (✓)



$2X + 3y = 240$

$X + 3y = 150$ −

$X = 90$

$90 + 3y = 150$

$-90 \quad -90$

$\frac{3y}{3} = \frac{60}{3}$

$y = \boxed{20}$

Covnes :- (0,0)
$p = 90 \times 0 + 120 \times 0$
$p = 0$

(120,0)
$p = 120 \cdot 90 + 120 \cdot 0$
$p = 10800$

(0,50)
$p = 90 \cdot 0 + 50 \cdot 120$
$p = 6000$

(90,20)
$p = 90 \cdot 90 + 120 \cdot 20$
8100 + ٢٤٠٠
10500

Mixamizer is 10800 when X = 120 / y = 0

حل سؤال الكتاب . ص 275

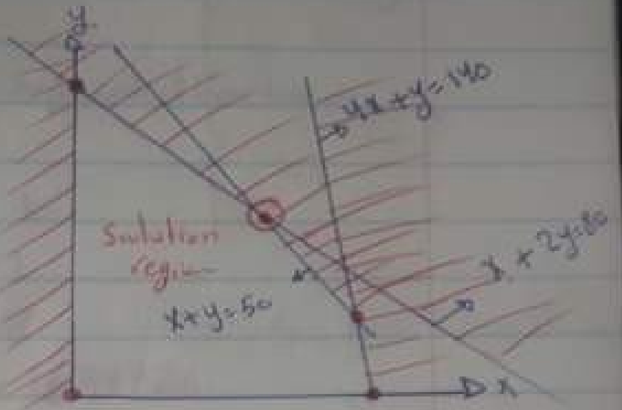
* In each problem 5-8 , Graph and find Mixamimand minimourns .

6) $f = 5x + 8y$

$4x + 2y = 180$
$- \quad x + y = 50$
$\frac{3\cancel{x}}{3} = \frac{90}{3} \rightarrow \boxed{x = 30}$
$\boxed{y = 20}$

$x + 2y = 80$
$0 + \frac{2y}{2} = \frac{80}{2}$
$y = 40$

$4x + y = 140$
$\frac{4x}{4} + 0 = \frac{140}{4}$
$x = 35$



Cornus :- (0,0)
$p = 5.0 + 8.0$
$p = 0$

(0,40)
$p = 5.0 + 8.40$
$p = 320$

(30,20)
$p = 30.5 + 20.8$
$p = 150 + 160$
$P = 310$

(35,0)
$p = 35.5 + 8.0$
$p = 175$

$x + 2y = 80$
$- \quad x + y = 50$
$y = 30 \rightarrow x = 20$

(20,30)
$p = 20.5 + 30.8$
$p = 100 + 240$
$p = 340$

mixamemes is 340 when $x = 20$, $y = 30$
minimousc is 0 whon $x = 0$, $y = 0$

Chapter 5

Section 5.1
Exponential Functions

* Consider a microorganism that doubles every houre. if we start with 1 organism and let y = number of organisms, that at

time	y
ohere	1
1 hour	1.2 = 2
2 houre	1.2.2 = 4
3 houre	2.2.2 = 8
X houre	2.2.2. X time.

exponential Function:

$y = 2^x$

$x = 2 \Rightarrow y = 2^2 = 4$

$x = 0 \Rightarrow y = 2^0 = 1$

$x = 3 \Rightarrow y = 2^3 = 8$

$x = 24 \Rightarrow y = 2^{24} = 16777216$

In general if $a > 0$, and $a \neq 1$, than $f(x) = c(a)^x$ is an exponential functions.

Ex: $y = 4^x$ (growth) $c = 1 / a = 4$

$y = (0.4)^x$ $c = 1 / a = 0.4$ (decay)

$y = 5(0.4)^x$ $c = 5 / a = 0.4$ (decay)

$y = 6(4)^{-x}$ $c = 6 / a = \frac{1}{4}$ (decay)

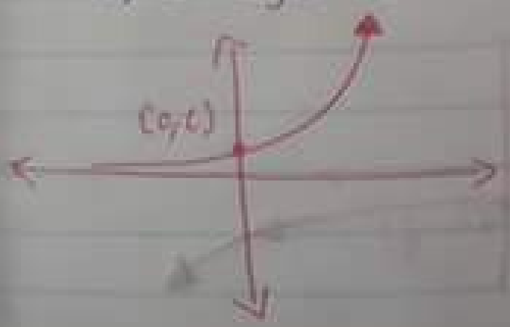
$y = (9/7)^{-x} \to (\frac{7}{9})^x$ $c = 1 / a = \frac{7}{9}$ (decay)

For $c > 0 \Rightarrow \begin{cases} ce^x \text{ is a growth } a > 1 \\ ce^t \text{ is a decay } a < 1 \end{cases}$

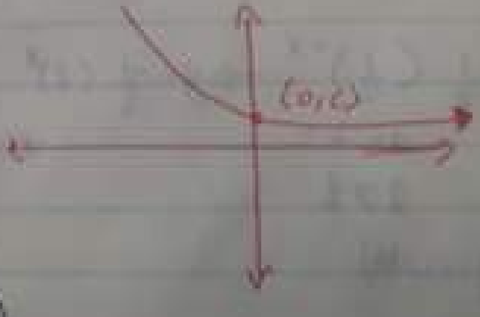
Graphing Exponential functions:

If $c > 0$ than the graph of $f(x) = c(a)^x$ is

$a > 1$ (growth)

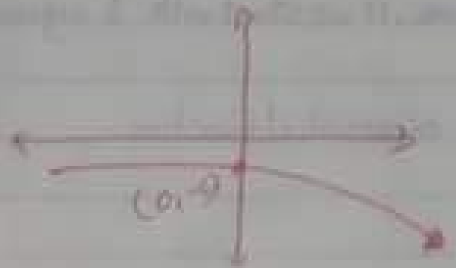


$a < 1$ (decay)

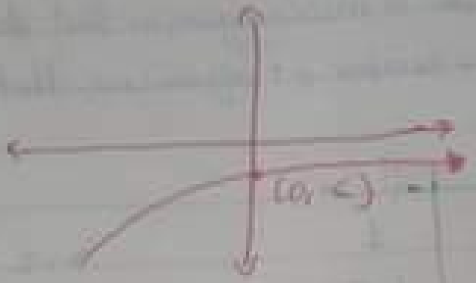


* if $\underline{c<0}$ then the graph is the same but reflected about the x-axis.

$a>1$



$a<1$



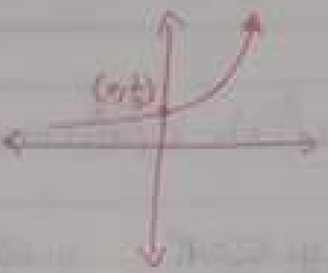
Example :- graph the following :- ملاحظة ← ميّز مقدار تراجع c / و a ← مقدار تراجع الجواب

1) $y=\frac{1}{3}6^{x}$

$c=\frac{1}{3}$ $a=6$

$\frac{1}{3}>0$ $6>1$

(growth)

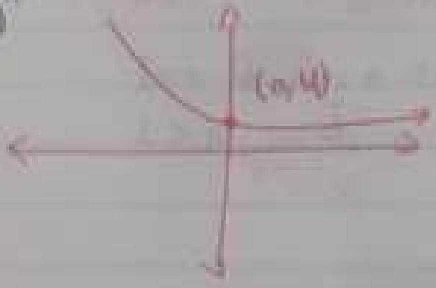


2.) $y=4\left(\frac{7}{3}\right)^{-x} \Rightarrow y=4\left(\frac{3}{7}\right)^{x}$

$c=4$ $a=\frac{3}{7}$

$4>0$ $\frac{3}{7}<1$

(decay)

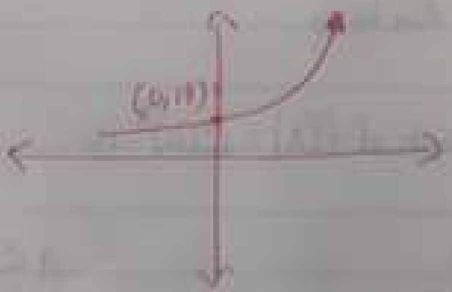


3.) $f(x)=17(\pi)^{x}$

$c=17$ $a=\pi\approx 3.14$

$17>1$ $3.14>1$

(growth)

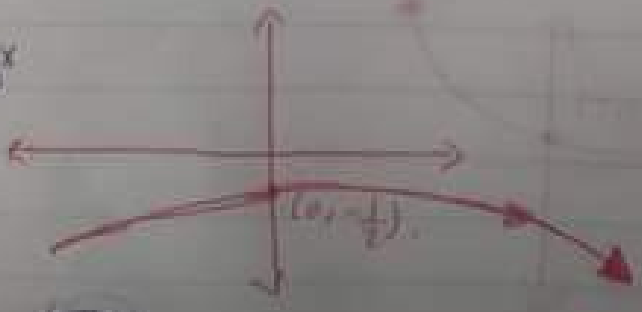


4.) $y=-\frac{1}{2}\left(\frac{1}{2}\right)^{-x} \rightarrow y=\frac{-1}{2}(2)^{x}$

$c=\frac{-1}{2}$ $a=2$

$\frac{-1}{2}<0$ $2>1$

(growth)



* A special exponential function is the base $e \approx 2.7$ function. Talk about it and graph it

$f(x) = e^x$

$c = 1$ / $a = 2.7$

$1 > 0$ / $2.7 > 1$

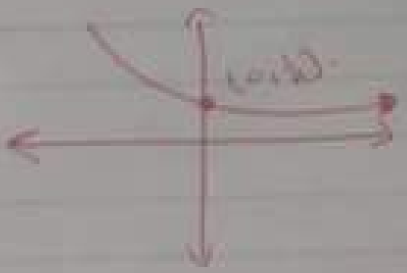
(growth)

Questions: indicate methode the following is a growth or decay funtion then graphit.

$y = \frac{1}{9}\left(\frac{\pi}{e}\right)^{-x} \rightarrow y = \frac{1}{9}\left(\frac{e}{\pi}\right)^{x}$

$c = \frac{1}{9}$ $\quad a = \frac{e}{\pi} < 1$

$\frac{1}{9} > 0$



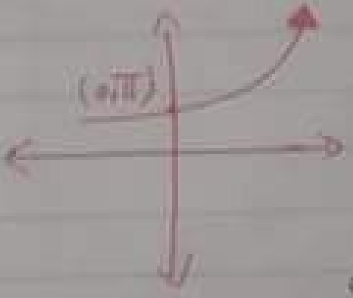
graht:-

$y = \pi 6^x$

$c = \pi$ $\quad a = 6$

$\pi > 0$ $\quad 6 > 1$

(growth).



outline 5.1

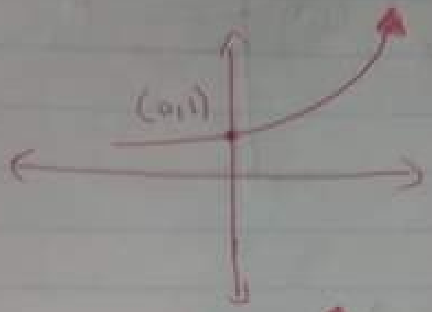
the Graph:- $f(x) = c(a)^x$ and sat, if the function growth or decay function.

6.) $y = 8^x$

$c = 1$ / $a = 8$

$1 > 0$ / $8 > 1$

(growth)



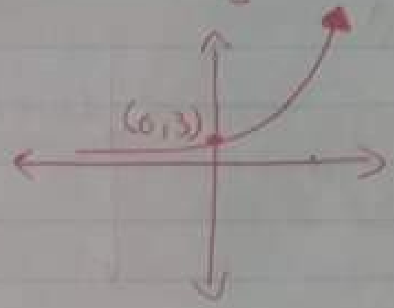
y-intersect $= (0,1)$

8.) $y = 3(2)^x$

$c = 3$ / $a = 2$

$3 > 1$ / $2 > 1$

(growth.)

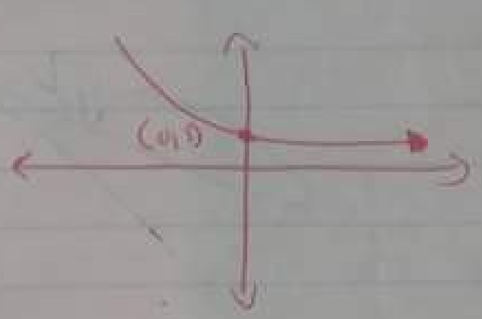


10.) $y = \left(\frac{2}{3}\right)^x$

$c = 1$ / $a = \frac{2}{3}$

$1 > 0$ / $\frac{2}{3} < 1$

(decay).



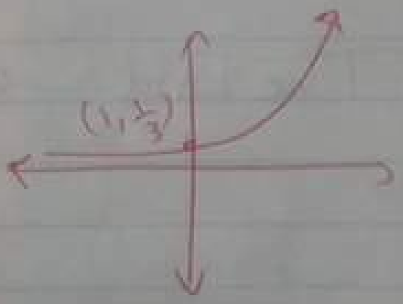
12.) $y = 3^{x-1} \rightarrow y = 3^x \cdot 3^{-1}$

$\frac{1}{3}(3^x)$

$c = \frac{1}{3}$ / $a = 3$

$\frac{1}{3} > 0$ / $3 > 1$

(growth)

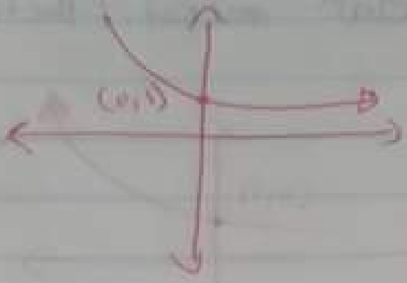


14-) $y = 3^{-x} \rightarrow y = (\frac{1}{3})^x$

$C = 1$ / $a = \frac{1}{3}$

$1 > 0$ / $\frac{1}{3} < 1$

(dacey)

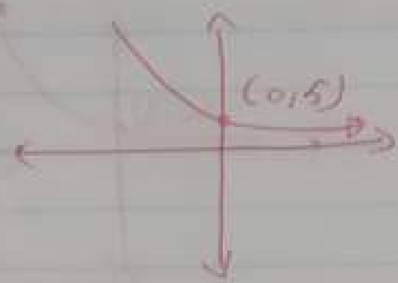


16-) $y = 5e^{-x} \rightarrow y = 5(\frac{1}{e})^x$

$C = 5$ / $a = \frac{1}{e}$

$5 > 0$ / $\frac{1}{e} < 1$

(dacey.)



The End.... ♡