Homework 6 (chapter 7) Reem Ali

Abistract 1.

3. let $H = \{0, \pm 3, \pm 6, \pm 9, \ldots\}$ Find all the left cosets of H in Z.

$0 + H = \{0, \pm 3, \pm 6, \pm 9, \ldots\} = H$.

$1 + H = \{\ldots, -8, -5, -2, 1, 4, 7, 10, \ldots\}$.

$2 + H = \{\ldots, -7, -4, -1, 2, 5, 7, 14, \ldots\}$.

$3 + H = \{\ldots, -6, -3, 0, 6, 9, 12, \ldots\} = H$.

So All cosets $0 + H$, $1 + H$ and $2 + H$.

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7. Find all of the left cosets of $\{1, 11\}$ in $U(30)$.

$U(30) = \{1, 7, 11, 13, 17, 19, 23, 29\}$, $H = \{1, 11\}$

$\#$ of cosets $= \frac{8}{2} = 4$.

- $1 . H = \{1, 11\} = 11 . H$
- $7 . H = \{7, 17\} = 17 . H$.
- $13 . H = \{13, 23\}$
- $19 . H = \{19, 29\}$.

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8. suppose that a has order 15. Find all the left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$. are there? list them.

$|a| = 15$

$H = \langle a^5 \rangle = \{e, a^5, a^{10}\}$

$|\langle a \rangle : \langle a^5 \rangle| = \frac{15}{3} = 5$

so there are 5 distinct cosets.

→ All left cosets :

$aH = \{a, a^6, a^{11}\}$ and $H = \{e, a^5, a^{10}\}$

$a^2H = \{a^2, a^7, a^{12}\}$

$a^3H = \{a^3, a^8, a^{13}\}$.

$a^4H = \{a^4, a^9, a^{14}\}$ . .

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14. suppose that K is a proper subgroup of H and H is a proper subgroup of G. If $|K| = 42$ and $|G| = 420$, what are the possible order of H?

By Lagrange Theorem :

we Know $|K|$ divides $|H|$ $\Rightarrow$ $|H| = 42K$ for some K

→ $42K$ divides 420 So K divides $420 \backslash 42 = 10$

→ since H is a proper subgroup of G, we Know $K < 10$ So

either $K = 2$ or $K = 5$ so,

$|H| = 2(42) = 84$ oR $|H| = 5(42) = 210$

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15. let G be a group with $|G| = pq$, where p and q are prime. Prove that every proper subgroup of G is cyclic :

If $H \leq G$ then $|H| / |G| = pq$

$\Rightarrow |H| = 1$ or p or q

$\Rightarrow H = \{e\} = \langle e \rangle$ or H is prime so H is cyclic.

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16. ... prove that if a is any integers less than n and relatively prime to n then $a^{\phi(n)} \bmod n = 1$.

$\phi(n) = |U(n)|$

and if $(a,n) = 1$ then $a \in U(n)$.

So

$$a^{|U(n)|} = e = a^{\phi(n)} = 1$$

or $a^{\phi(n)} \equiv 1 \bmod n$.

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20. suppose H and K are subgroups of a group G. If $|H| = 12$ and $|K| = 35$ Find $|H \cap K|$. Generalize.

$H \cap K \leq H$ and $H \cap K \leq K$

$\Rightarrow |H \cap K| / 12$ and $|H \cap K| / 35$

$\rightsquigarrow |H \cap K| = 1 \rightsquigarrow H \cap K = \{e\}$

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24. suppose that G is a group with more than one element and G has No proper nontrivial subgroups. prove that $|G|$ is prime (Do not assume at the outset that G is finite).

If $|G| = \infty$ then $a \neq e$, $\langle a \rangle \leq G$

$\Rightarrow$ $G = \langle a \rangle$ and $\langle a^2 \rangle \leq \langle a \rangle = G$ $\times$.

If $|G|$ is finite, $a \in G$, $a \neq e$.

Then $\{e\} \neq \langle a \rangle \leq G \Rightarrow \langle a \rangle = G$.

and $|a|$ is prime since otherwise it has a subgroup with order any divisor $\langle a \rangle$.

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28. let $|G| = 8$. show that G must have an element of order 2.

let $a \in G$, $a \neq e$

$|a| \mid |G| \rightsquigarrow |a| \mid 8$

$|a| = 2, 4, 8$.

$\rightsquigarrow$ a or a^2 or a^4 has order 2.

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31. let H and K be subgroups of a finite group G with $H \subseteq K \subseteq G$. prove that $|G:H| = |G:K||K:H|$.

$|G:H| = \frac{|G|}{|H|}$, $|G:K| = \frac{|G|}{|K|}$

So $|G:K| = \frac{|G|}{|K|} = \frac{|G|}{|H|} \cdot \frac{|H|}{|K|}$.