13.1: Factorial experiments.

exp:

Factor A		Factor B: College: Business		Eng.		Arts & Sci		
	3-Hower review	500, 580	540	540, 460	500	480, 400	440	493.33
Prep. program	one-day program	460, 540	500	560, 620	590	420, 480	450	513.33
	Ten week course	560, 600	580	600, 580	590	480, 460	445	538.33
			540		560		445	515

→ Factoral experement

Factor A : Prep. program

Factor B : College.

→ a = # of levels of factor a.

a = 3

→ b = # of levels of factor b

b = 3

→ r = # of replications.

r = 2

→ n_T : total # of observations.

$n_T = abr$

$n_T = (3)(3)(2) = 18$.

→

H_0^A : means of Factors A are equal.

$H_\#^A$: means of Factor A are not equal.

H_0^B : means of Factor B are equal.

H_B^B : means of Factor B are not equal.

H_0^{AB} : Factor A and Factor B have no interaction.

H_1^{AB} : Factor A and Factor B have an interaction.

→ $SST = SSA + SSB + SSAB + SSE$.

- $SST = \sum_{k=1}^{r}\sum_{j=1}^{b}\sum_{i=1}^{a}(x_{ijk} - \bar{\bar{x}})^2$

- $SSA = br\sum_{i=1}^{a}(\bar{x}_{i\bullet} - \bar{\bar{x}})^2$

- $SSB = ar\sum_{j=1}^{b}(\bar{x}_{\bullet j} - \bar{\bar{x}})^2$

- $SSAB = r\sum_{j=1}^{b}\sum_{i=1}^{a}(\bar{x}_{ij} - \bar{x}_{i\bullet} - \bar{x}_{\bullet j} + \bar{\bar{x}})^2$

- $SSE = SST - SSA - SSB - SSAB$.

Cont. exp:

SSA = 6100

SSB = 45300

SSAB = 11200

SSE = 19850

SST = 82450 .

→ ANOVA table : Two Factor Factorial experement.

ANOVA upper abc

Source of variance	df	SS	MS	F	F_α	P-value
Factor A	$a-1$	SSA	$MSA = \frac{SSA}{a-1}$	$F^A = \frac{MSA}{MSE}$	F_α with $a-1$ $ab(r-1)$	upper with df
Factor B	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F^B = \frac{MSB}{MSE}$	F_α with $b-1$ $ab(r-1)$	" "
Interaction AB	$(a-1)(b-1)$	SSAB	$MAB = \frac{SSAB}{(a-1)(b-1)}$	$F^{AB} = \frac{MSAB}{MSE}$	F_α with $(a-1)(b-1)$ $ab(r-1)$	" "
Error	$ab(r-1)$	SSE	$MSE = \frac{SSE}{ab(r-1)}$	—	—	—
Total	$n_T - 1$	SST	—	—	—	—

$n_T = abr$

$r \geq 2$

exp:

$a = 3$
$b = 3$
$r = 2$
$n_T = 18$
$\alpha = 0.05$

	df	SS	MS	F	F_α	P-value
	2	6100	3050	1.38	4.26	greater than α 0.1
	2	45300	22650	10.27	4.26	less than 0.1
	4	11200	2800	1.27	3.63	greater than 0.1
	9	19850	2205.56			
	17	82450	—			

Conclusion :

1. don't reject H_0^A ($\alpha = 0.05$)
2. Reject H_0^B ($\alpha = 0.05$)
3. dont reject H_0^{AB} ($\alpha = 0.05$)

Notes :

1. Don't reject H_0^A ($\alpha = 0.05$) : means of factor A are equal ($\alpha = 0.05$)

   $\Rightarrow$ Factor A (Prep. program) is not a singificant factor ($\alpha = 0.05$).

2. Reject H_0^B ($\alpha = 0.05$) : means of factor B are not equal ($\alpha = 0.05$)

   $\Rightarrow$ Factor B (college) is a singificant Factor ($\alpha = 0.05$)

3. Don't Reject H_0^{AB} ($\alpha = 0.05$) : Factor A and Factor B have No interaction ($\alpha = 0.05$).

   $\Rightarrow$ Interaction between A and B is not significant ($\alpha = 0.05$).