سارة بكار

Principles of physics (10th eddition)

phy 132

CH 23: Gauss's law

Problems: 1, 6, 17, 18, 23, 39

P1: An infinite line of charge produces a field of magnitude 1.7×10^4 N/C at distance 9.0 m. Find the field magnitude at distance 2.0 m

Sol:

The magnitude of electric field produced by uniformly charged infinit lines is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$E = 1.7 \times 10^4$ at $r = 9.0$ m

$$\lambda = E\, 2\pi\epsilon_0 r$$
$$= 1.7 \times 10^4 \times 2 \times 3.14 \times 8.85 \times 10^{-12} \times 9$$
$$= 8.5 \times 10^{-6} \text{ c/m}$$

E at $r = 2$ m

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{8.5 \times 10^{-6}}{2 \times 3.14 \times 8.85 \times 10^{-12} \times 2}$$

$$E = 76500 \text{ N/c}$$

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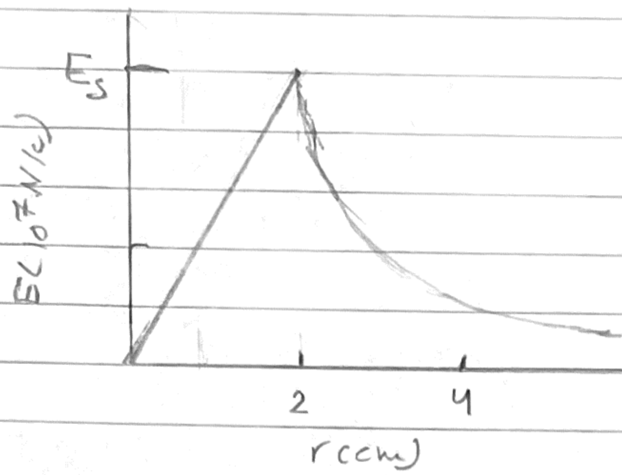
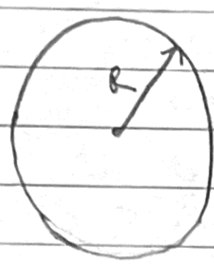
P_6 : Figure 23-23 gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. The scale of the vertical axis is set by $E_s = 10 \times 10^7$ N/C. (a) What is the charge on the sphere? (b) What is the field magnitude at $r = 8.0$ m

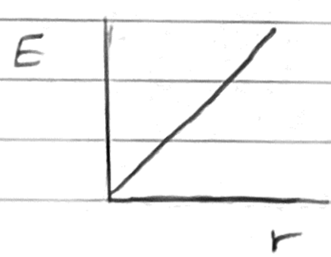


Sol :



The electric field inside a uniform sphere of charge

$$E = \frac{kq}{R^3} r \qquad r \le R$$



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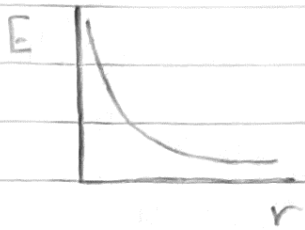
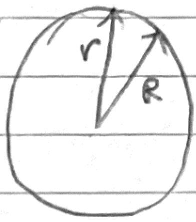
$$E = \frac{kq}{r^2} \qquad r > R$$



So from the figure the raduce of the sphere is 2 cm

a)

r=R



$$E = \frac{kq}{R^3} r$$

$$E = \frac{kq\,R}{R^3}$$

$$E = \frac{kq}{R^2}$$

$$q = \frac{ER^2}{k} = \frac{10\times10^7 \times (2\times10^{-2})^2}{9\times10^9}$$

$$= 4.44 \times 10^{-6}\ C$$

$$q = 4.44\ \mu C$$

b) E at $r = 8\,m$ $(r > R)$

$$E = \frac{kq}{r^2}$$

$$= \frac{9\times10^9 \times 4.44\times10^{-6}}{(8)^2}$$

$$= 625\ N/C$$

$$= 6.25 \times 10^2\ N/C$$

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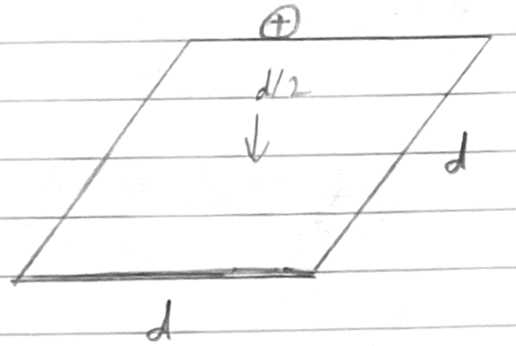
P17: In Figure 23-30, a proton is a distance d/2 directly above the center of a squre of side d. What is the magnitude of the electric flux through the squre? (Hint: Think of the squre as one face of a cube with edge d)



sol: Imagine a closed Gaussian surface in the shape of cube of edge d, with a proton charge $q = 1.6 \times 10^{-19}$ C located at the center of the cube

$$\Phi_{total\,cube} = \frac{q}{\varepsilon_0} = \frac{1.6 \times 10^{-19}}{8.85 \times 10^{-12}} = 1.808 \times 10^{-8}\ N.m^2/C$$

* The flux through the squre is one-sixth of the $\Phi_{total\,cube}$.

$$\Phi_{one\,surface} = \frac{\Phi_{total\,cube}}{6} = \frac{1.808 \times 10^{-8}}{6} = 3.013 \times 10^{-9}\ N.m^2/C$$

P18: A long nonconducting solid cylinder of radius 4.0 cm has a nonuniform volum charge density ρ that is a function of radial distance r from the cylinder axis: $\rho = Ar^2$. For $A = 6.3\ \mu C/m^5$ what is the magnitude of the electric field at (a) $r = 3.0$ cm and (b) $r = 5.0$ cm?

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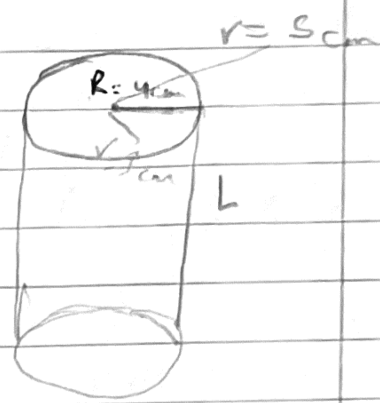

a) $R = 4\,cm = 0.04\,m$
$\rho = Ar^2$, $A = 6.3\ \mu C/m^5$

for cylinder

$V = \pi r^2 L$
$dv = 2\pi r L\, dr$

$Q = \int \rho\, dv$

$= \int_0 Ar^2\, 2\pi r L\, dr$

$= \int_0^{r_1} 2\pi A r^3 L\, dr$

$Q = 2\pi L A \frac{r^4}{4}$

$\Rightarrow \boxed{Q_{en} = \frac{\pi L A r_1^4}{2}}$

Gauss law $\Phi = E.A$

$\frac{Q_{en}}{\epsilon_0} = E.A$

$\frac{Q_{en}}{\epsilon_0} = E\, 2\pi r L$

$E = \frac{Q_{en}}{\epsilon_0} \frac{1}{2\pi r L}$

$= \frac{\pi L A r^4}{2\epsilon_0} \frac{1}{2\pi r L}$

$\boxed{E = \frac{Ar^3}{4\epsilon_0}}$

(6)

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E at r = 3 cm = 3×10^{-2} m r < R

$$E = \frac{A r^3}{4 \epsilon_0} = \frac{6.3 \times 10^{-6} (0.03)^3}{4 \times 8.85 \times 10^{-12}} = 4.8 \ N/C$$

b) E at r = 5 cm = 5×10^{-2} m r > R

$$Q_{total} = \int_0^R \rho \, dv$$

$$= \frac{\pi L A R^4}{2}$$

$$= \frac{3.14 \times 6.3 \times 10^{-6} \times (4 \times 10^{-2})^4 L}{2}$$

$$Q_{tot} = 2.53 \times 10^{-11} L$$

$$\frac{Q_{tot}}{L} = \lambda = 2.53 \times 10^{-11} \ c/m$$

$$\Rightarrow E.A = \frac{Q_{tot}}{\epsilon_0}$$

$$E . 2\pi r L = \frac{Q_{tot}}{\epsilon_0}$$

λ

$$E = \frac{1}{2\pi r L \epsilon_0} Q_{tot}$$

or $\frac{\lambda}{2\pi r \epsilon_0}$

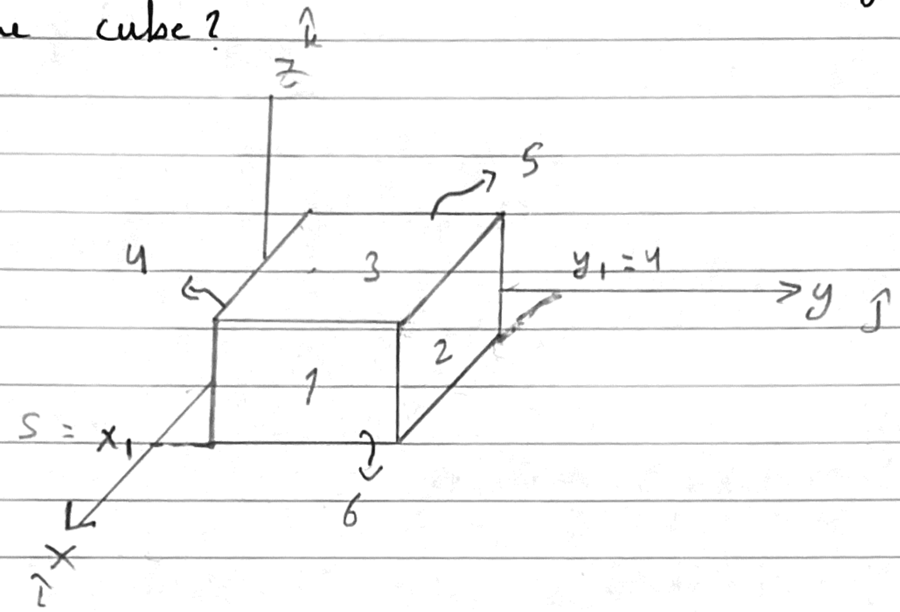
$$= \frac{1}{2\pi r \epsilon_0} 2.5 \times 10^{-11}$$

$$= \frac{2.5 \times 10^{-11}}{2 \times 3.14 \times 0.05 \times 8.85 \times 10^{-12}}$$

$$= 8.9 \ N/C$$

سؤال مهم

P23: Figure 23-34 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at $x_1 = 5.00$ m, $y_1 = 4.00$ m. The cube lies in a region where the electric field vectors is given by $\vec{E} = +23\hat{i} - 2y^2\hat{j} - 16\hat{k}$ N/C with y in meters. What is the net charge contained by the cube?



sol

$$q_{enc} = \varepsilon_0 \Phi \quad , \quad \Phi = \oint E \cdot dA$$

Face (1) at $x = 5$ m , $A = \int d\vec{A} = 2 \times 2 = 4\,m^2\,\hat{i}$

$$\Phi_1 = E \cdot A$$
$$= (+23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot 4\hat{i}$$
$$= 23\hat{i} \cdot 4\hat{i} - 0 - 0$$
$$= +92 \ N \cdot m^2/C$$

Face (2) at $y = 4$m $\quad A = 4\,m^2\,\hat{j}$

$$\Phi_2 = (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot 4\hat{j}$$
$$= 0 - 2y^2\hat{j} \cdot 4\hat{j} - 0$$
$$= -8y^2$$
$$= -8(4)^2$$
$$= -128 \ Nm^2/C$$

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ساعة/ركاز

Face(3) at $z = 2\ m$ $A = 4m^2\hat{k}$

$$\Phi_3 = (+23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot 4\hat{k}$$
$$= 0 - 0 - 16\hat{k} \cdot 4\hat{k}$$
$$= -64\ Nm^2/C$$

Face(4) at $y = 2m$ $A = -4m^2\hat{j}$

$$\Phi_4 = (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot -4\hat{j}$$
$$= 0 - 2y^2\hat{j} \cdot -4\hat{j} - 0$$
$$= 8y^2$$
$$= 8(2)^2$$
$$= 32\ Nm^2/C$$

Face(5) at $x = 3m$ $(S-2) \Rightarrow$ 2 لايوجد
$A = -4m^2\hat{i}$

$$\Phi_5 = (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot -4\hat{i}$$
$$= 23\hat{j} \cdot -4\hat{i}$$
$$= -92\ Nm^2/C$$

Face 6 at $z = 0$ $A = -4m^2\hat{k}$

$$\Phi_6 = (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot -4\hat{k}$$
$$= -16\hat{k} \cdot -4\hat{k}$$
$$= 64\ Nm^2/C$$

$$\Rightarrow \Phi_{tot} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6$$
$$= \cancel{92} + -128 + \cancel{-64} + 32 + \cancel{-92} + \cancel{64}$$
$$= -96\ Nm^2/C$$

$$Q_{enc} = \varepsilon_0 \Phi = 8.85 \times 10^{-12} \times -96 = -8.496 \times 10^{-10}\ C$$
$$= -0.8496\ nC$$

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P39 : A uniformly charged conducting sphere of 0.60 m diameter has surface charge density 5.7 $\mu C/m^2$. Find (a) the net charge on the sphere and (b) the total electric flux leaving the surface. (c) What is the net flux through a concentric Gaussian sphere of radius 2.0 m?

Sol: diameter $d = 0.60m$, radius $r = \frac{d}{2} = 0.30\,m$

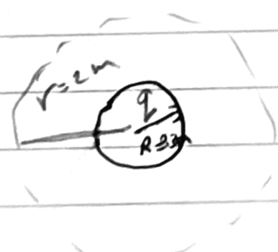
surface charge density $\sigma = 5.7\,\mu C/m^2 = 5.7 \times 10^{-6}\,C/m^2$

a) The net charge on the sphere = charge density × surface area

$$Q = \sigma \times A$$
$$= \sigma \times 4\pi r^2$$
$$= 5.7 \times 10^{-6} \times 4 \times 3.14 \times (0.30)^2$$
$$= 6.44 \times 10^{-6}\,C$$
$$= 6.44\,\mu C$$

b) $$\Phi = \frac{Q}{\epsilon_0} = \frac{6.44 \times 10^{-6}}{8.85 \times 10^{-12}} = 728054.2$$
$$\approx 7.3 \times 10^5\,Nm^2/C$$

c)



$$\phi = \frac{q}{\epsilon_0}$$
$$= \frac{6.44 \times 10^{-6}}{8.85 \times 10^{-12}}$$
$$= 7.3 \times 10^5\,Nm^2/C$$