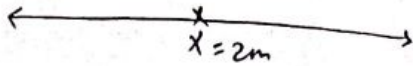


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Motion along a straight line
الحركة في بعد واحد

1 Position $[\vec{x}]$
الموقع



position of the Body = 2m

$$[x] \equiv m$$

لا تعتبر المسافة نقطة في البداية ولا النهاية

2 Displacement
الانزياح



$$\Delta \vec{x} = \vec{x}_2 - \vec{x}_1$$

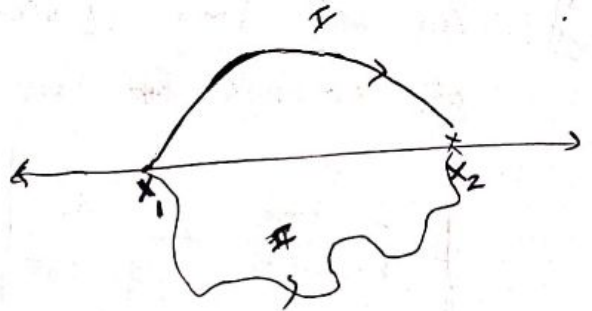
$$[\Delta x] = m$$

متجه vector
① does not depend on the Path.

متجه vector

3 distance
مسافة

- ① scalar quantity
- ② it depends on the path



4 average velocity
متوسط السرعة

\vec{v}_{avg} is a vector and Displacement

$$\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1}$$

$$[\vec{v}_{avg}] = \frac{m}{s}$$

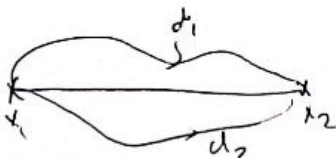


5 speed
سرعة

- * scalar
- * distance

$$* S = \frac{\text{total distance}}{\text{total time}}$$

$$S = d_1 + d_2$$

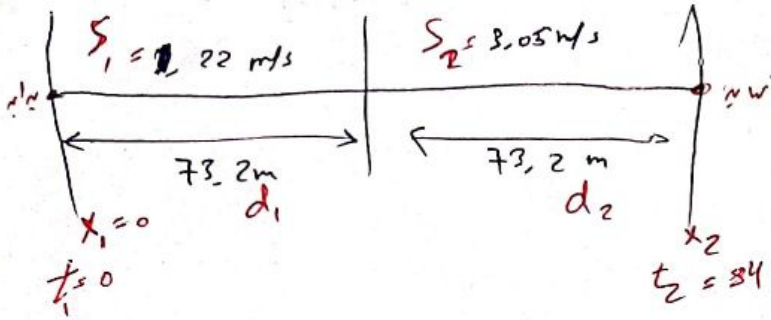


Q2
P 32

Compute V_{avg} in these two cases:

(I) You walk 73,2 m at speed 1,22 m/s and then you run 73,2 m at speed 3,05 m/s

$$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

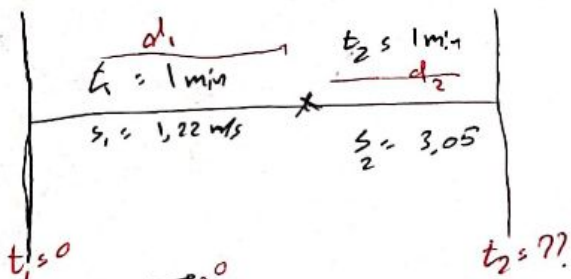


$$s_1 = \frac{d_1}{t_1} \Rightarrow t_1 = \frac{d_1}{s_1} = \frac{73,2}{1,22} = 60 \text{ sec}$$

$$s_2 = \frac{d_2}{t_2} \Rightarrow t_2 = \frac{73,2}{3,05} = 24 \text{ sec}$$

$$V_{avg} = \frac{146,4 - 0}{84 - 0} = \frac{146,4}{84} = 1,74 \text{ m/s}$$

(II) You walk 1 min at speed 1,22 m/s and then run for 1 min at 3,05 m/s find V_{avg}



$$t_2 = 1 \text{ min} + 1 \text{ min} = 2 \text{ min} = 120 \text{ sec}$$

$$s_1 = \frac{d_1}{t_1} \Rightarrow d_1 = s_1 t_1 = 73,2 \text{ m}$$

$$s_2 = \frac{d_2}{t_2} \Rightarrow d_2 = s_2 t_2 = 183 \text{ m}$$

$$x_2 = d_1 + d_2 = 256,2 \text{ m}$$

$$V_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{256,2 - 0}{120 - 0} = 2,14 \text{ m/s}$$

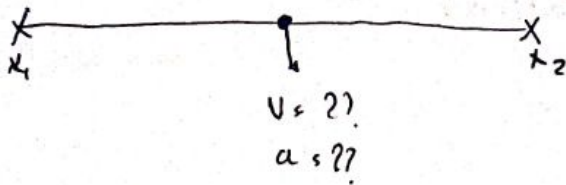
6 avg acceleration
* vector also

* depends on the velocity [change in velocity] $\Rightarrow [\vec{a}] = \frac{m}{s^2}$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

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x_1
 v_{avg}
 a_{avg}

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$v \equiv$ instantaneous velocity

$a \equiv$ instantaneous acceleration

$$v_{inst} = \frac{dx}{dt}$$

$$x = \int v dt$$

Exp. if $x(t) = t^2 - 2t$ find v at $t = 2$ sec?

$$v_{inst} = \frac{dx}{dt} = 2t - 2$$

$$v|_{t=2} = 2(2) - 2 = 2 \text{ m/s}$$

Exp2 if $x(t) = t^2 - t$

find v_{avg} between $t_1 = 3$ sec and $t_2 = 5$ sec

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$$

Exp3 if $v(t) = t + 1$ and $x(t=0) = 2$ find $x(t) = ?$

$$x(t) = \int v(t) dt$$

$$= \int t + 1 dt \Rightarrow x = \frac{t^2}{2} + t + C$$

$$x|_{t=0} = 2 \Rightarrow C = 2$$

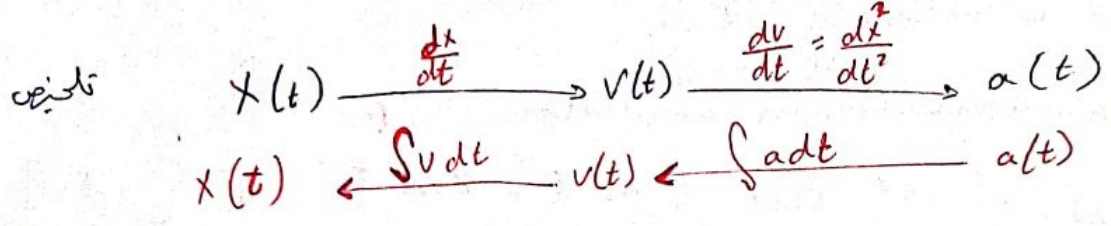
$$x(t) = \frac{t^2}{2} + t + 2$$

مشتقات

Instantaneous acceleration?

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$$a_{inst} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



Q14
P33

electron $x(t) = 16t e^{-t}$ How far is the electron from the origin when it stops?

stop $\Rightarrow v = 0$

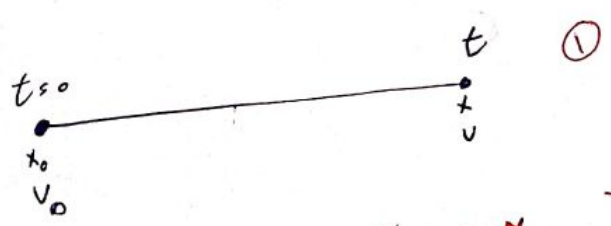
$$v = \frac{dx}{dt} = -16t e^{-t} + 16e^{-t} = 0$$

$$16e^{-t} [-t + 1] = 0$$

$\neq 0$

$$-t + 1 = 0 \Rightarrow t = 1 \text{ sec} \Rightarrow x = 16e^{-1} = \frac{16}{e} \text{ m}$$

Constant acceleration :-



$$a = \frac{dv}{dt} = \frac{v - v_0}{t - 0}$$

$$v = v_0 + at \quad (1)$$

- x_0 \equiv initial position
- v_0 \equiv initial velocity
- x \equiv final position
- v \equiv final velocity

$$(2) \quad v_{avg} = \frac{dx}{dt} = \frac{x - x_0}{t - 0}$$

$$x - x_0 = (v_{avg}) t \quad , \quad v_{avg} = \frac{v + v_0}{2}$$

$$x - x_0 = \left(\frac{v + v_0}{2}\right) t$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 \quad (2)$$

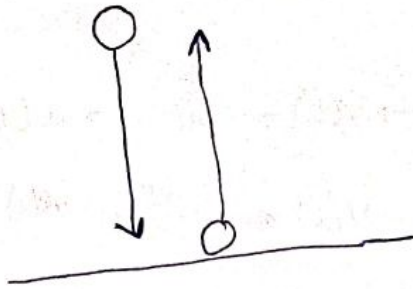
$$v^2 = v_0^2 + 2a \Delta x \quad (3)$$

* free falling

السقوط الحر

$a \rightarrow g$
 $ox \rightarrow oy$

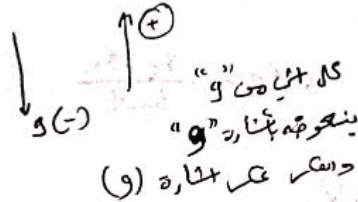
$a = g = 9,8 \text{ m/s}^2$



① $v = v_0 + gt$

② $y - y_0 = v_0 t + \frac{1}{2} g t^2$

③ $v^2 = v_0^2 + 2g \Delta y$



P58: An object falls a distance h from rest if it travels $0,5 h$ in the last 1 sec find (a) the time (b) the height of it is fall

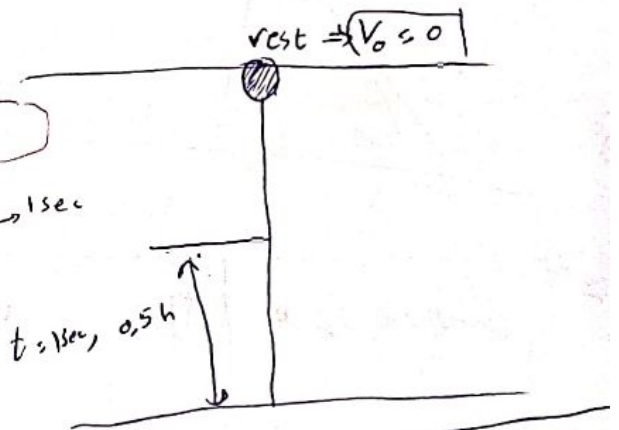
$\Delta y = v_0 t + \frac{1}{2} g t^2$

$h = \frac{1}{2} g t^2$

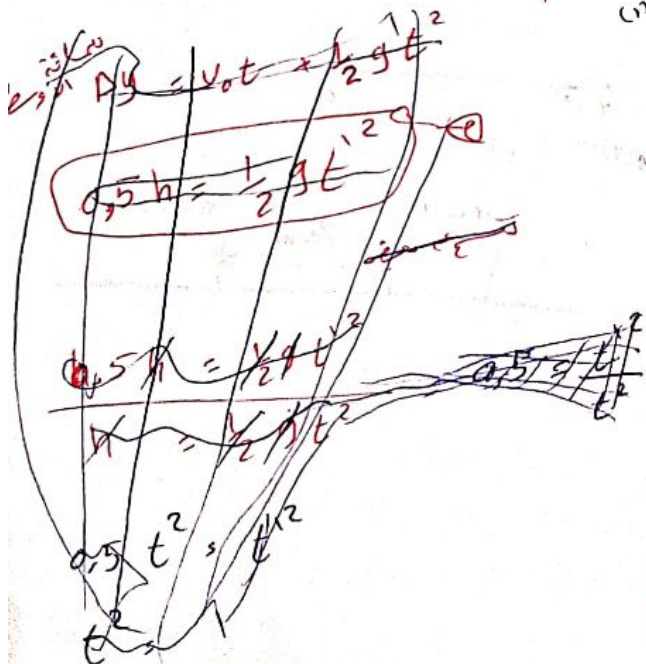
$v^2 = v_0^2 + 2g \Delta y$

$v = \sqrt{2g(0,5h)}$

$0,5h = v_0 t + \frac{1}{2} g t^2 \rightarrow 1 \text{ sec}$

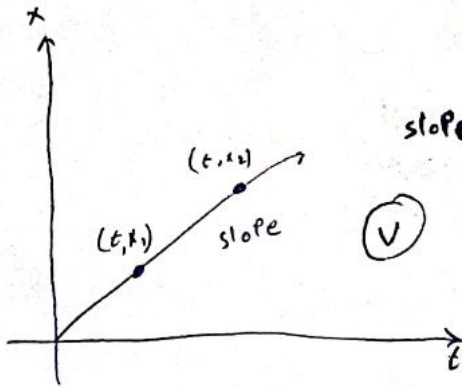
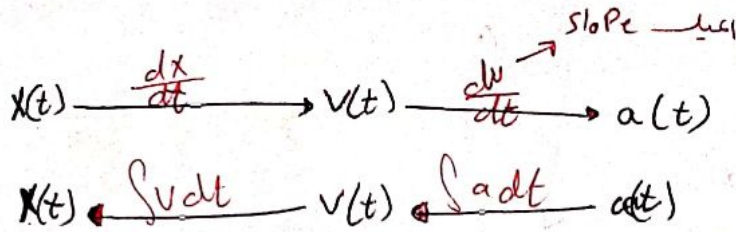


$t = 3,4 \text{ sec}$

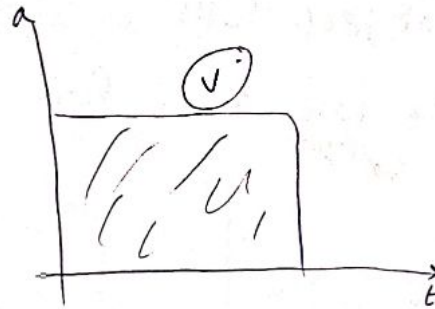
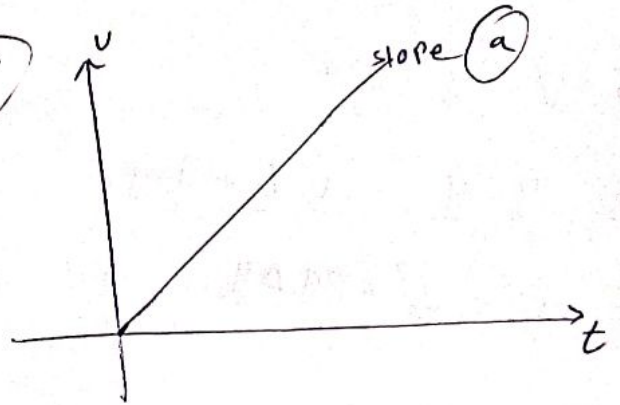


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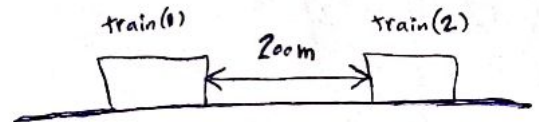
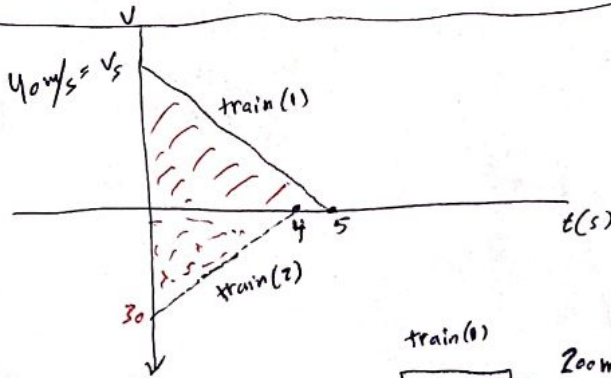
Graphical integration in motion analysis.



$$\text{slope} = \frac{x_2 - x_1}{t_2 - t_1}$$



P35



$$x(\text{Train (1)}) = \frac{1}{2} [5] [40] = 100 \text{ m}$$

$$x(\text{Train (2)}) = \frac{1}{2} [4] [30] = 60 \text{ m}$$

$$\Delta x = 40 \text{ m}$$

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Chapter 2

① ↓

Variables

- * distance, displacement
- * speed, avg velocity
- * avg acceleration
- * Inst, a , v ,

Constant a

x-motion

y-motion

$a = g$

$dx = dy$

Graphical

Chapter 2

Discussion

Q3

$V_{avg} = ??$

① V_{Avg}

$$V_{Avg} = \frac{x_f - x_i}{t_f - t_i}$$

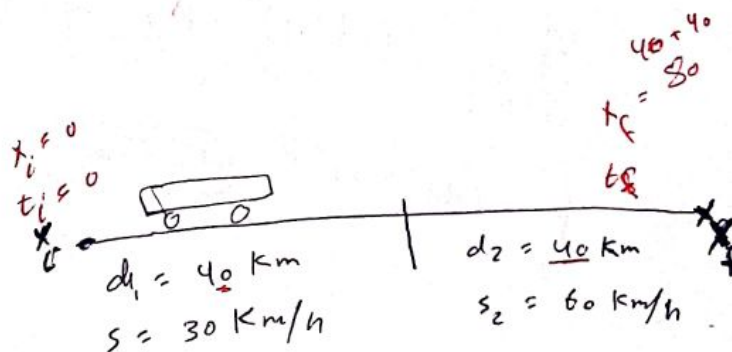
$$t_1 = \frac{d_1}{s_1} = \frac{40 \text{ km}}{30 \text{ km/h}} = 1.33 \text{ h}$$

$$t_2 = \frac{d_2}{s_2} = \frac{40}{60} = 0.67 \text{ h}$$

$$t_f = \text{total time} = 2 \text{ h}$$

$$V_{avg} = \frac{80 - 0}{2 - 0} = 40 \text{ km/h}$$

② $s = \frac{\text{total } d}{\text{total } t} = \frac{80}{2} = 40 \text{ km/h}$



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Q 17: $x = 9,75 + 1,5t^3$

find (a) the v_{avg} during $t = 2\text{sec}$ to $t = 3\text{sec}$ [2,3]

(b) instantaneous v at $t = 2\text{sec}$

(c) $t = 3\text{sec}$

(d) -- when the particle is midway at $t = 2\text{sec}$ and 3sec

(a) $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x(t=3) - x(t=2)}{3 - 2}$
 $= \frac{50,25\text{ cm} - 21,75\text{ cm}}{3 - 2} = 28,5\text{ cm/s}$

(b) $v_{ins} = \frac{dx}{dt} = 4,5t^2$
 $v|_{t=2} = (4,5)(2)^2 = 18\text{ cm/s}$

(c) $v|_{t=3} = (4,5)(3)^2 = 40,5\text{ cm/s}$

(d) $x_{mid} = \frac{x_1 + x_2}{2} = 36\text{ cm}$

$36 = 9,75 + 1,5t^3$
 $t = 2,546$
 $v|_t = 30,3\text{ m/s}$

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7 22

$$x(t) = ct^2 - bt^3$$

a) what unit, y , c , and b ? $c \left[\frac{m}{s^2} \right], b \left[\frac{m}{s^3} \right]$

b) $C = 3, b = 2$ at what time the particle need the maximum position from $x=0$ to $t=4$?
 position positive

c) what distance does the particles move.

d) = = displacement

e) find v at $t = 1 \text{ sec}$

- 1) $t = 2, 3, 4 \text{ s}$
- 2) find a at $t = 1, 2, 3, 4 \text{ s}$.

f) when particle reach max $\Rightarrow v = 0$

$$v = \frac{dx}{dt} = 2ct - 3bt^2 = 0, t[2c - 3bt] = 0$$

$$t \neq 0$$

$$2c - 3bt = 0$$

$$t = \frac{2c}{3b} = 1 \text{ sec}$$

g) $x(t=4) = -80 \text{ m}$
 move to $x=1$ then

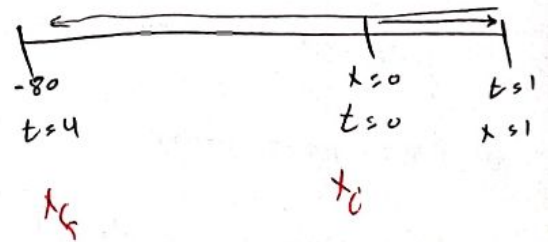
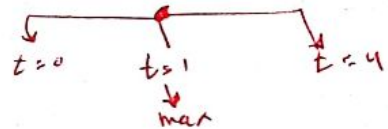
$$\text{total path} = |x| + 80 \text{ m} = 82 \text{ m}$$

h) displacement

$$x(t=0) = 0$$

$$x(t=4) = -80$$

$$\Delta x = -80 - 0 = -80 \text{ m}$$



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Q 35 $x_{g0} = 270 \text{ m}$
 $x_{r0} = -35 \text{ m}$

green-car (speed constant)
 $= v_{g0} = 20 \text{ m/s}$ ($a_g = 0$)

red-car begin from rest [$v_{r0} = 0$]

what is a of red car ??

$$x_r(t) = x_{r0} + \frac{1}{2} a_r t^2 + v_{r0} t$$

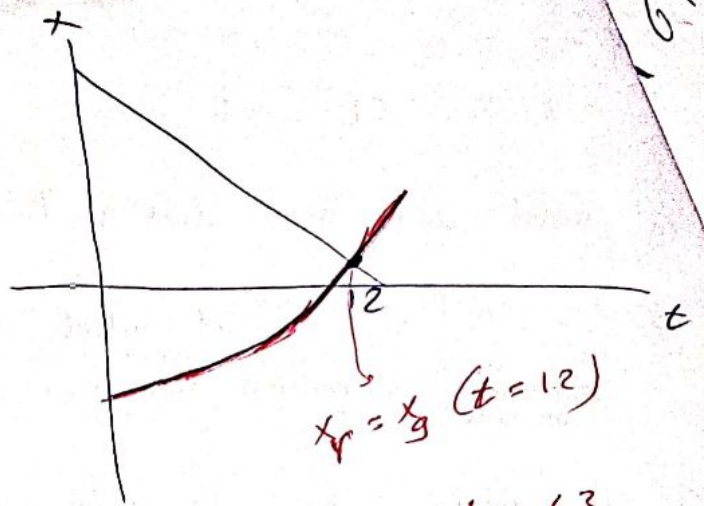
$$x_r(t) = x_{r0} + \frac{1}{2} a_r t^2$$

$$x_g = x_{g0} + v_g t, a = 0$$

$$x_r = x_g \text{ (when } t = 12)$$

$$270 - 20(12) = -35 + \frac{1}{2} a_r (12)^2$$

$$a = 0,9 \text{ m/s}^2$$



$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$x(t_0) = v_0 t + \frac{1}{2} a t^2$$

$$x = x_0 + \frac{1}{2} a t^2 + v_0 t$$

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Q 51

$v = 19,6 \text{ m/s}$
 vertical velocity

@ max-height above

b) how high is break-face point above the ground.

$$v = v_0 + g t$$

$$0 = v_0 - (9,8)(2) = 19,6 \text{ m/s}$$

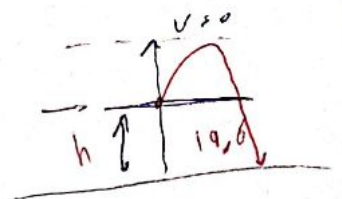
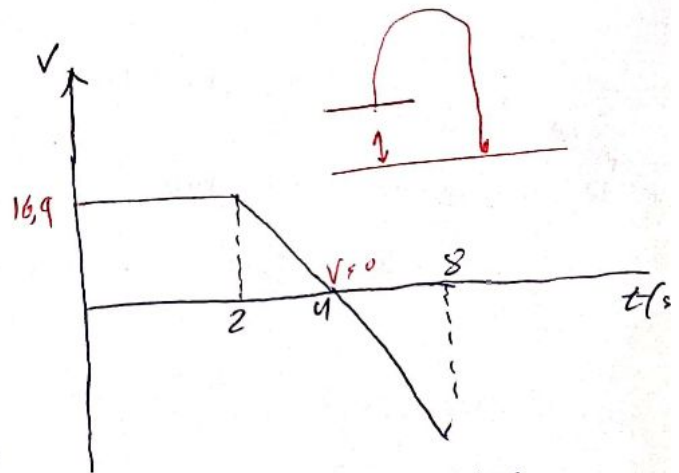
$$\Delta y = v_0 t + \frac{1}{2} g t^2$$

$$(19,6)(2) + \frac{1}{2} (-9,8)(2)^2 \approx 20 \text{ m}$$

$$\Delta y = v_0 t + \frac{1}{2} g t^2$$

$$= (19,6)(6) + \frac{1}{2} (-9,8)(6)^2 \approx 59 \text{ m}$$

$$|\Delta y| = 59, \text{ m}$$

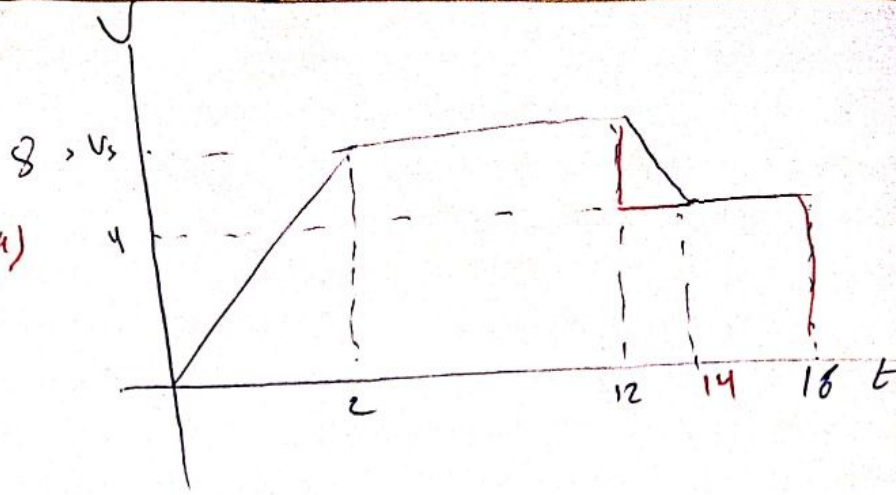


X 69

$$\Delta x = \text{Area}(v-t)$$

$$s = \frac{1}{2}(2)(8) + 8(8) + \frac{1}{2}(7)(4) + 4(4)$$

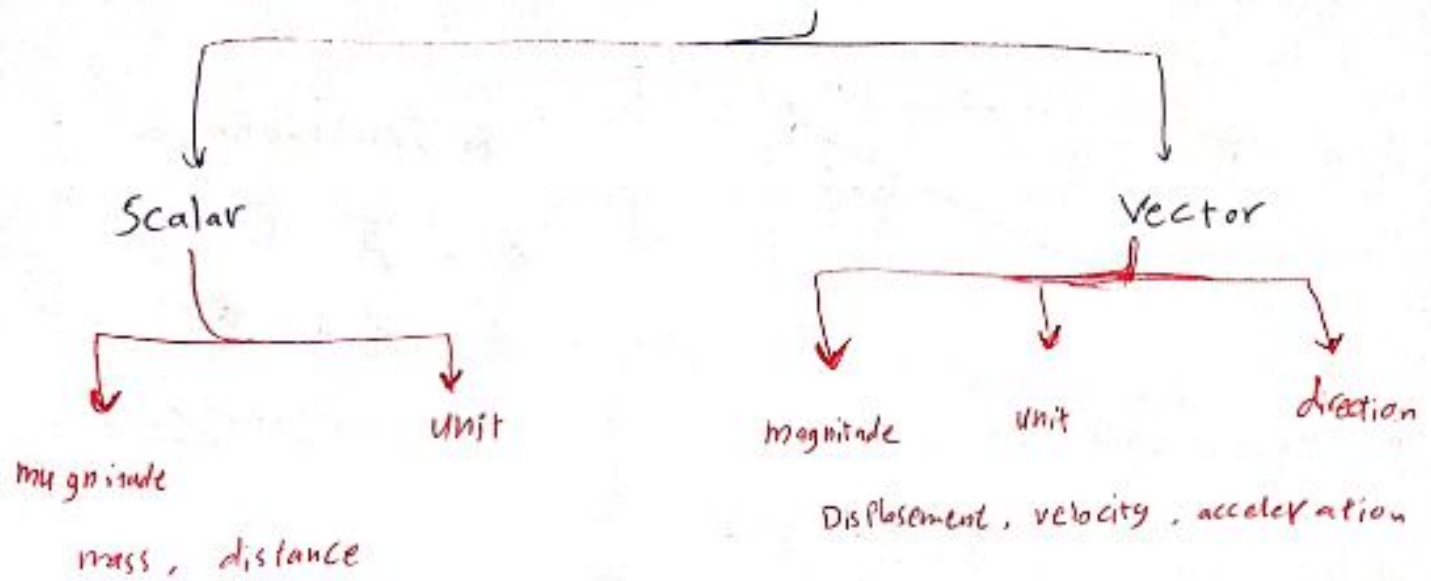
s 100 m



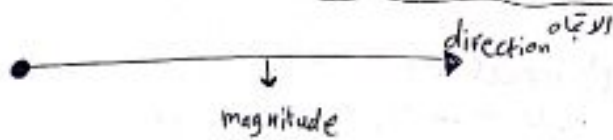
Good Luck
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Chapter 3 Vectors

Physical quantities



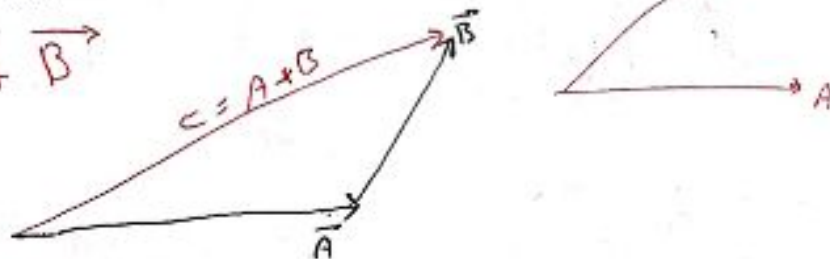
* Vectors



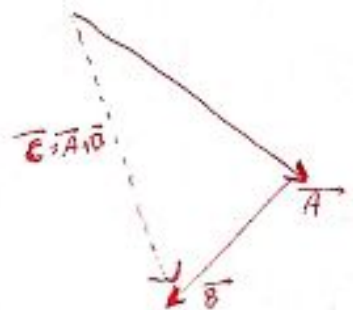
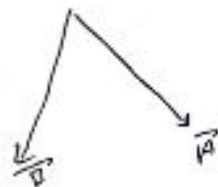
vector ① \hat{i} ①
 ② \hat{j} ②
 ③ \hat{k} ③

* Adding vectors Geometrically

$$\vec{C} = \vec{A} + \vec{B}$$



Exmp 1 find $\vec{C} = \vec{A} + \vec{B}$



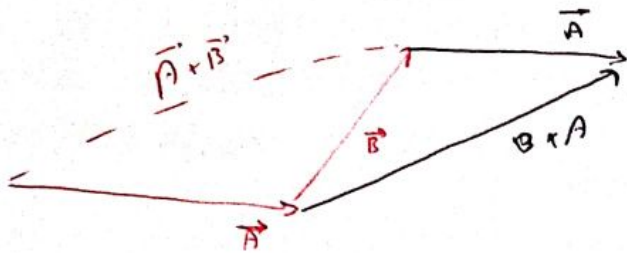
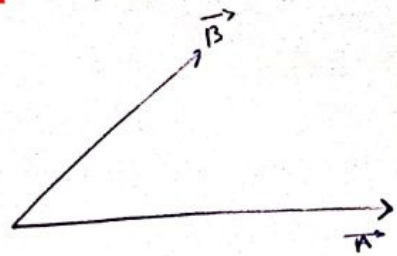
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القوانين

① Commutative التبادلية

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



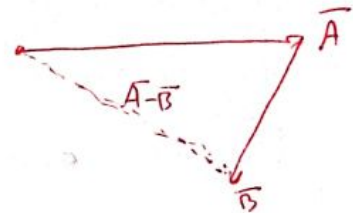
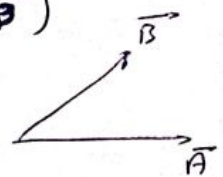
② associative law :
القانون التجميعي

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

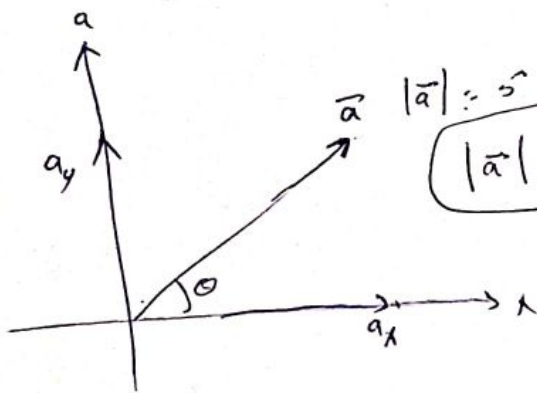
* Subtraction

$$\vec{C} = \vec{A} - \vec{B}$$

$$= \vec{A} + (-\vec{B})$$



* Components of vectors
المكونات المتجهية



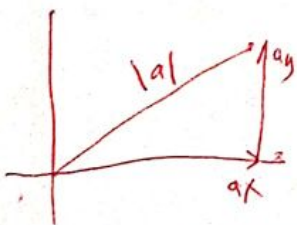
$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$a_x = |\vec{a}| \cos \theta$$

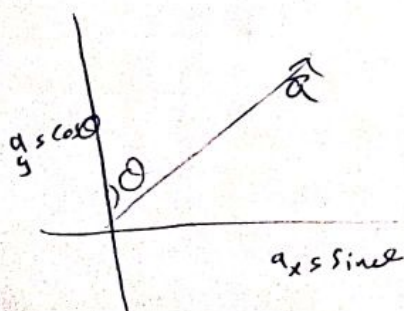
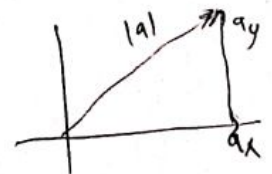
$$a_y = |\vec{a}| \sin \theta$$

$$\sqrt{a_x^2 + a_y^2} = \sqrt{|\vec{a}|^2 \cos^2 \theta + |\vec{a}|^2 \sin^2 \theta}$$

$$= \sqrt{|\vec{a}|^2 [\cos^2 \theta + \sin^2 \theta]} = \sqrt{|\vec{a}|^2} = |\vec{a}|$$



$$\tan \theta = \frac{a_y}{a_x}$$



$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$\tan \theta = \frac{a_y}{a_x}$$

$$\sin(\theta + 2\theta) = 2 \sin \theta$$

STP

$$AB \cos \theta = A \cdot B$$

$$\cos B = \cos(B + \theta)$$

$$AB \cos \theta = A \cdot B$$

* Adding Vectors :-

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \quad \text{find } \vec{c} = \vec{a} + \vec{b} \text{ : ??}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\vec{c} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j} + (a_z + b_z) \hat{k}$$

Exp: if $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 2\hat{k}$ find $\vec{a} + \vec{b} = ??$

$$\vec{a} + \vec{b} = (2+1)\hat{i} + (3-4)\hat{j} + (-1+2)\hat{k}$$

$$\vec{a} + \vec{b} = 3\hat{i} - \hat{j} + \hat{k} \Rightarrow |\vec{a} + \vec{b}| = \sqrt{(3)^2 + (-1)^2 + (1)^2} = \sqrt{9+1+1} = \sqrt{11}$$

Chapter 3 Part 2

$|\vec{a}|, |\vec{b}|, \theta$; addition

$$|\vec{c}| = |\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$$

$|\vec{a}| = |\vec{b}|$ ① \vec{a}, \vec{b} in the same direction

$$\begin{array}{l} |\vec{a}| = 5 \\ |\vec{b}| = 6 \end{array} \quad \vec{c} = |\vec{a}| + |\vec{b}|$$

② \vec{a}, \vec{b} in opposite direction

$$\begin{array}{l} \vec{b} = 7 \\ \vec{a} = 5 \end{array} \quad |\vec{c}| = |\vec{b} - \vec{a}|$$

in the same direction of the largest vector

③ \vec{a}, \vec{b} , Normal \perp



$$|\vec{c}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2}$$

$$\tan\theta = \frac{|\vec{a}|}{|\vec{b}|}$$

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* ضرب المتجهات Multiplying Vectors

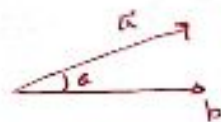
① dot Product (الضرب النقطي)

② cross Product (الضرب المتقاطع)

* dot product

①

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$



if $(\vec{a} \perp \vec{b}) \Rightarrow \vec{a} \cdot \vec{b} = 0$

② $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$

find $\vec{a} \cdot \vec{b} = ??$

$\hat{i} \cdot \hat{j} = 0$
 $\hat{j} \cdot \hat{k} = 0$
 $\hat{i} \cdot \hat{k} = 0$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

$\hat{i} \cdot \hat{i} = |\hat{i}| |\hat{i}| \cos 0 = 1$

$\hat{j} \cdot \hat{j} = 1$
 $\hat{k} \cdot \hat{k} = 1$

EXP: if $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, find $\vec{a} \cdot \vec{b}$?
 $\vec{b} = 3\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{a} \cdot \vec{b} = 6 + 12 - 5 = 13$

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نفس $\vec{a} \cdot \vec{b}$

Case I $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$

Case II $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

② find θ between \vec{a} and \vec{b} ?

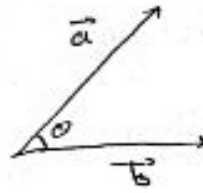
$|\vec{a}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$
 $|\vec{b}| = \sqrt{3^2 + 3^2 + 5^2} = \sqrt{50}$

$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$
 $13 = \sqrt{14} \sqrt{50} \cos \theta \Rightarrow \cos \theta = \frac{13}{\sqrt{14} \sqrt{50}} = 0.49 \Rightarrow \theta \approx 60$

* Cross Product .

Case I..

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta$$



$$\begin{aligned} \hat{i} \times \hat{i} &= 0 \\ \hat{j} \times \hat{j} &= 0 \\ \hat{k} \times \hat{k} &= 0 \end{aligned}$$



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Case II :- $\vec{a} = a_x \hat{i} + a_y \hat{j}$
 $\vec{b} = b_x \hat{i} + b_y \hat{j}$

$$\vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j}) \times (b_x \hat{i} + b_y \hat{j}) \Rightarrow \boxed{a_x b_y \hat{k} - a_y b_x \hat{k}}$$

Exp. : if $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$
 $\vec{b} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

find $\vec{a} \times \vec{b} = ??$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix}$$

$$= (3 \times 2 - 4 \times 1) \hat{i} - (2 \times 2 - 3 \times 1) \hat{j} + (2 \times 4 - 3 \times 3) \hat{k}$$

$$= \boxed{\vec{a} \times \vec{b} = 10\hat{i} - 7\hat{j} + \hat{k}}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(10)^2 + (-7)^2 + (1)^2}$$

Exo 10 : $\vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{c} \perp \vec{a}$
 $\vec{c} \perp \vec{b}$

$$P_{34} \quad \vec{a} = 3\hat{i} + 5\hat{j}$$

$$\vec{b} = 2\hat{i} + 4\hat{j}$$

find (a) $\vec{a} \times \vec{b}$ (b) $\vec{a} \cdot \vec{b}$ (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$

(d) the component of \vec{a} along the direction of \vec{b}

$$\text{(a) } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 0 \\ 2 & 4 & 0 \end{vmatrix} = (0) \hat{i} - (0) \hat{j} + (2) \hat{k} = \boxed{2\hat{k}}$$

$$\text{(b) } \vec{a} \cdot \vec{b} = 6\hat{i} + 20\hat{j} = \boxed{26}$$

$$\text{(c) } \vec{a} + \vec{b} = 5\hat{i} + 9\hat{j}$$

$$(\vec{a} + \vec{b}) \cdot \vec{b} = 5 \times 2 + 9 \times 4 = \boxed{46}$$

$$\text{(d) } \vec{a} \cdot \hat{b}$$

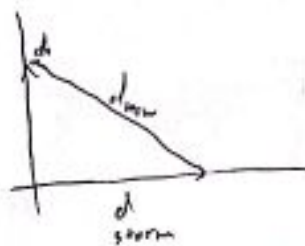
$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{4 + 16}} = \left(\frac{2}{\sqrt{20}} \hat{i} + \frac{4}{\sqrt{20}} \hat{j} \right)$$

$$\vec{a} \cdot \hat{b} = (3\hat{i} + 5\hat{j}) \cdot \left(\frac{2}{\sqrt{20}} \hat{i} + \frac{4}{\sqrt{20}} \hat{j} \right) = 1,34 + 2,68 = \boxed{4}$$

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Chapter 3 Discussion

Q5: $\vec{d}_0 = 120 \text{ km } \hat{j}$
 $d_{\text{stream}} = 100 \hat{i}$



$$\vec{d}_0 = \vec{d}_{\text{new}} + \vec{d}_{\text{stream}}$$

$$\vec{d}_{\text{new}} = \vec{d}_0 - \vec{d}_{\text{stream}}$$

$$\vec{d}_{\text{new}} = 120 \hat{j} - 100 \hat{i}$$

$$\vec{d}_{\text{new}} = -100 \hat{i} + 120 \hat{j}$$

$$|\vec{d}_{\text{new}}| = \sqrt{(-100)^2 + (120)^2} = \underline{156 \text{ km}}$$

(b) direction = $\tan^{-1} \left(\frac{120}{-100} \right) \Rightarrow \theta = \underline{-59.2}$

Q12: $\vec{A} = 50 \hat{i}$
 $\vec{B} = 30 \hat{j}$

$$\vec{C} = 25 \cos 60 \hat{i} + 25 \sin 60 \hat{j}$$

(a) $\vec{r} = \vec{A} + \vec{B} + \vec{C} = 62.5 \hat{i} + 51.7 \hat{j}$

$$|\vec{r}| = \sqrt{(62.5)^2 + (51.7)^2} = \underline{81 \text{ km}}$$

(b) $\tan \theta = \frac{51.7}{62.5} \Rightarrow \theta = \underline{40^\circ}$ north of east

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Q36 if $\vec{d}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$
 $\vec{d}_2 = -5\hat{i} + 2\hat{j} - \hat{k}$

what is $\frac{(\vec{d}_1 + \vec{d}_2) \cdot (\vec{d}_1 \times \vec{d}_2)}{|\vec{d}_1 + \vec{d}_2| |\vec{d}_1 \times \vec{d}_2|}$?

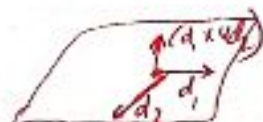
(a) $\vec{d}_1 + \vec{d}_2 = \underline{-2\hat{i} + 3\hat{k}}$

$\vec{d}_1 \times \vec{d}_2$	$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ -5 & 2 & -1 \end{vmatrix}$
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$$= \underline{-24\hat{i} - 68\hat{j} - 16\hat{k}}$$

$$= \underline{(A \cdot 0) = 0}$$

\vec{d}_1, \vec{d}_2 in the plane
 $\vec{d}_1 + \vec{d}_2$ is perpendicular so



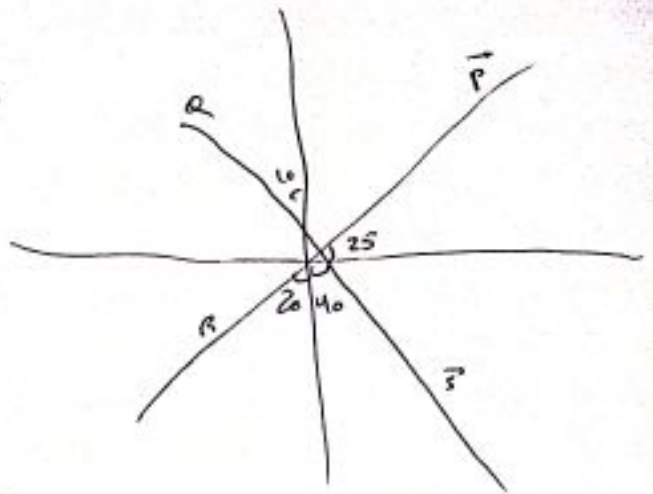
P. 56

\vec{P} : 10 m at 25° counter clock wise x - x

\vec{Q} : 12 m \sim 10 clock wise $+y$

\vec{R} : 8 m \sim 20 counter clock wise $-y$

\vec{S} : 9 m \sim 40 counter clock wise $-y$



$$\vec{P} = 10 \cos 25^\circ \hat{i} + 10 \sin 25^\circ \hat{j}$$

$$\vec{Q} = 12 \cos 100^\circ \hat{i} + 12 \sin 100^\circ \hat{j}$$

$$\vec{R} = 8 \cos 250^\circ \hat{i} + 8 \sin 250^\circ \hat{j}$$

$$\vec{S} = 9 \cos 310^\circ \hat{i} + 9 \sin 310^\circ \hat{j}$$

$$\vec{P} + \vec{Q} + \vec{R} + \vec{S} = \boxed{10\hat{i} + 1.63\hat{j}}$$

$$\sqrt{(10)^2 + (1.63)^2} = 10.2 \text{ m}$$

$$\tan \theta = \frac{1.63}{10} \Rightarrow \theta = 9.24^\circ$$

P. 73) Two vectors $\vec{a} = 3\hat{i} + 5\hat{j}$, $\vec{b} = 2\hat{i} + 4\hat{j}$ find @ $\vec{a} \times \vec{b}$
 (b) $\vec{a} \cdot \vec{b}$ (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$ (d) the component of \vec{a} along \vec{b}

$$\text{(a) } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 0 \\ 2 & 4 & 0 \end{vmatrix} = 2\hat{k}$$

$$\text{(b) } \vec{a} \cdot \vec{b} = 3(2) + 5(4) = 26$$

$$\text{(c) } (\vec{a} + \vec{b}) \cdot \vec{b} = (5\hat{i} + 9\hat{j}) \cdot (2\hat{i} + 4\hat{j}) = 10 + 36 = \boxed{46}$$

$$\text{(d) } \hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{(2)^2 + (4)^2}} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{20}} = \frac{2\hat{i}}{\sqrt{20}} + \frac{4\hat{j}}{\sqrt{20}}$$

$$\vec{a} \cdot \hat{b} = \frac{3(2) + 5(4)}{\sqrt{20}} = 5.81$$

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Chapter 4

motion in two and three dimensions

① Position Vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Example: $\vec{r} = -3\hat{i} + 2\hat{j} + 5\hat{k}$

② Displacement Vector

If the object is displaced from position r_1 to r_2

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

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Sample problem 4.1

$\vec{r}_1 = -3\hat{i} + 2\hat{j} + 5\hat{k}$, $\vec{r}_2 = 9\hat{i} + 2\hat{j} + 8\hat{k}$ find $\Delta \vec{r}$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= (9 - (-3))\hat{i} + (2 - 2)\hat{j} + (8 - 5)\hat{k} = 12\hat{i} + 3\hat{k}$$

Q. : A Particle goes from $x = -2, y = 3, z = 1$ to $x = 3, y = -1, z = 4$
It's displacement is.

(a) $1\hat{i} + 2\hat{j} + 5\hat{k}$

(b) $5\hat{i} + 4\hat{j} + 3\hat{k}$

(c) $-5\hat{i} + 4\hat{j} - 3\hat{k}$

(d) $-5\hat{i} - 2\hat{j} - 3\hat{k}$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (3 - (-2))\hat{i} + (-1 - 3)\hat{j} + (4 - 1)\hat{k} = 5\hat{i} - 4\hat{j} + 3\hat{k}$$

سؤال 16 نست بائو

$$x = v_0 \sqrt{\frac{2y}{g}}$$

Q. $r_1 = 2\hat{i} + 5\hat{j} - 12\hat{k}$
 $r_2 = 2\hat{i} + 5\hat{j} + 8\hat{k}$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= (2 - 2)\hat{i} + (5 - 5)\hat{j} + (8 - (-12))\hat{k} = 20\hat{k}$$

* Average velocity

$$v_{avg} = \frac{\text{displacement}}{\text{interval time}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}$$

$$= v_{avg,x} \hat{i} + v_{avg,y} \hat{j} + v_{avg,z} \hat{k}$$

* Instantaneous velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$$

$$= v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

* Average acceleration

$$\vec{a}_{avg} = \frac{\text{change in velocity}}{\text{interval time}} = \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} + \frac{\Delta v_z}{\Delta t} \hat{k}$$

$$= a_{avg,x} \hat{i} + a_{avg,y} \hat{j} + a_{avg,z} \hat{k}$$

* Instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k}$$

$$= \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} + \frac{d^2z}{dt^2} \hat{k}$$

$$= a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

* Sample Problem 4, 2

$$x = -0,31t^2 + 7,2t + 28$$

$$y = 0,22t^2 - 9,1t + 30$$

@ At $t = 15$ s, what is the rabbit's position vector \vec{r} in unit vector notation and in magnitude-angle notation?

$$\vec{r} = x\hat{i} + y\hat{j} = (-0,31t^2 + 7,2t + 28)\hat{i} + (0,22t^2 - 9,1t + 30)\hat{j}$$

$$\vec{r}(15) = (-0,31(15)^2 + 7,2(15) + 28)\hat{i} + (0,22(15)^2 - 9,1(15) + 30)\hat{j}$$

$$= (66\text{ m})\hat{i} + (-57\text{ m})\hat{j}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(66)^2 + (-57)^2} = 87\text{ m}$$

$$\theta = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{-57}{66} = -41^\circ$$



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b) find the velocity of the rabbit at the instant $t = 15$ s?

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$v_x = \frac{dx}{dt} = -0,62t + 7,2 \Rightarrow v_x(t=15) = \boxed{-3,1} \text{ m/s}$$

$$v_y = \frac{dy}{dt} = 0,44t - 9,1 \Rightarrow v_y(t=15) = \boxed{-2,5} \text{ m/s}$$

$$\vec{v} = (-2,5) \hat{i} + (-2,5) \hat{j}$$

c) find the acceleration at $t = 15$ s

$$v_x = -0,62t + 7,2 \Rightarrow a_x = \frac{dv_x}{dt} = -0,62 \text{ m/s}^2$$

$$v_y = 0,44t - 9,1 \Rightarrow a_y = \frac{dv_y}{dt} = 0,44 \text{ m/s}^2$$

$$\vec{a} = -0,62 \hat{i} + 0,44 \hat{j}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0,62)^2 + (0,44)^2} = 0,76 \text{ m/s}^2$$

$$\theta = \tan^{-1} \frac{0,44}{-0,62} = \boxed{-35^\circ}$$

$-35 + 180 = 145^\circ$

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1) $\Delta \vec{r} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$$\vec{r}_f = 3\hat{j} - 4\hat{k}$$

$$\Delta r = \vec{r}_f - \vec{r}_i \Rightarrow r_0 = \vec{r}_f - \Delta r$$

$$2\hat{i} - 3\hat{j} + 6\hat{k} = 3\hat{j} - 4\hat{k} - \vec{r}_0$$

$$r_0 = r - \Delta r$$

$$= (3\hat{j} - 4\hat{k}) - (2\hat{i} - 3\hat{j} + 6\hat{k}) = -2\hat{i} + 6\hat{j} + (-10)\hat{k}$$

(5) $\vec{r} = 5\hat{i} - 6\hat{j} + 2\hat{k}$ $t = 10\text{s}$ $\vec{r} = -2\hat{i} + 8\hat{j} - 2\hat{k}$



$$V_{\text{avg}} = \frac{d\vec{r}}{dt} = \frac{\vec{r}_2 - \vec{r}_1}{\Delta t} = \frac{(-2\hat{i} + 8\hat{j} - 2\hat{k}) - (5\hat{i} - 6\hat{j} + 2\hat{k})}{10\text{s}}$$

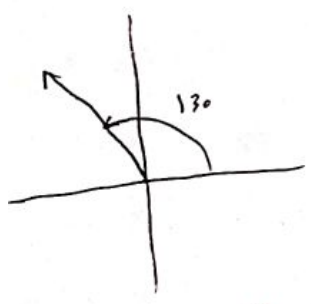
$$= \boxed{-0,7\hat{i} + 1,40\hat{j} - 0,4\hat{k}} \text{ m/s}$$

(11) $\vec{r} = \hat{i} + 4t^2\hat{j} + t\hat{k}$ (a) velocity (b) acceleration

(a) $\vec{v} = \frac{d}{dt} = \boxed{8t\hat{j} + \hat{k}}$ (b) $\vec{a} = \frac{d}{dt} = 8\hat{j} + \hat{k} = \boxed{8\hat{j}}$

Sample Problem 4.5

$\vec{v}_0 = -2\hat{i} + 4\hat{j}$ at $t = 0$ $a = 3\text{ m/s}^2$ $\theta = 130^\circ$ \vec{v} at $t = 5\text{s} = ??$



$a_x = a \cos \theta = 3 \cos 130^\circ = \boxed{-1,93 \text{ m/s}^2}$
 $a_y = a \sin \theta = 3 \sin 130^\circ = \boxed{2,30 \text{ m/s}^2}$

$\vec{a} = \boxed{-1,93\hat{i} + 2,30\hat{j}}$

$v = v_0 + at$

$v_x = -2 + (-1,93)(5) = \boxed{-11,65 \text{ m/s}}$
 $v_y = 4 + (2,30)(5) = \boxed{15,50 \text{ m/s}}$

$\vec{v} = \boxed{-12\hat{i} + 16\hat{j}}$

$v = \sqrt{(-12)^2 + (16)^2} = 19,4 \approx 19$
 $\theta = \tan^{-1} \frac{v_y}{v_x} = \frac{12,7}{19} = \boxed{130^\circ}$



Q: At $t = 0$ a car moves with velocity $v_0 = 2\hat{i} + \hat{j}$ and $\vec{a} = 2\hat{i}$ the velocity of the car at $t = 2\text{ s}$:

$v_{0x} = 2$ $v_{0y} = 1$ $a_x = 2$ $a_y = 0$

$v_x = v_{0x} + a_x t = 2 + 2 \times 2 = \boxed{6}$ $v_y = v_{0y} + a_y t = 1 + 0 \times 2 = \boxed{1}$

$\vec{v} = v_{0x}\hat{i} + v_{0y}\hat{j} = \boxed{2\hat{i} + 5\hat{j}}$

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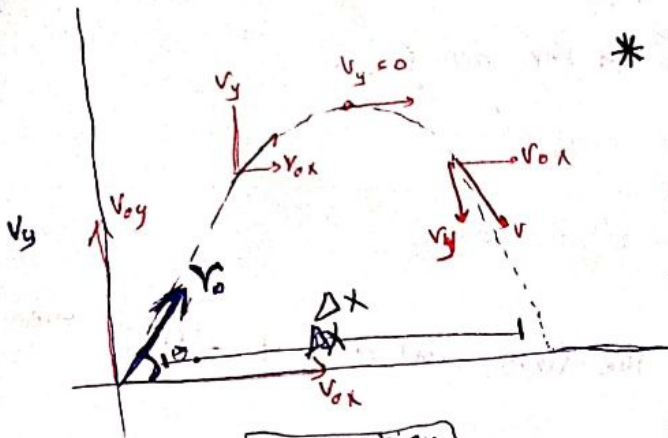
$$x = v_0 \sqrt{\frac{2y}{g}}$$

Chapter 4 Part 2

Projectile motion

حركة المقذوفات

* X-motion, y-motion are independent on each other
 كل حركة مستقلة عن الأخرى



$$v_{0x} = v_0 \cos \theta$$

$$v_{0y} = v_0 \sin \theta = (v_0 \cos \theta) t$$

ملاحظة

X-motion

$$1) v_{0x} \Rightarrow \text{Constant } (a_x = 0)$$

$$2) \Delta x = (v_{0x})(t)$$

Y-motion

$$1) v_y = v_{0y} + g t$$

$$2) v_y^2 = v_{0y}^2 + 2a \Delta y$$

$$3) \Delta y = v_{0y} t + \frac{1}{2} g t^2$$

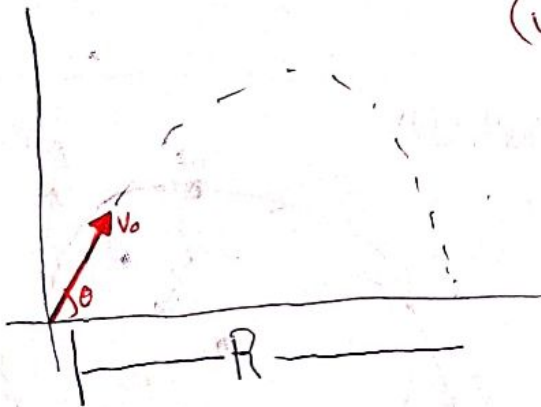
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* Range

$$\Delta x = R = (v_0 \cos \theta)(t) \quad (1)$$

$$\Delta y = 0 = v_{0y} t + \frac{1}{2} g t^2 = 0$$

$$v_0 \sin \theta(t) + \frac{1}{2} g t^2 = 0 \quad (2)$$



$$R = \frac{v_0^2 \sin 2\theta}{g}$$

* Range max $\rightarrow \theta_0 = 45^\circ$

بعض الأحيان

* Equation of the Path

$$y = (\tan \theta_x) x - \frac{g x^2}{2(v_0 \cos \theta)^2}$$

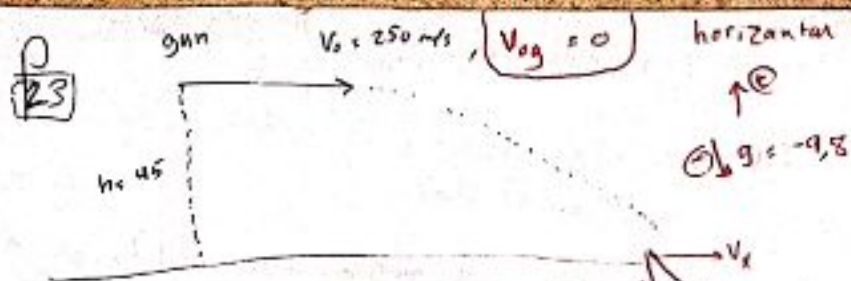
$$t = \sqrt{\frac{2y}{g}}$$

$$x = v_0 t$$

$$R = v_x t$$

$$v_f = v_0 \sin \theta - g t$$

$$y = \frac{1}{2} g t^2$$



a) how long does the projectile remain in the air? ($t = ?$)

$$\Delta y = v_{y0}t + \frac{1}{2}gt^2 \Rightarrow -h = \frac{1}{2}(-9.8)t^2$$

$$t = \sqrt{\frac{2h}{g}} = \boxed{3.03 \text{ Sec}}$$

b) At what horizontal distance from the firing point does it strike the ground.

$$\Delta x = v_{0x}t = 250(3.03) = \underline{\underline{758 \text{ m}}}$$

c) what is the magnitude of the vertical velocity when it strikes the ground

$$v_x = v_{0x} \Rightarrow v_y = v_{y0} + gt$$

$$v_y = gt = (9.8)(3.03) = \boxed{29.6} \Rightarrow |v_y| = \underline{\underline{29.6}}$$

d) مقدار السرعة عند الاصطدام
 $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

2) Uniform Circular motion (الحركة الدائرية المنتظمة)

$\vec{v} \Rightarrow$ in the direction of the tangent

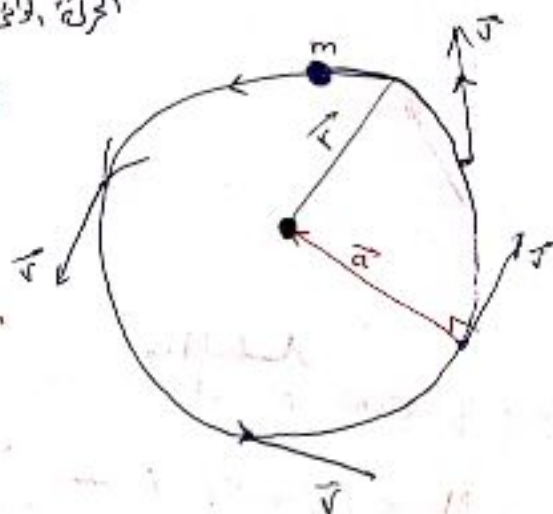
Uniform $\Rightarrow |\vec{v}| = \text{constant}$

$$|\vec{a}_t| = 0$$

$$|\vec{v}| = \frac{2\pi r}{\text{Time}}$$

$T \equiv$ Period
 الزمن الدوري

$a_r \equiv$ centripetal acceleration
 $(a_r \perp v)$ $a_r = \frac{v^2}{r}$



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$$a_{avg} = \frac{v_2 - v_1}{\Delta t}$$

$$\vec{v} = \frac{2\pi r}{T}$$

$$a_r = \frac{v^2}{r}$$

$$\vec{a} = 6\hat{i} - 4\hat{j}$$

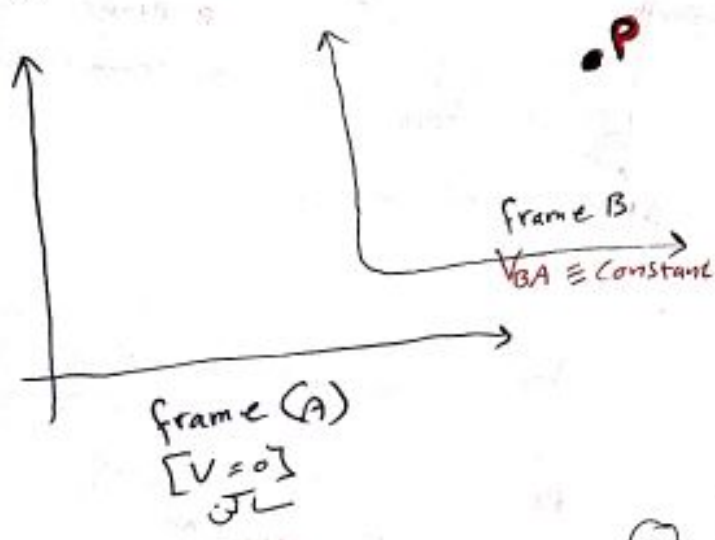
Ⓐ $\vec{v} \cdot \vec{a}$
 $\Rightarrow 0$ $[\vec{a} \perp \vec{v}]$

Ⓑ $\vec{r} \times \vec{b}$
 $\downarrow \sin 180^\circ = 0$
 $\Rightarrow 0$ $[\vec{a} \parallel \vec{r}]$

$$R = v\hat{e}_t$$

$$y = \frac{1}{2}gt^2$$

* Relative motion in 1-D



$$x_{PA} = x_{PB} + x_{BA}$$

- Ⓐ frame A
- Ⓑ frame B
- Ⓒ ~~frame B~~ Point t

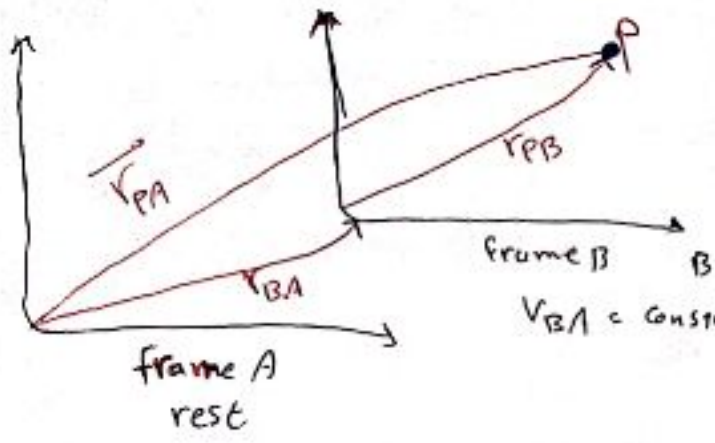
$$\frac{d}{dt}[x_{PA}] = \frac{d}{dt}[x_{PB} + x_{BA}]$$

$$v_{PA} = v_{PB} + v_{BA}$$

$$a_{PA} = a_{PB} + 0$$

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* Relative motion in 2-D



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

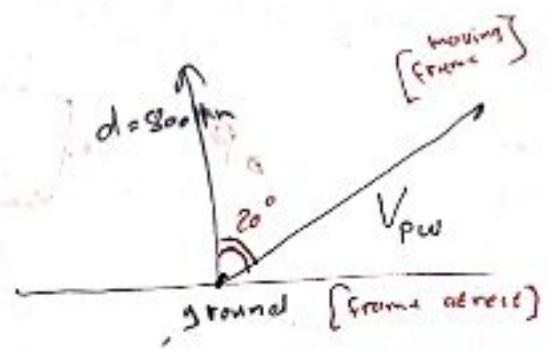
$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

$$\vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA}$$

P76)

$\vec{v}_{PB} = 500 \text{ km/h}$

$\vec{v}_{PG} = ??$



a: air 250
P: plane 250
g: ground 130

$\vec{d}_{PG} = 800 \text{ s [2h]}$

$\vec{v}_{PG} = \vec{v}_{Pa} + \vec{v}_{ag}$

$|\vec{v}_{PG}| = \frac{800 \text{ km}}{2\text{h}} = \underline{400 \text{ km/h}}$

$\vec{v}_{Pa} = 500 \cos 70^\circ \hat{i} + 500 \sin 70^\circ \hat{j}$
 $= \underline{171 \hat{i} + 470 \hat{j}}$

$\vec{v}_{ag} = \vec{v}_{PG} - \vec{v}_{Pa}$
 $= 400 \hat{j} - (171 \hat{i} + 470 \hat{j})$

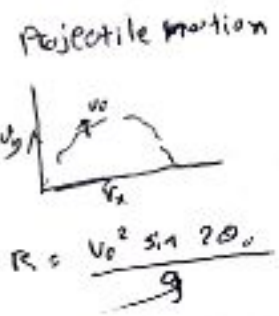
$\vec{v}_{ag} = -171 \hat{i} - 70 \hat{j} \quad \underline{185}$

Variables of motion

$d\vec{r} = \vec{v} dt = \vec{v}_i dt$

$v_{avg} = \frac{dr}{dt} \Rightarrow v_{in} = \frac{dr}{dt}$

$a_{avg} = \frac{dv}{dt} \Rightarrow a_{net} = \frac{dv}{dt}$



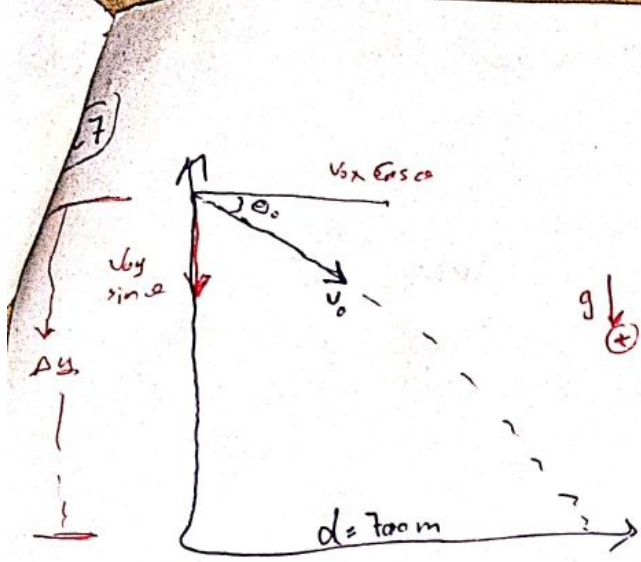
Uniform circular motion

$v = \frac{2\pi r}{T}$

$a = \frac{v^2}{r}$

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Discussion chapter 4



$$\Delta x = v_{0x} t$$

$$t = \frac{d}{v_{0x}} = \frac{d}{v_0 \cos \theta_0} = \frac{700}{80,5 \cos 30} = \boxed{10,5}$$

$$v_0 = 290 \text{ Km/h} \quad @ \quad t = ??$$

$$\theta_0 = 30^\circ \quad \rightarrow \quad \frac{290 \times 1000}{3600} = \boxed{80,5 \text{ m/s}}$$

(b) $\Delta y = ??$

$$\Delta y = v_{0y} t + \frac{1}{2} g t^2$$

$$\Delta y = v_0 \sin \theta_0 t + \frac{1}{2} g t^2$$

$$= 80,5 \sin 30 (10) + \frac{1}{2} (9,8) (10)^2 = \boxed{892,5 \text{ m}}$$

P32

isi @ $\Delta y = v_{0y} t + \frac{1}{2} g t^2$

$$\Delta y = 25 \sin 40 (1,15) + \frac{1}{2} (9,8) (1,15)^2$$

$$= \boxed{12 \text{ m}}$$

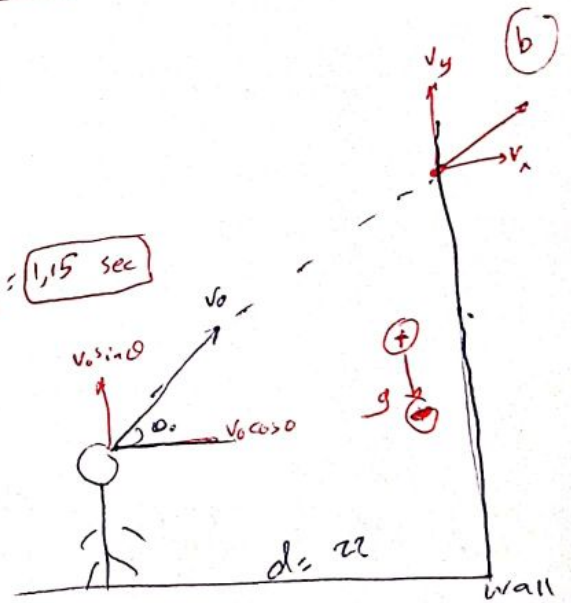
$$d = v_{0x} t$$

$$t = \frac{d}{v_0 \cos \theta_0} = \frac{??}{25 \cos 40} = \boxed{1,15 \text{ sec}}$$

(b) $v_x = v_{0x} = 25 \cos 40 = \boxed{19,2 \text{ m/s}}$

$$v_y = v_{0y} - g t = 25 \sin 40 - 9,8 (1,15) = \boxed{4,8 \text{ m/s}}$$

4,8 m/s = kecepatan vertikal saat itu

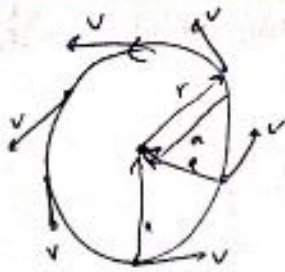


$$v_0 = 25$$

$$\theta_0 = 40^\circ$$

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P60



uniform $|v| = \text{constant}$

$r = 3\text{m}$

$\vec{a} = 6\hat{i} - 4\hat{j}$ find a) $\vec{v} \cdot \vec{a}$ b) $\vec{r} \times \vec{a}$

$|v| = \frac{2\pi r}{T}$

$|\vec{a}_r| = \frac{v^2}{r}$

a) $\vec{v} \cdot \vec{a} = |\vec{v}| |\vec{a}| \cos 90^\circ = 0$

b) $\vec{r} \times \vec{a} = |\vec{r}| |\vec{a}| \sin 180^\circ = 0$

P76

find $\vec{V}_{ag} = ??$

$V_{Pg} = \frac{500}{2} = 400 \hat{j}$

$V_{Pa} = 500 \cos 70^\circ \hat{i} + 500 \sin 70^\circ \hat{j}$

$\vec{V}_{Pg} = \vec{V}_{Pa} + \vec{V}_{ag}$

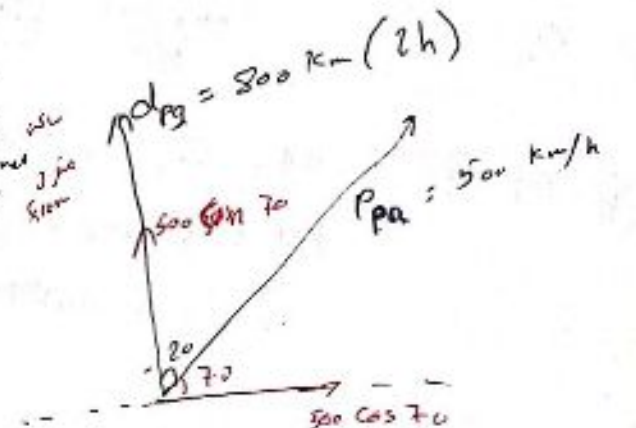
$\vec{V}_{ag} = \vec{V}_{Pg} - \vec{V}_{Pa} = -171 \hat{i} - 70 \hat{j}$

$|\vec{V}_{ag}| = \sqrt{(-171)^2 + (-70)^2} = 185 \text{ km/h}$

air
P = Plane
g = ground
a = air

$V_{Pg} = 500 \text{ km/h}$

$V_{Pa} = 500 \text{ km/h}$



سرعت نسبی = سرعت نسبی + سرعت نسبی (Constant)



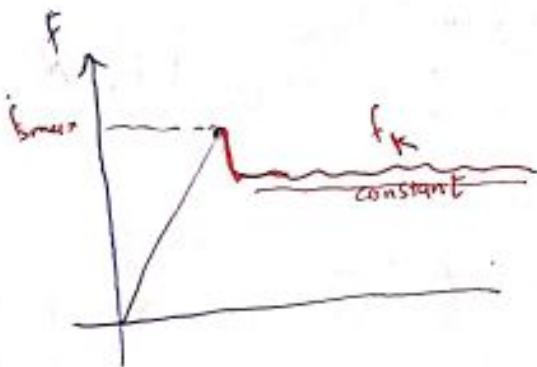
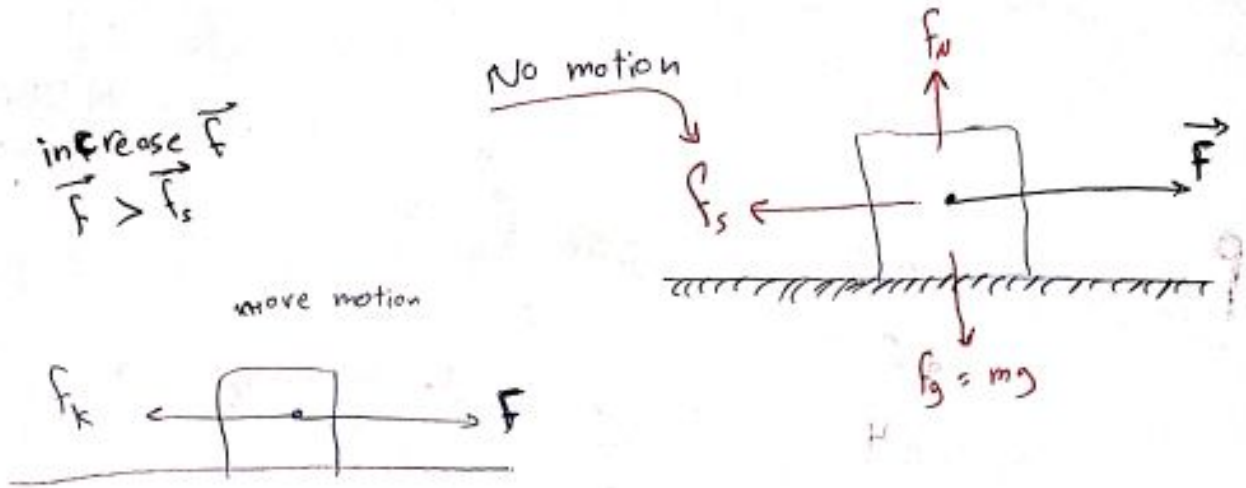
Good Luck
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Chapter 6 Forces and Motion II

98 friction

① Static frictional force f_s (القوة الاحتكاكية الساكنة) $(F_{s, \max})$

② Kinetic frictional force f_k (القوة الاحتكاكية الحركية)



① $F_{s, \max} = \mu_s F_N$, $\mu_s =$ coeff static friction

$F_N =$ Normal force

② $f_k = \mu_k F_N$, $\mu_k =$ coeff kinetic friction

$\mu_k < \mu_s$

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إذا كانت $F > f_{s, \max}$ $f_s = \mu_k N$ \rightarrow a

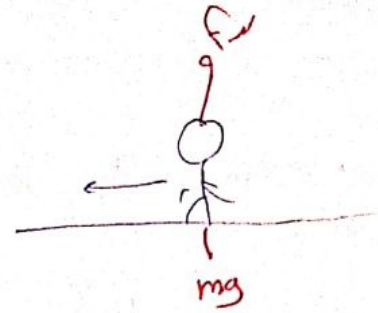
إذا كانت $F < f_{s, \max}$ $f_s = F$ \rightarrow $a = 0$

$mg - \mu_k g = a$

Pg : $m = 79 \text{ kg}$, $f_k = 470 \text{ N}$, $\mu_k = ?$

$$f_k = \mu_k f_N$$

$$\mu_k = \frac{f_k}{f_N} = \frac{470}{774,2} = 0,61$$



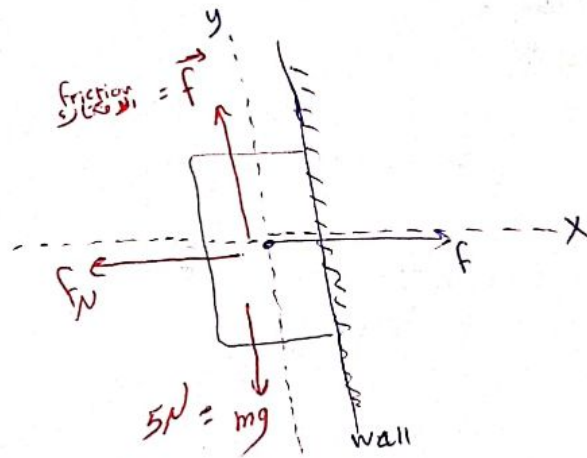
$$f_N - mg = 0$$

$$f_N = mg$$

$$= 79(9,8) = 774,2 \text{ N}$$

μ_s μ_k
 0,6 0,4
 0,6 0,4

Pg : $|\vec{F}| = 12 \text{ N}$
 $\mu_s = 0,6$
 $\mu_k = 0,4$



f_N f_k
 0,6 0,4

ⓐ Will the block move?

if $f >> f_{smax}$ [moving]

$$f_{smax} = \mu_s f_N = 0,6(12) = 7,2 \text{ N}$$

$$\vec{F} = mg = 5 \text{ N}$$

$f < f_{smax}$ No moving

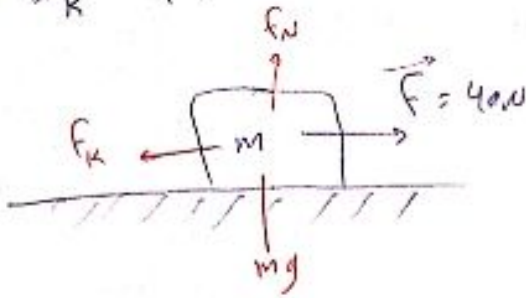
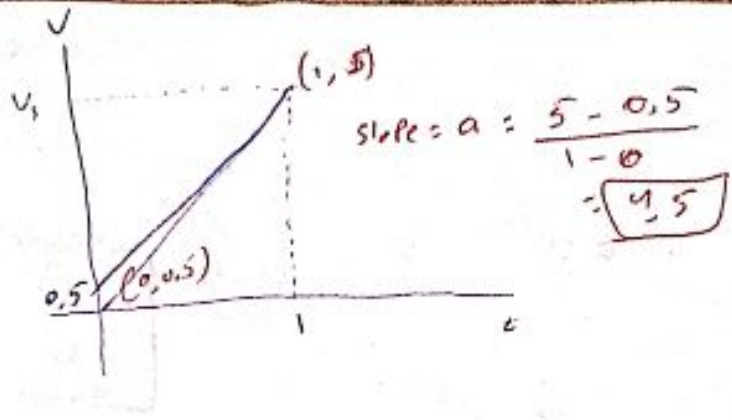
ⓑ In unit vector notation what is the force from the wall on the block?

$$\vec{F}_{wall} = -f_N \hat{i} + f \hat{j}$$

$$\vec{F}_{wall} = -12 \hat{i} + 5 \hat{j}$$

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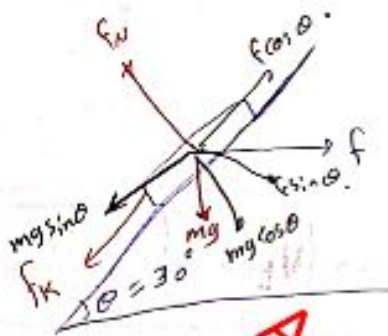
$m = 4,1 \text{ kg}$
 $v_s = 5 \text{ m/s}$
 $f = 40 \text{ N}$
 $a_k = ??$



$F - f_k = ma$
 $F_k = f - ma$
 $= 40 - 4,1(4,5)$
 $= 21,5 \text{ N}$

$F_k = \mu_k f_n \Rightarrow \mu_k = \frac{F_k}{f_n} = \frac{21,5}{41} = 0,53$

Exp. :



$F = 12 \text{ N}$
 $m = 1 \text{ kg}$
 $\mu_k = 0,25$

Find the acceleration of the block?

$f \cos \theta - (mg \sin \theta + F_k) = ma$

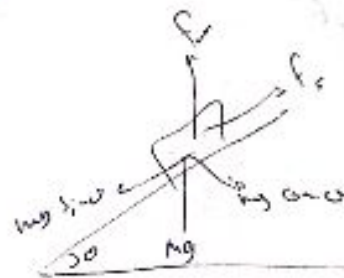
$F_k = \mu_k f_n = \mu_k [f \sin \theta + mg \cos \theta]$

$mg \sin \theta = f_s = \mu_s f_n$

$f_n = mg \cos \theta$

$mg \sin \theta = \mu_s mg \cos \theta$

$\mu_s = \frac{\sin \theta}{\cos \theta} = \tan \theta$

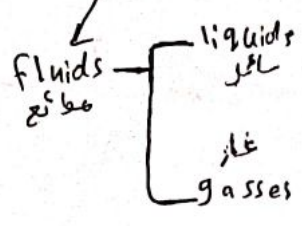


$\mu_s = \tan \theta$

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Drag force, terminal speed.

$1 < C < 1$



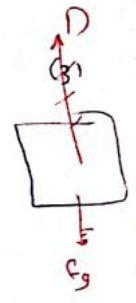
$$D = \frac{1}{2} C \rho A v^2$$

D → Drag force
 C → Drag coeff
 ρ → density of fluid
 A → cross sectional Area
 v → speed

~~F_{net}~~ = 0
 |a| = 0

$F_{net} = 0$
 $D - F_g = 0$

$D = F_g = \frac{1}{2} C \rho A v_t^2 = F_g \Rightarrow v_t = \sqrt{\frac{2 F_g}{C \rho A}}$



Sample Problem 6.3

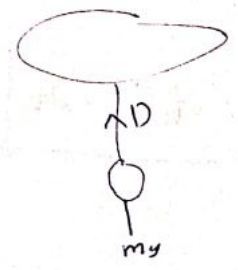
- R = 1.5 mm
- h = 1200 m
- C = 0.6

Assume the drop is sphere, find the terminal speed if $\rho_w = 1000 \text{ kg/m}^3$ and $\rho_a = 1.2 \text{ kg/m}^3$

$v_t = \sqrt{\frac{2 F_g}{C \rho A}}$

$v_t = 7.94 \text{ m/s}$

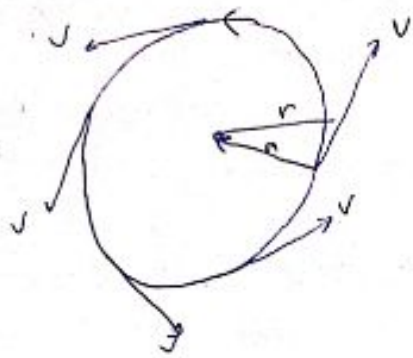
$F_g = mg = (\rho V)g$
 $= \frac{4}{3} \pi R^3 \rho_w g$
 $F_g = 0.14 \text{ N}$



$A = \pi r^2$
 $= (3.14) () = 7.06 \times 10^{-4} \text{ m}^2$

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Uniform Circular motion.



$$|\vec{v}| = \text{constant}$$

$$|\vec{v}| = \frac{2\pi R}{T}$$

$$a_r = \frac{v^2}{R}$$

$$f_r = ma$$

$$f_r = \frac{mv^2}{R}$$

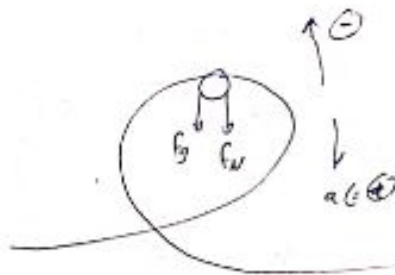
$$\tan \theta = \frac{a}{g}$$

Case one 1

① Vertical loop

$$f_g + f_N = \frac{mv^2}{R}$$

f_r

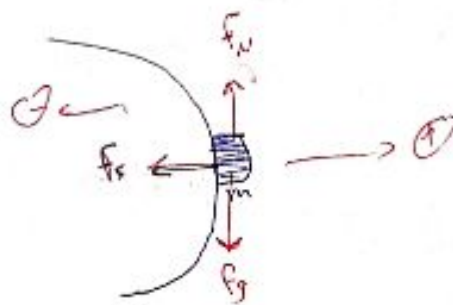


② flat circular motion :-

$$-f_s = \frac{mv^2}{R}$$

$$f_s = \frac{mv^2}{R}$$

$$f_N - f_g = 0$$



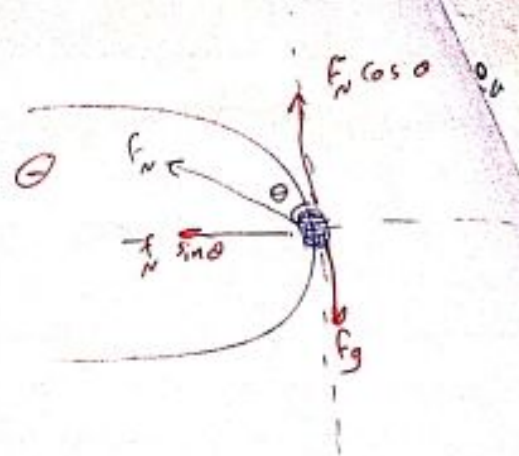
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3) Banked circular motion 1-

$$-f_N \sin \theta = -\frac{mv^2}{R}$$

$$f_N \sin \theta = \frac{mv^2}{R}$$

$$f_N \cos \theta - f_g = 0 \quad \tan \theta = \frac{v^2}{gR}$$



Discussion

1) $m = 2,5 \text{ kg}$ (at rest)

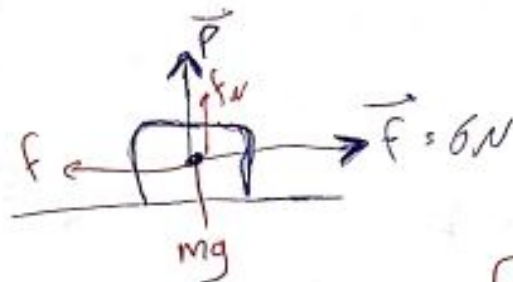
$$\mu_s = 0,4$$

$$\mu_k = 0,25$$

a) $|P| = 8 \text{ N}$ f_s ??

$$f_{s, \max} = \mu_s f_N = 0,4 (16,5) = \boxed{6,6 \text{ N}}$$

$f < f_{s, \max}$ no motion



$$f_N = mg - P = \boxed{16,5 \text{ N}}$$

b) $|P| = 10 \text{ N}$

$$f_N = mg - P = 14,5 \text{ N}$$

$$f_{s, \max} = \mu_s f_N = \boxed{5,8} \text{ moving}$$

$$f_k = \mu_k f_N = 0,25 (14,5) = \boxed{3,6 \text{ N}}$$

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c) $P = 12 \text{ N}$

$$f_N = 12,5 \text{ N}$$

$$f_{s, \max} = 5 \text{ N}$$

$$f_k = \mu_k f_N = 0,25 (12,5) = \boxed{3,1 \text{ N}}$$

10 at rest

$$f = 0,5 mg$$

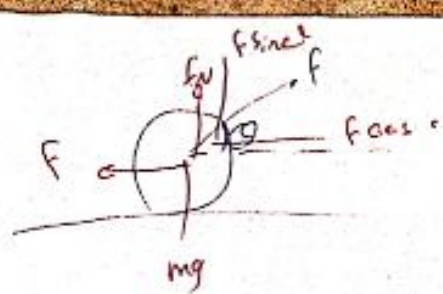
$$\theta = 20$$

$$\textcircled{a} \mu_s = 0,6, \mu_k = 0,5$$

$$F_{max} = \mu_s f_N$$

$$= 0,6 \times 82 mg = \boxed{49 mg}$$

$$f_{max} < 0,46 \text{ No moving}$$



$$\textcircled{a} f_N = mg - f \sin \theta$$

$$= mg - 0,5 mg \sin 20$$

$$= \underline{82 mg}$$

$$\textcircled{a} f \cos \theta = (0,5 mg) (\cos 20) = \boxed{0,46 mg}$$

$$\textcircled{b} \mu_s = 0,4, \mu_k = 0,3$$

$$F_{max} = \mu_s f_N$$

$$= 0,4 (0,82 mg) = \boxed{3,328 mg}$$

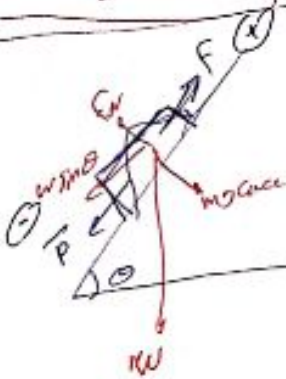
moving $f > f_{max}$

$$f \cos \theta - f_k = ma$$

$$0,46 mg - 0,3 = 1,82 mg = ma$$

$$\boxed{a = 2,17 \text{ m/s}^2}$$

P17



$$w = 45 \text{ N} \quad \mu_s = 0,5$$

$$\theta = 15^\circ \quad \mu_k = 0,34$$

$$\textcircled{a} \vec{P} = -5 \hat{i}$$

$$f_{max} = \mu_s f_N$$

$$= 0,5 w \cos \theta = 14 (45) (\cos 15) = \boxed{21,7 \text{ N}}$$

$$w \sin \theta + P = 45 \sin 15 + 5 = \boxed{16,6 \text{ N}}$$

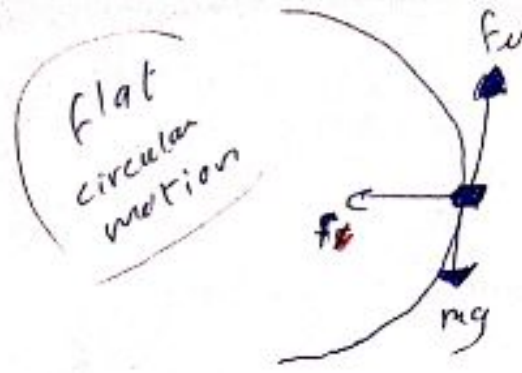
no moving

$$f_s = |P| + w \sin \theta$$

$$= 16,6 \text{ N} = \boxed{16,6 \hat{i}}$$

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Paz : $\mu_s = 0,6$
 $r = 30,5 \text{ m}$



$$f \leq f_{\max}$$

$$\frac{mv^2}{R} \leq \mu_s f_w$$

$$\frac{mv^2}{R} \leq \mu_s mg$$

$$v^2 \leq \mu_s g R \Rightarrow v \leq \sqrt{\mu_s g R} \Rightarrow \frac{v = \sqrt{\mu_s g R}}{\cancel{v = \sqrt{\mu_s g R}}} = \boxed{13 \text{ m/s}}$$

$$f = ma$$
$$f = \frac{mv^2}{R}$$

Good Luck
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