11.4

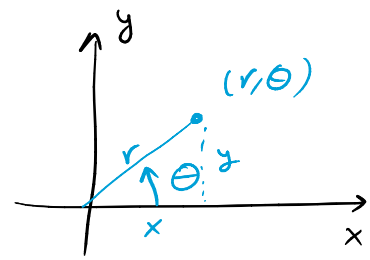
$y = f(x) \Rightarrow$ slope $\left.\frac{dy}{dx}\right|_{(x_0, y_0)}$

$r = f(\theta)$

$\Downarrow$

$r' = f'(\theta)$

$x = r\cos\theta$
$= f(\theta)\cos\theta$



$y = r\sin\theta$
$= f(\theta)\sin\theta$

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + \sin\theta\, r'}{r(-\sin\theta) + \cos\theta\, r'}$$

$$= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

Exp Find slope of $r = \cos 2\theta$ at $\theta = 0, \frac{\pi}{2}$

$\boxed{\theta = 0} \Rightarrow r = \cos 2(0) = \cos 0 = 1 \Rightarrow (r, \theta) = (1, 0)$

$$\left.\frac{dy}{dx}\right|_{(1,0)} = \left.\frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}\right|_{(1,0)} = \left.\frac{-2\sin 2\theta \sin\theta + r\cos\theta}{-2\sin 2\theta \cos\theta - r\sin\theta}\right|_{(1,0)}$$

$r' = -2\sin 2\theta$

(1,0) (1,0) (1,0)

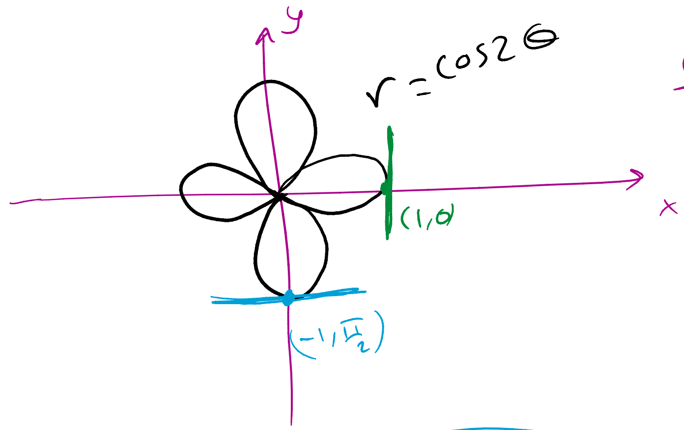
$$= \frac{0 + (1)\cos 0}{0 - (1)(0)} = \frac{1}{0} \quad \text{undefined}$$

المماس عند النقطة (1,0) سيكون عمودي

$$\boxed{\theta = \frac{\pi}{2}} \Rightarrow r = \cos 2\frac{\pi}{2} = -1 \Rightarrow (r, \theta) = \left(-1, \frac{\pi}{2}\right)$$

$$\left.\frac{dy}{dx}\right|_{\left(-1, \frac{\pi}{2}\right)} = \frac{-2\sin\pi \, \sin\frac{\pi}{2} + (-1)\cos\frac{\pi}{2}}{-2\sin\pi \, \cos\frac{\pi}{2} - (-1)\sin\frac{\pi}{2}} = \frac{0 + 0}{0 + 1} = \frac{0}{1} = 0$$

المماس عند النقطة $\left(-1, \frac{\pi}{2}\right)$ سيكون أفقي



To draw $r = f(\theta) \Rightarrow$ it is important to Know the symmetry

- (A) x-axis
- (B) y-axis
- (C) origin

• if $r = f(\theta)$ is symmetric about A and B $\Longrightarrow$ C

〃 〃 〃 〃

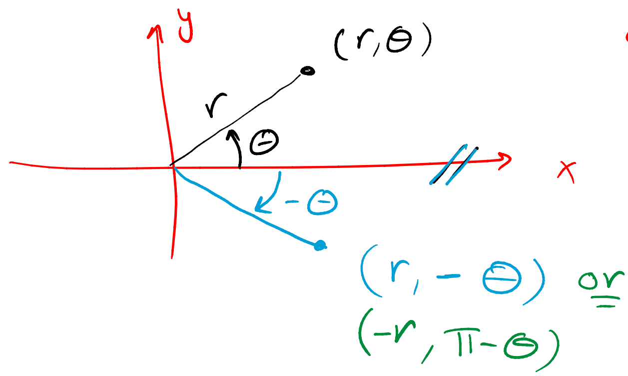
• if $r = f(\theta)$ is not 〃 〃 A = B $\Longrightarrow$

〃 〃 〃 〃 〃

A : x-axis

if (r,θ) on the graph

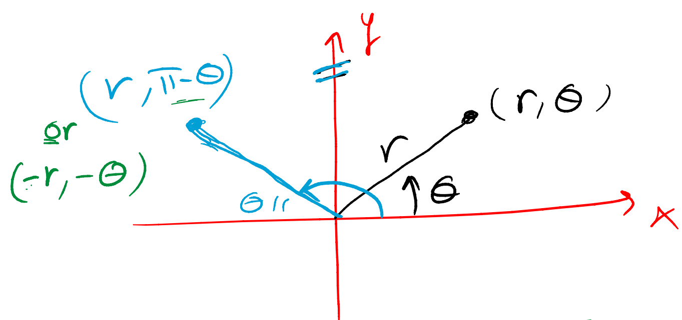
then $(r,-\theta)$ or $(-r, \pi-\theta)$ on the graph



B : symmetry about y-axis

if (r,θ) on the graph

then $(r,\pi-\theta)$ or $(-r,-\theta)$ on the graph

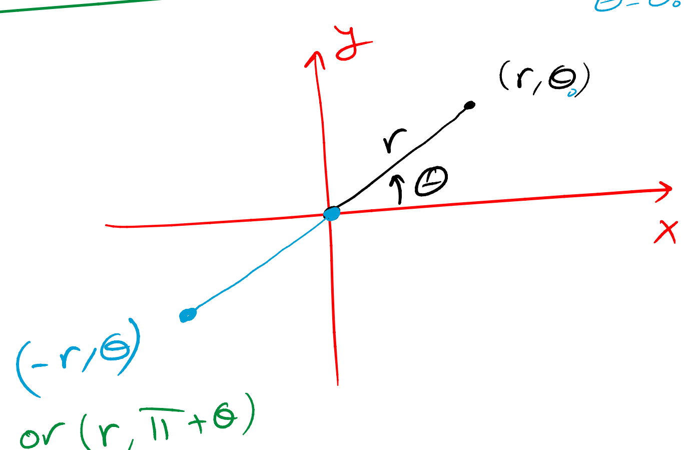


$\theta = \theta_0$

C Symmetry about origin

if (r,θ) on the graph

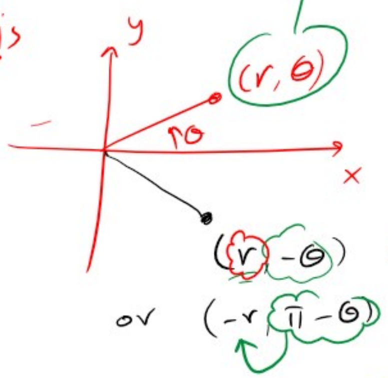
then $(-r,\theta)$ or $(r,\pi+\theta)$ on the graph



on the graph $\qquad$ or $(r, \pi+\theta)$

Exp sketch $r = 1 + \sin\theta$ $\dashrightarrow$ Identify the symmetry

x-axis



substitute $-\theta \Rightarrow 1 + \sin(-\theta)$

$= 1 - \sin\theta$

$\neq r$

---

substitute $\pi - \theta \Rightarrow 1 + \sin(\pi - \theta)$ $\qquad$ $A = \pi$, $B = -\theta$

$= 1 + \sin\pi \cos(-\theta) + \sin(-\theta)\underbrace{\cos\pi}_{-1}$

$= 1 + 0 + 1 \sin\theta$

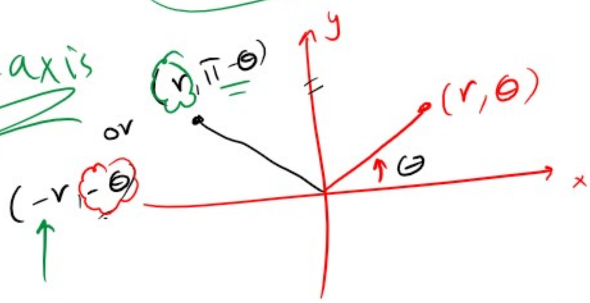
$= 1 + \sin\theta$

$\neq -r$

$$\boxed{\begin{aligned} \sin(A+B) &= \sin A \cos B + \sin B \cos A \\ \cos(A+B) &= \cos A \cos B - \sin A \sin B \end{aligned}}$$

---

y-axis



substitute $-\theta \Rightarrow 1 + \sin(-\theta)$

$= 1 - \sin\theta$

$\neq -r$

---

substitute $\pi - \theta \Rightarrow 1 + \sin(\pi - \theta)$

$= 1 + \underbrace{\sin\pi}_{0} \cos(-\theta) + \sin(-\theta)\underbrace{\cos\pi}_{-1}$

$= 1 + 0 + \sin\theta$

$= 1 + \sin\theta$

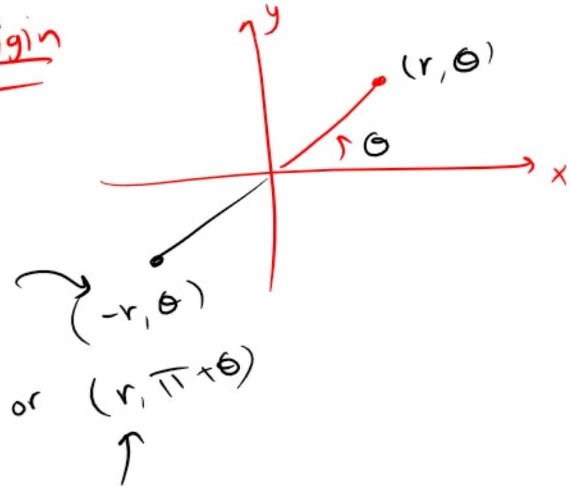
$= r$

Hence, $r = 1 + \sin\theta$ is symmetric about y-axis

---

substitute $\theta \Rightarrow 1 + \sin\theta = r$

origin



substitute $\theta \Rightarrow 1 + \sin\theta = r$

$\neq -r$

$= \pi + \theta \Rightarrow 1 + \sin(\pi + \theta)$

$= 1 + 0 + \sin\theta \underbrace{\cos\pi}_{=-1}$

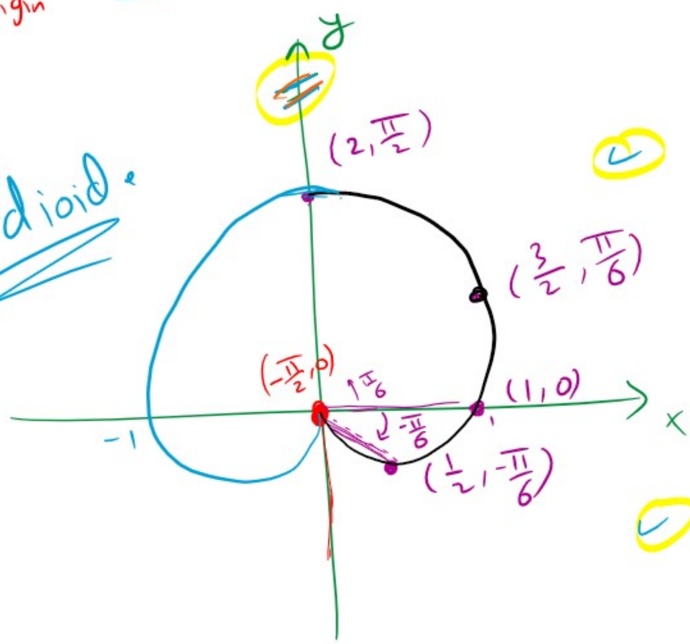
$= 1 - \sin\theta$

$\neq r$

$r = 1 + \sin\theta$

$r = 0 \Rightarrow$ origin

Cardioid.



θ	r
$-\frac{\pi}{2}$	$1 + \sin -\frac{\pi}{2} = 1 - 1 = 0$
$-\frac{\pi}{6}$	$1 + \sin -\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$
$\rightarrow 0$	$1 + \sin 0 = 1 + 0 = 1$
$\frac{\pi}{6}$	$1 + \sin\frac{\pi}{6} = 1 + \frac{1}{2} = \frac{3}{2}$
$\frac{\pi}{2}$	$1 + \sin\frac{\pi}{2} = 1 + 1 = 2$

$1 + \sin -\frac{\pi}{6}$

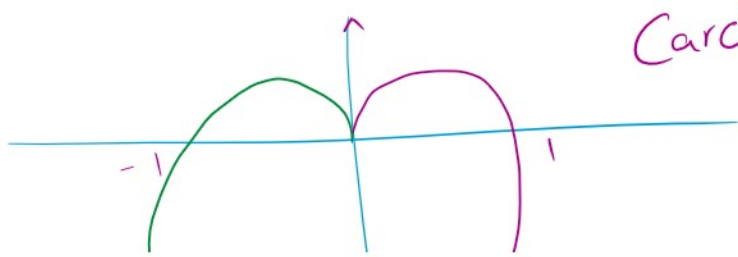
$1 - \sin\frac{\pi}{6}$

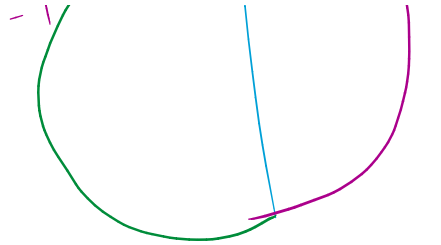
$1 - \frac{1}{2}$

$\frac{1}{2}$

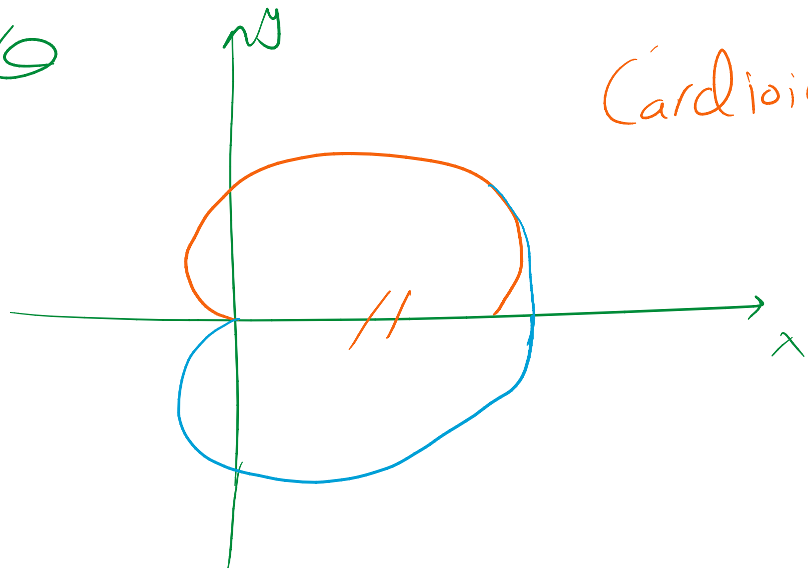
$r = 1 - \sin\theta$

Cardioid





$r = 1 + \cos\theta$



Cardioid

$r = 1 - \cos\theta$