



Birzeit University

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

First Semester – 2023/2024

ENCS2340 - Digital Systems

Homework # 1 - Due Thursday October 19, 2023

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**Section
A**

**THIS SECTION CONSISTS OF SIX QUESTIONS.
YOU MUST ANSWER ALL THE QUESTIONS IN THIS SECTION.**

Question 1 (12 points):

Find the value of X that satisfies the following equations:

- $(739)_{10} = (X)_7$
- $(110010.101)_2 = (X)_9$
- $(10101.10101)_4 = (X)_{16}$
- $(2699)_{10} = (BEE)_X$
- $(24)_X + (17)_X = (40)_X$
- $X = \text{the } 10\text{'s complement of } (935)_{11}$

Question 2 (21 points):

Perform the following arithmetic operations on the given *unsigned numbers* using the designated bases without converting to decimal (Don't use r 's complement or $(r - 1)$'s complement methods). You may verify your result by converting the numbers to decimal and then performing the operation in decimal:

- $(10111011)_2 - (1001111)_2$
- $(1101)_2 * (1011)_2$
- $(52E9)_{16} - (133F)_{16}$
- $(11011.0111)_2 + (11.1101)_2$
- $(27.61)_{16} + (25.9F)_{16}$
- $(AE.F3)_{16} - (103.111)_4 = (X)_2$ (Find X)
- $(110100.01)_4 - (111.101)_2 = (Y)_2$ (Find Y)

Question 3 (20 points):

Conversion between different numbering systems. Fill the table below with the different representations of numbers. BCD stands for binary-coded decimal. For fractions, stop after 4 digits after the base point.

Decimal	Binary	Octal	Hexadecimal	BCD
19.22				
	10001.10011			
		745.347		
			1DF.23	
				10010011.0010

Question 4 (6 points):

Perform $(669)_{10} + (835)_{10}$ using BCD addition. That is, convert the two decimal numbers to BCD code and add the two to get the result in BCD.

Question 5 (5 points):

A new university has 2,000 students. It is required to give each student a unique binary code (inside the registration software):

- How many bits would we need?
- If it is anticipated that the number of students will double every 3 years, how many bits would we need after 15 years?
- For the sake of documentation, how many Hex digits would we need to represent these codes (now and after 15 years)? Comment on the number of Hex digits needed as compared to the binary bits needed.

Question 6 (12 points):

If 6-bit registers are used, perform the following signed 2's complement arithmetic operations on the provided signed 2's complement binary numbers. Check for overflow and mark clearly any overflow occurrences.

- $101101 + 110011$
- $01101 - 101101$
- $010011 - 01101$
- $1011 - 110000$

Section B	YOU MUST ANSWER TWO QUESTIONS FROM THIS SECTION. ANSWER ALL THE PARTS FOR THE TWO QUESTIONS YOU CHOOSE.
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Question 7 (12 points):

Determine the decimal value of the 7-bit binary numbers **A** = (1001100) and **B** = (0101010) when interpreted as:

- a) Unsigned numbers.
- b) Signed-magnitude numbers.
- c) Signed 1's complement numbers.
- d) Signed 2's complement numbers.

Question 8 (12 points):

If you type the phrase 'ENC52340' on your keyboard, what is the binary sequence sent to the computer using 8-bit ASCII with the 8th most-significant bit being an even parity bit. Note that the 7-bit ASCII code of 'A' and '0' in hexadecimal are 41 and 30, respectively.

Question 9 (12 points):

If 6-bit registers are used, show the binary number representation of the decimal numbers 23 and -23 using the following representation systems:

- a) Unsigned system.
- b) Signed-magnitude system.
- c) Signed 1's complement system.
- d) Signed 2's complement system.

Section C	EXTRA CREDIT, THE FOLLOWING QUESTION IS BONUS.
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Question 10 (5 points):

In 2's complement representation with 5-bits.

- a) What is the largest positive number that can be added to $(10)_{10}$ without causing an overflow.
- b) What is the largest positive number that can be subtracted from $(10)_{10}$ without causing overflow.

GOOD LUCK

Question 18-

a)
$$\begin{array}{r|l} 739 & 7 \\ 15 & 4 \\ 2 & 1 \\ 0 & 2 \end{array}$$

$X = (214)_7$

b) $(110010.101)_2 = (X)_9$
32 16 8 4 2 1 -1 -2 -3

- Decimal to Binary is direct ①

$$= 2 + 16 + 32 \quad \left| \quad 2^{-1} + 2^{-3} = \frac{1}{2} + \frac{1}{8} = 0.625 \Rightarrow (50.625)_{10}$$

$$= 50$$

$(50.625)_{10} \rightarrow (55.\bar{5})_9$

$\begin{array}{r} 50 \\ 5 \\ 0 \end{array}$	$\begin{array}{r} 9 \\ 5 \\ 5 \end{array}$	$\left. \begin{array}{l} 0.625 \times 9 = 5.625 \\ 0.625 \times 9 = \dots \\ \vdots \end{array} \right\} \begin{array}{l} 5 \\ 4 \\ 2 \\ 5 \end{array}$
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c) $(10101.10101)_4 = (X)_{16}$

$(10101.10101)_4 \rightarrow (0100010001.0100010001)_2 \quad 2^4 = 16$

$(111.444)_{16}$

d) $(2699)_{10} = (BEE)_x \quad \leftarrow \begin{array}{l} 14 \\ 11 \\ 14 \end{array}$

$(BEE)_x \rightarrow (2699)_{10}$

$x^2(11) + x(14) + x^0(14) = 2699$

$11x^2 + 14x + 14 = 2699$

$11x^2 + 14x - 2685 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{(14)^2 + (4)(11)(2685)}}{22} = \frac{-14 \pm 344}{22} = \begin{cases} x = 15 \\ x < 0 \end{cases}$$

e) $(24)_x + (17)_x = (40)_x$

$$\begin{array}{r} 24 \\ 17 \\ \hline 40 \end{array} \quad + \quad \boxed{X=11}$$

f) $X = \text{the 10's complement of } (935)_4$

$$\begin{aligned} X &= r^n - 1 - N \\ &= (11)^3 - 1 - 935 \\ &= 1331 - 1 - 935 \\ &= 395 \end{aligned}$$

Question 2 a) $(10111011)_2 - (1001111)_2$

$$\begin{array}{r}
 \begin{array}{cccccccc}
 & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\
 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & \\
 \hline
 1 & 0 & 0 & 1 & 1 & 1 & 1 & & \\
 \hline
 1 & 1 & 0 & 1 & 1 & 0 & 0 & &
 \end{array}
 \qquad
 \begin{array}{r}
 187 \\
 - 79 \\
 \hline
 108
 \end{array}
 \end{array}$$

b) $(1101)_2 * (1011)_2 =$

$$\begin{array}{r}
 \begin{array}{r}
 1101 \\
 1011 \times \\
 \hline
 1101 \\
 11010 \\
 00000 \\
 1101000 \\
 \hline
 10001111
 \end{array}
 \qquad
 \begin{array}{r}
 13 * \\
 11 \\
 \hline
 143
 \end{array}
 \end{array}$$

c) $(52E9)_{16} - (133F)_{16}$

$$\begin{array}{r}
 \begin{array}{cccc}
 & 4 & 16 & 13 & 16 \\
 5 & 2 & E & 9 & \\
 \hline
 1 & 3 & 3 & F & \\
 \hline
 1 & 3 & F & A & A
 \end{array}
 \end{array}$$

ABCDEF
10 11 12 13 14 15

d) $(11011.0111)_2 + (11.1101)_2$

$$\begin{array}{r}
 \begin{array}{cccc}
 000 & 000 & & \\
 11011 & . & 0111 & \\
 \hline
 11 & . & 1101 & + \\
 \hline
 11111 & . & 0100 &
 \end{array}
 \end{array}$$

e) $(27.61)_{16} + (25.9F)_{16}$

$$\begin{array}{r}
 \begin{array}{r}
 0 & 0 \\
 27 & . & 61 \\
 + & & \\
 25 & . & 9F \\
 \hline
 4D & . & 00
 \end{array}
 \end{array}$$

f) $(AE.F3)_{16} - (103.111)_4 = (X)_2$

$(10101110.11110011)_2 - (010011.010101)_2$

$$\begin{array}{r}
 \begin{array}{cccccccc}
 10 & 10 & 11 & 10 & . & 11 & 11 & 00 & 11 \\
 \hline
 00 & 01 & 00 & 1 & . & 01 & 01 & 0 & 100 \\
 \hline
 10011011 & . & 10011111 & & & & & &
 \end{array}
 \end{array}$$

$(10011011.10011111)_2$

$$(g) (110100.01)_4 - (111.101)_2 = (Y)_2$$

$$\begin{array}{r} (010100010000.0001)_2 \\ - (111.1010)_2 \\ \hline (010100001000.0111)_2 \end{array}$$

Question 3 g-

Decimal	Binary	Octal	Hexadecimal	BCD
19.22	10011.0011	23.14	13.3	0011001.00100010
17.5625	10001.10011	21.46	11.98	0010111.0101110
485.251	011100010.01110001	745.347	E2.718	01001000101.01100101001
479.1367	0001101111.0010001	0727.106	1DF.23	010001111001.00100110110
93.2	1011101.0011	135.14	5D.3	<u>10010011</u> , <u>0010</u>

Question 4 g-

$$(669)_{10} + (835)_{10}$$

$$\begin{array}{r} 0110 \quad 0110 \quad 1001 \\ 1000 \quad 0011 \quad 0101 \quad + \\ 1111 \quad 1010 \quad 1110 \\ 0110 \quad 0110 \quad 0110 \quad + \\ \hline 0001 \quad 0101 \quad 0000 \quad 0100 \\ \hline 1504 \end{array}$$

Question 5 g-

(a) $2^{10} < 2000 < 2^{11}$
so the number of bits equals 11.

(b) $2 \times 2^5 < 2000 < 2^5 \times 2^5$

so we need 16 bit after 15 years.

(c) $\underbrace{0000}_4 \underbrace{0000}_3 \underbrace{0000}_2 \underbrace{0000}_1 \Rightarrow$ for 15 years we need 16 bit in binary and 4 bit for Hex

Question 6 :

(a)
$$\begin{array}{r} 1111 \\ 101101 \\ + 110011 \\ \hline 100000 \end{array} \rightarrow \text{Carry } 1$$

 overflow 0
 result (100000)

(b)
$$\begin{array}{r} 001101 \\ 010011 \\ \hline 100000 \end{array}$$

 $2's \text{Comp}(101101) = 010011$
 overflow = $1 \oplus 0 = 1$
 carry = 0
 result = 100000

(c)
$$\begin{array}{r} 010011 \\ + 110011 \\ \hline 000110 \end{array}$$

 carry = 1
 overflow = $0 \oplus 1 = 1$
 result = (000110)

(d)
$$\begin{array}{r} 111011 \\ + 010000 \\ \hline 001011 \end{array}$$

 carry = 1
 overflow = $1 \oplus 1 = 0$
 result = (001011)₂

Question 7 :

$A = (1001100)$, $B = (\underline{0}101010)$

a) $A = +76$, $B = +42$

b) $A = -12$, $B = +42$

c) $A = -51$, $B = +42$
 0110011

d)
$$\begin{array}{l|l} 0110100 & \\ \hline A = -52 & B = +42 \end{array}$$

Question 9 :

23 , -23

(a) $(0101110)_2 = (23)_{10}$, $(-23)_{10} = (\text{not exist})_2$

(b) $(23)_{10} = (010111)_2$, $(-23)_{10} = (110111)_2$

(c) $(23)_{10} = (010111)_2$, $(-23)_{10} = (101000)_2$

(d) $(23)_{10} = (010111)_2$, $(-23)_{10} = (101001)_2$

Question 10 8-

2's Complement representation with 5-bit.

A)

$$\text{Range } -2^{n-1} \rightarrow 2^{n-1}-1$$
$$\underline{-16} \rightarrow \underline{+15}$$

$$(10)_{10} = (01010)_2$$

$$\begin{array}{r} 01010 \\ + \\ 01111 \\ \hline 01111 \\ + 1010 \\ \hline 00101 \end{array}$$

the largest positive number.

B)

the largest positive number that could be subtract from $(10)_{10}$ without causing overflow.

$$(15)_{10} = (01111)_2$$

$$\begin{array}{r} 01010 \\ - \\ 01111 \\ \hline 11011 \end{array}$$

Carry = 0, overflow = 0