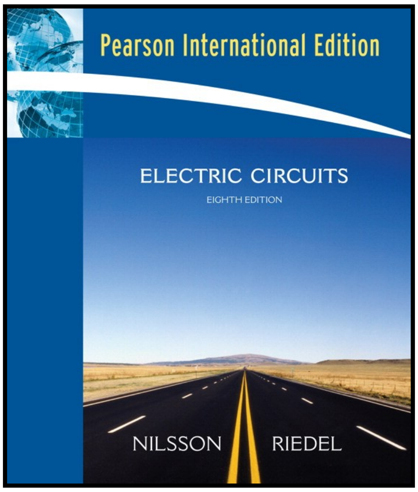


JAMES W. NILSSON
&
SUSAN A. RIEDEL

ELECTRIC CIRCUITS

EIGHTH EDITION

PEARSON Education Asia

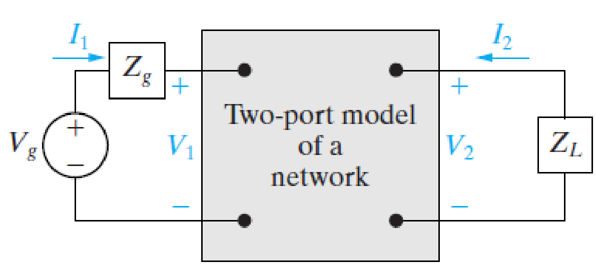
CHAPTER 18

Two-Port Circuits
Part 2

CONTENTS

18.3 Analysis of the Terminated Two-Port Circuit

- In the typical application of a two-port model, the circuit is driven at port 1 and loaded at port 2.
- The goal is to solve $\{V_1, I_1, V_2, I_2\}$ as functions of given parameters V_g, Z_g, Z_L, and matrix elements of the two-port circuit.



18.3 Analysis of the Terminated Two-Port Circuit

- Six characteristics of the terminated two-port circuit define its terminal behavior:
  1. the input impedance $Z_{in} = V_1/I_1$, or the admittance $Y_{in} = I_1/V_1$.
  2. the output current $\boldsymbol{I_2}$.
  3. the Thévenin voltage and impedance ($\boldsymbol{V_{TH}}$, $\boldsymbol{Z_{TH}}$) with respect to port 2
  4. the current gain $\boldsymbol{I_2/I_1}$
  5. the voltage gain $\boldsymbol{V_2/V_1}$
  6. the voltage gain $\boldsymbol{V_2/V_g}$

Analysis in terms of [*Z*]

- Four equations are needed to solve the four unknowns $\{V_1, I_1, V_2, I_2\}$.

$$\left.\begin{cases} V_1 = z_{11}I_1 + z_{12}I_2 \cdots (1) \\ V_2 = z_{21}I_1 + z_{22}I_2 \cdots (2) \end{cases}\right\} \textbf{Two-Port equations}$$

$$\left.\begin{cases} V_1 = V_g - I_1 Z_g \cdots (3) \\ V_2 = -I_2 Z_L \cdots (4) \end{cases}\right\} \textbf{Constraint equations due to terminations}$$

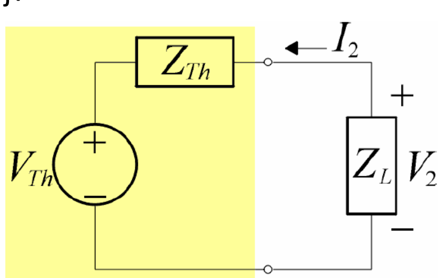
Analysis in terms of [*Z*]

$$\Rightarrow \begin{bmatrix} -1 & 0 & z_{11} & z_{12} \\ 0 & -1 & z_{21} & z_{22} \\ 1 & 0 & Z_g & 0 \\ 0 & 1 & 0 & Z_L \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_g \\ 0 \end{bmatrix},$$

$\{V_1, I_1, V_2, I_2\}$ are derived by inverse matrix method.

Thévenin equivalent circuit with respect to port 2

- Once $\{V_1, I_1, V_2, I_2\}$ are solved, $\{V_{Th}, Z_{Th}\}$ can be determined by Z_L and $\{V_2, I_2\}$:



$$\begin{cases} V_2 = \dfrac{Z_L}{Z_{Th} + Z_L} V_{Th} \cdots (1) \\ I_2 = \dfrac{V_2 - V_{Th}}{Z_{Th}} \cdots (2) \end{cases}$$

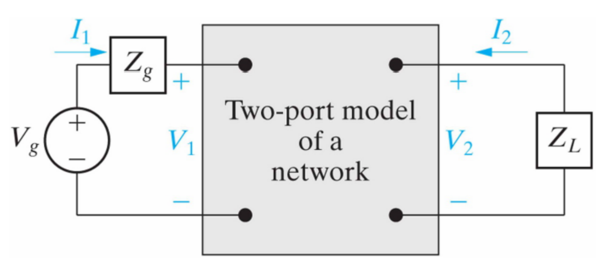
$$\Rightarrow \begin{bmatrix} Z_L & -V_2 \\ 1 & I_2 \end{bmatrix} \times \begin{bmatrix} V_{Th} \\ Z_{Th} \end{bmatrix} = \begin{bmatrix} V_2 Z_L \\ V_2 \end{bmatrix}; \quad \begin{bmatrix} V_{Th} \\ Z_{Th} \end{bmatrix} = \begin{bmatrix} Z_L & -V_2 \\ 1 & I_2 \end{bmatrix}^{-1} \times \begin{bmatrix} V_2 Z_L \\ V_2 \end{bmatrix}.$$

Terminal behavior (1)

- The terminal behavior of the circuit can be described by manipulations of $\{V_1, I_1, V_2, I_2\}$:
- Input impedance: $Z_{in} \equiv \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$;
- Output current: $I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$;
- Current gain: $\frac{I_2}{I_1} = -\frac{z_{21}}{z_{22} + Z_L}$;
- Voltage gains: $\begin{cases} \frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}; \\ \frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}; \end{cases}$

Terminal behavior (2)

- Thévenin voltage: $V_{Th} = \frac{z_{21}}{z_{11} + Z_g} V_g$;
- Thévenin impedance: $Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$;



Analysis in term of a two-port matrix [T]≠[Z]

- If the two-port circuit is modeled by [T]≠[Z]…

  T={Y, A, B, H, G},

  the terminal behavior can be determined by two methods:

1. Use the 2 two-port equations of [T] to get a new 4X4 matrix in solving {V_1 , I_1 , V_2 , I_2 } (Table 18.2);
2. Transform [T] into [Z] by Table 18.1, borrow the formulas derived by analysis in terms of [Z].

TABLE 18.1 Parameter Conversion Table

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$$

$$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$$

$$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$$

$$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$$

$$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$$

$$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$$

$$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$$

$$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$$

$$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$$

$$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$$

$$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$$

$$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$$

TABLE 18.1 Parameter Conversion Table

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$$

$$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$$

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$$

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$$

$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$$

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

TABLE 18.2 Terminated Two-Port Equations

z Parameters	y Parameters
$Z_{\text{in}} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$	$Y_{\text{in}} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$
$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$
$V_{\text{Th}} = \frac{z_{21}}{z_{11} + Z_g}V_g$	$V_{\text{Th}} = \frac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$
$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$	$Z_{\text{Th}} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$
$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$	$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$
$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$	$\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$
$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$

TABLE 18.2 Terminated Two-Port Equations

***a* Parameters**	***b* Parameters**
$Z_{in} = \dfrac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$	$Z_{in} = \dfrac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$
$I_2 = \dfrac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_g Z_L + a_{22}Z_g}$	$I_2 = \dfrac{-V_g \Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$
$V_{Th} = \dfrac{V_g}{a_{11} + a_{21}Z_g}$	$V_{Th} = \dfrac{V_g \Delta b}{b_{22} + b_{21}Z_g}$
$Z_{Th} = \dfrac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$	$Z_{Th} = \dfrac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$
$\dfrac{I_2}{I_1} = \dfrac{-1}{a_{21}Z_L + a_{22}}$	$\dfrac{I_2}{I_1} = \dfrac{-\Delta b}{b_{11} + b_{21}Z_L}$
$\dfrac{V_2}{V_1} = \dfrac{Z_L}{a_{11}Z_L + a_{12}}$	$\dfrac{V_2}{V_1} = \dfrac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$
$\dfrac{V_2}{V_g} = \dfrac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$	$\dfrac{V_2}{V_g} = \dfrac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$

TABLE 18.2 Terminated Two-Port Equations

***h* Parameters**	***g* Parameters**
$Z_{in} = h_{11} - \dfrac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$	$Y_{in} = g_{11} - \dfrac{g_{12}g_{21}}{g_{22} + Z_L}$
$I_2 = \dfrac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L}$	$I_2 = \dfrac{-g_{21}V_g}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$
$V_{Th} = \dfrac{-h_{21}V_g}{h_{22}Z_g + \Delta h}$	$V_{Th} = \dfrac{g_{21}V_g}{1 + g_{11}Z_g}$
$Z_{Th} = \dfrac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$	$Z_{Th} = g_{22} - \dfrac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$
$\dfrac{I_2}{I_1} = \dfrac{h_{21}}{1 + h_{22}Z_L}$	$\dfrac{I_2}{I_1} = \dfrac{-g_{21}}{g_{11}Z_L + \Delta g}$
$\dfrac{V_2}{V_1} = \dfrac{-h_{21}Z_L}{\Delta h Z_L + h_{11}}$	$\dfrac{V_2}{V_1} = \dfrac{g_{21}Z_L}{g_{22} + Z_L}$
$\dfrac{V_2}{V_g} = \dfrac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$	$\dfrac{V_2}{V_g} = \dfrac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$

Example 18.4 Analyzing a Terminated Two-Port Circuit

The two-port circuit shown in Fig. 18.8 is described in terms of its b parameters, the values of which are

$$b_{11} = -20, \qquad b_{12} = -3000\ \Omega,$$
$$b_{21} = -2\ \text{mS}, \qquad b_{22} = -0.2.$$

a) Find the phasor voltage $\mathbf{V}_2$.
b) Find the average power delivered to the 5 kΩ load.
c) Find the average power delivered to the input port.
d) Find the load impedance for maximum average power transfer.
e) Find the maximum average power delivered to the load in (d).

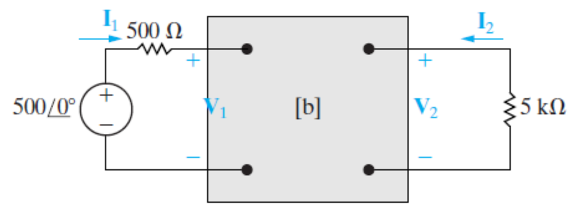


Figure 18.8 ▲ The circuit for Example 18.4.

Solution

a) To find $\mathbf{V}_2$, we have two choices from the entries in Table 18.2. We may choose to find $\mathbf{I}_2$ and then find $\mathbf{V}_2$ from the relationship $\mathbf{V}_2 = -\mathbf{I}_2 Z_L$, or we may find the voltage gain $\mathbf{V}_2/\mathbf{V}_g$ and calculate $\mathbf{V}_2$ from the gain. Let's use the latter approach. For the b-parameter values given, we have

$$\Delta b = (-20)(-0.2) - (-3000)(-2 \times 10^{-3})$$
$$= 4 - 6 = -2.$$

From Table 18.2,

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$$
$$= \frac{(-2)(5000)}{-3000 + (-20)500 + (-0.2)5000 + [-2 \times 10^{-3}(500)(5000)]}$$
$$= \frac{10}{19}.$$

Then,

$$\mathbf{V}_2 = \left(\frac{10}{19}\right)500 = 263.16\underline{/0^\circ}\ \text{V}.$$

b) The average power delivered to the 5000 Ω load is

$$P_2 = \frac{263.16^2}{2(5000)} = 6.93\ \text{W}.$$

c) To find the average power delivered to the input port, we first find the input impedance Z_{in}. From Table 18.2,

$$Z_{\text{in}} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$$
$$= \frac{(-0.2)(5000) - 3000}{-2 \times 10^{-3}(5000) - 20}$$
$$= \frac{400}{3} = 133.33\ \Omega.$$

Now $\mathbf{I}_1$ follows directly:

$$\mathbf{I}_1 = \frac{500}{500 + 133.33} = 789.47\ \text{mA}.$$

The average power delivered to the input port is

$$P_1 = \frac{0.78947^2}{2}(133.33) = 41.55\ \text{W}.$$

d) The load impedance for maximum power transfer equals the conjugate of the Thévenin impedance seen looking into port 2. From Table 18.2,

$$Z_{Th} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$$

$$= \frac{(-20)(500) - 3000}{(-2 \times 10^{-3})(500) - 0.2}$$

$$= \frac{13{,}000}{1.2} = 10{,}833.33\ \Omega.$$

Therefore $Z_L = Z_{Th}^* = 10{,}833.33\ \Omega$.

e) To find the maximum average power delivered to Z_L, we first find $\mathbf{V}_2$ from the voltage-gain expression $\mathbf{V}_2/\mathbf{V}_g$. When Z_L is 10,833.33 Ω, this gain is

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = 0.8333.$$

Thus

$$\mathbf{V}_2 = (0.8333)(500) = 416.67\ \text{V},$$

and

$$P_L(\text{maximum}) = \frac{1}{2}\frac{416.67^2}{10{,}833.33}$$

$$= 8.01\ \text{W}.$$

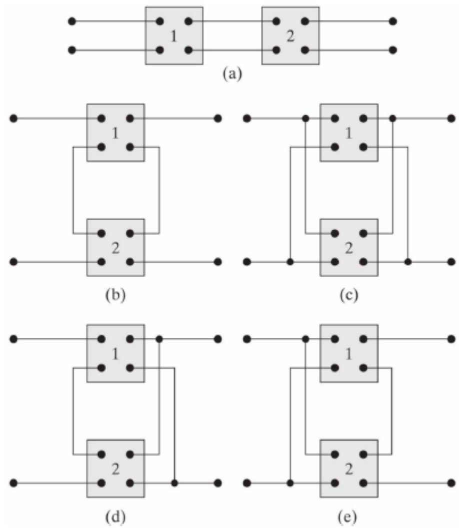
Section 18.4 Interconnected Two-Port Circuits

- Why interconnected?

Design of a large system is simplified by first designing subsections (usually modeled by two-port circuits), then interconnecting these units to complete the system.

Section 18.4 Interconnected Two-Port Circuits

Five types of interconnections of two-port circuits



a. Cascade: Better use $[A]$.

b. Series: $[Z]$

c. Parallel: $[Y]$

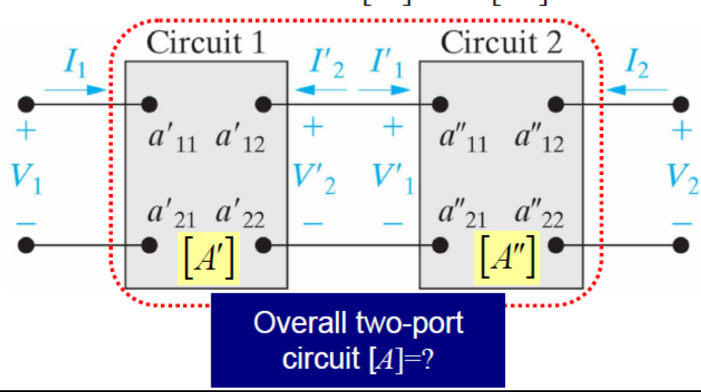
d. Series-parallel: $[H]$.

e. Parallel-series: $[G]$.

Section 18.4 Interconnected Two-Port Circuits

Analysis of cascade connection

- Goal: Derive the overall matrix $[A]$ of two cascaded two-port circuits with known transmission matrixes $[A']$ and $[A'']$.



Section 18.4 Interconnected Two-Port Circuits

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [A'] \times \begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = [A'] \times \begin{bmatrix} V_1' \\ -I_1' \end{bmatrix} \cdots (1)$$

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = [A''] \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} a_{11}'' & -a_{12}'' \\ \boxed{a_{21}'' \quad -a_{22}''} \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix},$$

$$\Rightarrow \begin{bmatrix} V_1' \\ -I_1' \end{bmatrix} = \begin{bmatrix} a_{11}'' & -a_{12}'' \\ \boxed{-a_{21}'' \quad a_{22}''} \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = [\underline{A}''] \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \cdots (2)$$

$$\text{By (1), (2),} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [A'] \times [\underline{A}''] \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = [A] \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix},$$

$$\Rightarrow [A] = [A'] \times [\underline{A}''], \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}'a_{11}'' + a_{12}'a_{21}'' & -(a_{11}'a_{12}'' + a_{12}'a_{22}'') \\ a_{21}'a_{11}'' + a_{22}'a_{21}'' & -(a_{21}'a_{12}'' + a_{22}'a_{22}'') \end{bmatrix}.$$

Section 18.4 Interconnected Two-Port Circuits

Key points

- How to calculate the 6 possible 2X2 matrices of a two-port circuit?
- How to find the 4 simultaneous equations in solving a terminated two-port circuit?
- How to find the total 2X2 matrix of a circuit consisting of interconnected two-port circuits?

Example 18.5 **Analyzing Cascaded Two-Port Circuits**

Two $\boxed{\text{identical}}$ amplifiers are connected in cascade, as shown in Fig. 18.11. Each amplifier is described in terms of its *h* parameters. The values are $h_{11} = 1000\ \Omega$, $h_{12} = 0.0015$, $h_{21} = 100$, and $h_{22} = 100\ \mu S$. Find the voltage gain V_2/V_g.

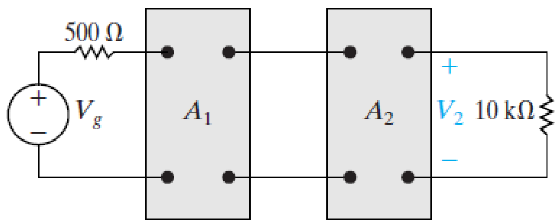


Figure 18.11 ▲ The circuit for Example 18.5.

Solution

The first step in finding V_2/V_g is to convert from *h* parameters to *a* parameters. The amplifiers are identical, so one set of *a* parameters describes the amplifiers:

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{+0.05}{100} = 5 \times 10^{-4},$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{100} = -10\ \Omega,$$

$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-100 \times 10^{-6}}{100} = -10^{-6}\,\text{S},$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{100} = -10^{-2}.$$

Next we use Eqs. 18.74–18.77 to compute the *a* parameters of the cascaded amplifiers:

$$\begin{aligned}
a_{11} &= a'_{11}a'_{11} + a'_{12}a'_{21} \\
&= 25 \times 10^{-8} + (-10)(-10^{-6}) \\
&= 10.25 \times 10^{-6}, \\
a_{12} &= a'_{11}a'_{12} + a'_{12}a'_{22} \\
&= (5 \times 10^{-4})(-10) + (-10)(-10^{-2}) \\
&= 0.095\ \Omega, \\
a_{21} &= a'_{21}a'_{11} + a'_{22}a'_{21} \\
&= (-10^{-6})(5 \times 10^{-4}) + (-0.01)(-10^{-6}) \\
&= 9.5 \times 10^{-9}\,\text{S}, \\
a_{22} &= a'_{21}a'_{12} + a'_{22}a'_{22} \\
&= (-10^{-6})(-10) + (-10^{-2})^2 \\
&= 1.1 \times 10^{-4}.
\end{aligned}$$

From Table 18.2,

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{10^4}{[10.25 \times 10^{-6} + 9.5 \times 10^{-9}(500)]10^4 + 0.095 + 1.1 \times 10^{-4}(500)}$$

$$= \frac{10^4}{0.15 + 0.095 + 0.055}$$

$$= \frac{10^5}{3}$$

$$= 33{,}333.33.$$

Thus an input signal of 150 μV is amplified to an output signal of 5 V. For an alternative approach to finding the voltage gain V_2/V_g, see Problem 18.41.