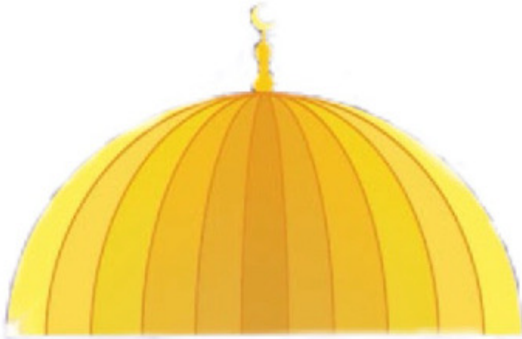

من للقدس إلا أنت

كلُّ محاولاتك عند الله أجور

استعن بالله ولا تعجز

(b) $x_1 + x_2 = 5$
$2x_1 + x_2 - x_3 = 6$
$3x_1 - 2x_2 + 2x_3 = 7$

Ans :- $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

9. Let

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

(a) Write **b** as a linear combination of the column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$.

(b) Use the result from part (a) to determine a solution of the linear system $A\mathbf{x} = \mathbf{b}$. Does the system have any other solutions? Explain.

(c) Write **c** as a linear combination of the column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$.

Ans :- a) $c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$\begin{bmatrix} c_1 + 2c_2 \\ c_1 - 2c_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$c_1 + 2c_2 = 4$
$+\ c_1 - 2c_2 = 0$

$2c_1 = 4 \rightarrow \boxed{c_1 = 2}$

$\boxed{c_2 = 1}$

$\begin{bmatrix} 4 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 4 \\ 0 \end{bmatrix} = 2a_1 + a_2$

b) $\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & -2 & 0 \end{array}\right] \xrightarrow{R_1 - R} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 4 & 4 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 1 \end{array}\right]$

$\boxed{x_2 = 1}$, $x_1 + 2 = 4$
$\boxed{x_1 = 2}$

the same answer of Part (a)
then it has unique sol.

c) $\begin{bmatrix} -3 \\ -2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} c_1 + 2c_2 \\ c_1 - 2c_2 \end{bmatrix}$

$2 = c_1 - 2c_2$
$-3 = c_1 + 2c_2 \quad +$

$\boxed{c_1 = -\frac{1}{2}}$, $c_2 = \frac{5}{4}$

$\begin{bmatrix} -3 \\ 2 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{5}{4}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -3 \\ 2 \end{bmatrix} = -\frac{1}{2}a_1 + \frac{5}{4}a_2$

11. Let A be a 5×3 matrix. If

$$\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_2 + \mathbf{a}_3$$

then what can you conclude about the number of solutions of the linear system $A\mathbf{x} = \mathbf{b}$? Explain.

$b = a_1 + a_2 + 0a_3$
$b = 0a_1 + 1a_2 + 1a_3$ $\longrightarrow$ $\left.\begin{array}{l} x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\ x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \end{array}\right\}$ infinite number of sol.

12. Let A be a 3×4 matrix. If

$$\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4$$

then what can you conclude about the number of solutions to the linear system $A\mathbf{x} = \mathbf{b}$? Explain.

$X = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $A_{3\times4}$ is under determin

inconsistant — infinite # of sol.

so, it has infinte # of sol

13. Let $A\mathbf{x} = \mathbf{b}$ be a linear system whose augmented matrix $(A|\mathbf{b})$ has reduced row echelon form

$$\left(\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

(a) Find all solutions to the system.

(b) If

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{a}_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}$$

determine $\mathbf{b}$.

a) x_2, x_4, x_5 are free variabels

$x_2 = \alpha$, $x_4 = \beta$, $x_5 = \gamma$

$x_3 + 2x_4 + 4x_5 = 5$
$x_3 + 2\beta + 4\gamma = 5$
$x_3 = 5 - 4\gamma - 2\beta$

$x_1 + 2x_2 + 3x_4 + x_5 = -2$

$x_1 = -2 - 2\alpha - 3\beta - \gamma$

b) if $\alpha, \beta, \gamma = 0 \Rightarrow (-2, 0, 5, 0, 0)$

$= a_1 x_1 + a_3 x_3$

$= -2 \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix} + 5 \begin{bmatrix} 2 \\ -1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -8 \\ -7 \\ -1 \\ 7 \end{bmatrix}$

15. Let A be an $m \times n$ matrix. Explain why the matrix multiplications A^TA and AA^T are possible.

A is an $m \times n$ matrix then A^T is $n \times m$ matrix
AA^T $m \times n, n \times m$ is valid multiplication.
and A^TA $n \times m, m \times n$ is valid multiplication.

16. A matrix A is said to be *skew symmetric* if $A^T = -A$. Show that if a matrix is skew symmetric, then its diagonal entries must all be 0.

Cosider that A is skew matrix, So $A^T = -A$
Which entries on A are a_{ij}, and b_{ji} entries in A^T
since $A^T = -A$, then $b_{ji} = -a_{ij}$, and when $i = j$
$a_{ji} = -a_{ij}$, $a_{ii} = -a_{ii}$, which give us that the diagonal is 0.
iff $a_{ii} = 0$

1.4 outline

4. Find nonzero matrices A, B, and C such that

$$AC = BC \quad \text{and} \quad A \neq B$$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$AC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $BC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ #

10. Let A and B be symmetric $n \times n$ matrices. For each of the following, determine whether the given matrix must be symmetric or could be nonsymmetric:

(a) $C = A + B$ **(b)** $D = A^2$
(c) $E = AB$ **(d)** $F = ABA$
(e) $G = AB + BA$ **(f)** $H = AB - BA$

a) $C^T = (A+B)^T = A^T + B^T$
$= A + B$
$= C$ → symm.

b) $D^T = (A.A)^T = A^T . A^T$
$= A . A$
$= A^2 = D$ Sym

c) $E^T = (AB)^T = B^T . A^T$
$= B . A$ non sym.

D) $F^T = (ABA)^T$ $AB = C$
$= (C.A)^T = A^T . C^T$
$= A^T . (B^T . A^T)$
$= A B A$
symm. $= F$

e) $G^T = (AB)^T + (BA)^T$
$= B^T . A^T + A^T . B^T$
$= BA + AB$
$= G$ symm.

f) $H^T = B^T A^T - A^T B^T$
$= BA - AB$
non symm.

12. Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Show that if $d = a_{11}a_{22} - a_{21}a_{12} \neq 0$, then

$$A^{-1} = \frac{1}{d}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$A \cdot A^{-1}$

$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} \frac{a_{22}}{d} & \frac{-a_{12}}{d} \\ \frac{-a_{21}}{d} & \frac{a_{11}}{d} \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ ⫽

14. Let A and B are $n \times n$ matrices. Show that if

$$AB = A \quad \text{and} \quad B \neq I$$

then A must be singular.

Suppose that A is nonsingular. $A \cdot A^{-1} = I$

$A^{-1} \cdot AB = A^{-1} \cdot A$

$\boxed{B = I}$ Contradiction

So A is singular.

15. Let A be a nonsingular matrix. Show that A^{-1} is also nonsingular and $(A^{-1})^{-1} = A$.

16. Prove that if A is nonsingular then A^T is nonsingular and

$$(A^T)^{-1} = (A^{-1})^T$$

Hint: $(AB)^T = B^T A^T$.

17. Let A be an $n \times n$ matrix and let $\mathbf{x}$ and $\mathbf{y}$ be vectors in $\mathbb{R}^n$. Show that if $A\mathbf{x} = A\mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$, then the matrix A must be singular.

15) $A^{-1} \cdot A = I$ ← A is the inverse of A^{-1}

$A^{-1} \cdot ? = I$ let $B = A^{-1}$

the inverse

$B \cdot \boxed{??} = I$

$B \cdot B^{-1} = I$

substitute B

$(A^{-1})^{-1}$ ⫽

16) $(A \cdot A^{-1})^T = (I)^T$

$(A^{-1})^T \cdot A^T (A^T)^{-1} = I^T \cdot (A^T)^{-1}$

$(A^{-1})^T = (A^T)^{-1}$ ⫽

17) Assume A is nonsingular.

$A^{-1}Ax = A^{-1}Ay$

$x = y \rightarrow$ Contradiction.

so, A is singular.

19. Let A be an $n \times n$ matrix. Show that if $A^2 = O$, then $I - A$ is nonsingular and $(I - A)^{-1} = I + A$.

$(I-A).(I-A)^{-1} = I \leftarrow$ we want to show that.

$(I-A).(I+A) = I^2 + I.A - A.I - \cancel{A^2}$ 0

$= I + \cancel{A - A}$ 0

$= I$ so, $(I-A)^{-1} = I + A$

25. Let A be an idempotent matrix.

(a) Show that $I - A$ is also idempotent.

(b) Show that $I + A$ is nonsingular and $(I + A)^{-1} = I - \frac{1}{2}A$

Idempotant means $\rightarrow A^2 = A$.

$\#$ $(I-A)^2 = (I-A) \leftarrow$ what we want to prove.

$= (I^2 - 2IA + A^2)$

$= I - 2A + A^2 \rightarrow$ $\underline{\underline{A^2 = A}}$ Idempotant matrix

$= I - 2A + A$

$= I - A$ $\#$

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28. Let A be an $m \times n$ matrix. Show that A^TA and AA^T are both symmetric.

تم الحل في التلخيص

29. Let A and B be symmetric $n \times n$ matrices. Prove that $AB = BA$ if and only if AB is also symmetric.

30. Let A be an $n \times n$ matrix and let

$$B = A + A^T \qquad \text{and} \qquad C = A - A^T$$

(a) Show that B is symmetric and C is skew symmetric.

(b) Show that every $n \times n$ matrix can be represented as a sum of a symmetric matrix and a skew-symmetric matrix.

30) تم الحل في التلخيص

29) $A^T = A, \quad B^T = B.$

$$(AB)^T = B^T . A^T = B \, A$$

* if AB symmetric will equals BA.

35. If A and B are singular $n \times n$ matrices, then $A + B$ is also singular.

36. If A and B are nonsingular matrices, then $(AB)^T$ is nonsingular and

$$((AB)^T)^{-1} = (A^{-1})^T(B^{-1})^T$$

35) Counter example :-

$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix},$

A is singular
B is singular

However $A+B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ nonsingular.

36) $\left((AB)\cdot(AB)^{-1}\right)^T = (I)^T$

$$\left((AB)^{-1}\right)^T \cdot (AB)^T = I^T \quad \cdot\left((AB)^T\right)^{-1}$$

$$\cdot\left((AB)^T\right)^{-1}$$

$$\left((AB)^{-1}\right)^T = \left((AB)^T\right)^{-1}$$

$$\left((AB)^T\right)^{-1} = \left(B^{-1}\cdot A^{-1}\right)^T$$

$$\left((AB)^T\right)^{-1} = \left(A^{-1}\right)^T \cdot \left(B^{-1}\right)^T$$

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