جامعة بيرزيت

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

First Semester – 2023/2024

ENCS2340 – Digital Systems

Answer Sheet – Homework # 1

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Question 1 : **Conversion between different numbering systems. Fill the table below with the different representations of numbers. BCD stands for binary-coded decimal. For fractions, stop after 4 digits after the base point.**

Decimal	Binary	octal	Hex.	BCD
154	10011010	232	9A	000101010100
29.25	11101.01	35.2	1D.4	00101001.00100101
93.1999	001011101.001100110011	135.1463	5D.333	10010011.0001100110011001
17.5	10001.1	21.4	11.8	10111.0101
0.125	0.001	0.1	0.2	0000.000100100101

Question 2 : **Using Boolean Algebraic manipulations, minimize the following fu**

nctions to *minimum* number of literals in *sum-of-products* representation (Show your work clearly step by step indicating the used properties of Boolean Algebra):

a) A'C' + A'BC + B'C'

A'C' + A'BC + B'C' = A'(C' + BC) + B'C' — By Distributive (.) over (+)

= A'((C' + B)(C' + C)) + B'C' — By Distributive (+) over (.)

= A'((C' + B).1) + B'C' — By Postulate : X + X' = 1

= A'(C' + B) + B'C' — By Postulate : X . 1 = X

= A'C' + A'B + B'C' — By Distributive (.) over (+)

= BA' + B'C' + A'C'

= A'B + B'C' — By consensus theorem (XY + X'Z + YZ = XY +X'Z)

b) BC + AC' + AB + BCD

BC + AC' + AB + BCD = BC + BCD + AC' + AB — By Commutative law

= BC (1 + D) + AC' + AB — By Distributive (.) over (+)

= BC . 1 + AC' + AB — By Theorem : 1 + X = 1

= BC + AC' + AB — By Postulate : X . 1 = X

= CB + C'A + BA

= BC + AC' — By consensus theorem (XY + X'Z + YZ = XY +X'Z)

c) ABC + A'B'C + A'BC + ABC' + A'B'C'

ABC + A'B'C + A'BC + ABC' + A'B'C' = BC(A+A') + AB(C+C') + A'B'(C+C') + A'C(B'+B)

By Distributive (.) over (+)

= (BC . 1) + (AB . 1) + (A'B' .1) + (A'C . 1) By Postulate : X + X' = 1

= BC + AB + A'B' + A'C By Postulate : X . 1 = X

= AB + A'C + BC + A'B'

= AB + A'C + A'B' By consensus theorem (XY + X'Z + YZ = XY +X'Z)

d) ((CD)'+A)' + A + CD + AB

((CD)'+A)' + A + CD + AB = CD.A' + A + CD + AB By DeMorgan's Theorem

= A'CD + A + CD + AB By Commutative law

= CD(A' + 1) + A(1 + B) By Distributive (.) over (+)

= (CD.1) + (A.1) By Theorem : X + 1 = 1

= A + CD By Postulate : X . 1 = X

e) (A + C + D)(A + C + D')(A + C' + D)(A + B')

(A + C + D)(A + C + D')(A + C' + D)(A + B')

= (AA + AC + AD' + AC + CC + CD' + AD + CD + DD')(A + C' + D)(A + B') By Distributive (.) over (+)

= (A + AC + AD' + AC + C + CD' + AD + CD + 0)(A + C' + D)(A + B')

By Theorem : X . X = X And Postulate : X . X' = 0

= (A + AC + AD' + C + CD' + AD + CD)(A + C' + D)(A+B')

By Theorem : X + X = X And Postulate : X + 0 = X

= (A(1 + C+D'+D) + C (A + 1 + D' + D) + D'(A + C) + D(A + C)) (A + C' + D)(A+B')

By Distributive (.) over (+)

= (A.1 + C.1 + (A + C)(D + D')) (A + C' + D)(A +B') By Theorem : X + 1 = 1 And Distributive (.) over (+)

= (A + C + ((A + C) .1))) (A + C' + D) (A + B') By Postulate : X . 1 = X And Postulate : X + X' = 1

= (A + C + A + C) (A + C' + D) (A + B') By Postulate : X . 1 = X

= (A + C) (A + C' + D) (A + B') By Theorem : X + X = X

= (A + AC' + AD + AC + 0 + CD)(A+B') By Distributive And Theorem : X.X=X And X.X'=0

= (A(1 + C' + D + C) + CD)(A + B') By Distributive And Postulate : X + 0 = X

= (A.1 + CD)(A + B') = (A + CD)(A + B') By Theorem : X + 1 = 1 And X . 1 = X

= A + AB' + ACD + B'CD By Distributive (.) over (+)

= A(1 + B' + CD) + B'CD By Distributive (.) over (+)

= A.1 + B'CD By Theorem : X + 1 = 1

= A + B'CD By Postulate : X . 1 = X

f) (WX(Y'Z + YZ') + W'X'(Y' + Z)(Y + Z'))'

(WX(Y'Z + YZ') + W'X'(Y' + Z)(Y + Z'))'

= (WX(Y'Z + YZ') + W'X'(Y'Z + YZ')')' Note : (Y'Z+YZ')' = (Y+Z')(Y'+Z)

= [W' + X' + (Y'Z + YZ')'] . [W + X + (Y'Z + YZ')] By DeMorgan's Theorem

= W'.W + W'.X + W'.(Y'Z + YZ') + X'.W + X'.X + X'.(Y'Z + YZ') + W.(Y'Z + YZ')' + X.(Y'Z+YZ')' + (Y'Z + YZ')'.(Y'Z + YZ') By Distributive (.) over (+)

= 0 + W'X + W'(Y'Z + YZ') +WX' + 0 + X'(Y'Z + YZ') + W(Y'Z + YZ')' + X(Y'Z + YZ')' + 0 By Postulate : X . X' = 0

= W'X + W'(Y'Z + YZ') +WX' + X'(Y'Z + YZ') + W(Y'Z + YZ')' + X(Y'Z + YZ')' By Postulate : X + 0 = X

= {W'X + W(Y'Z + YZ')' + X(Y'Z + YZ')'} + {WX' + W'(Y'Z + YZ') + X'(Y'Z + YZ')} By Commutative

= W'X + W(Y'Z + YZ')' + WX' + W'(Y'Z + YZ') By Consensus Theorem (XY + X'Z + YZ = XY + X'Z)

= W'X + W(Y + Z')(Y' + Z) + WX' + W'(Y'Z + YZ') By DeMorgan's Theorem

= W'X + W(Y.Y' + Y.Z + Y'.Z' + Z'.Z) + WX' + W'Y'Z + W'YZ' By Distributive (.) over (+)

= W'X + W(0 + YZ + Y'Z' + 0) + WX' + W'Y'Z + W'YZ' By Postulate : X . X' = 0

= W'X + W(YZ + Y'Z') + WX' + W'Y'Z + W'YZ' By Postulate : X + 0 = X

= W'X + WYZ + WY'Z' + WX' + W'Y'Z + W'YZ' By Distributive (.) over (+)

Question 3 : Given the truth table for functions $F1$ and $F2$:

a) Draw a 2-level AND-OR circuit that implements F_1

b) Draw a 2-level OR-AND circuit that implements F_2

c) Write an algebraic expression for the dual of function F_1

W	X	Y	Z	F_1	F_2
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	0	0
1	1	1	0	1	1
1	1	1	1	1	0

a) to draw a 2 level AND – OR circuit that implements F_1,

we should find the expression of F_1 in SOM / SOP :

$F_1(\mathrm{W,X,Y,Z})$ = Sum of Minterm entries that evaluate to 1

$F_1 = \sum(1, 6, 7, 8, 11, 12, 14, 15)$

$= m_1 + m_6 + m_7 + m_8 + m_{11} + m_{12} + m_{14} + m_{15}$

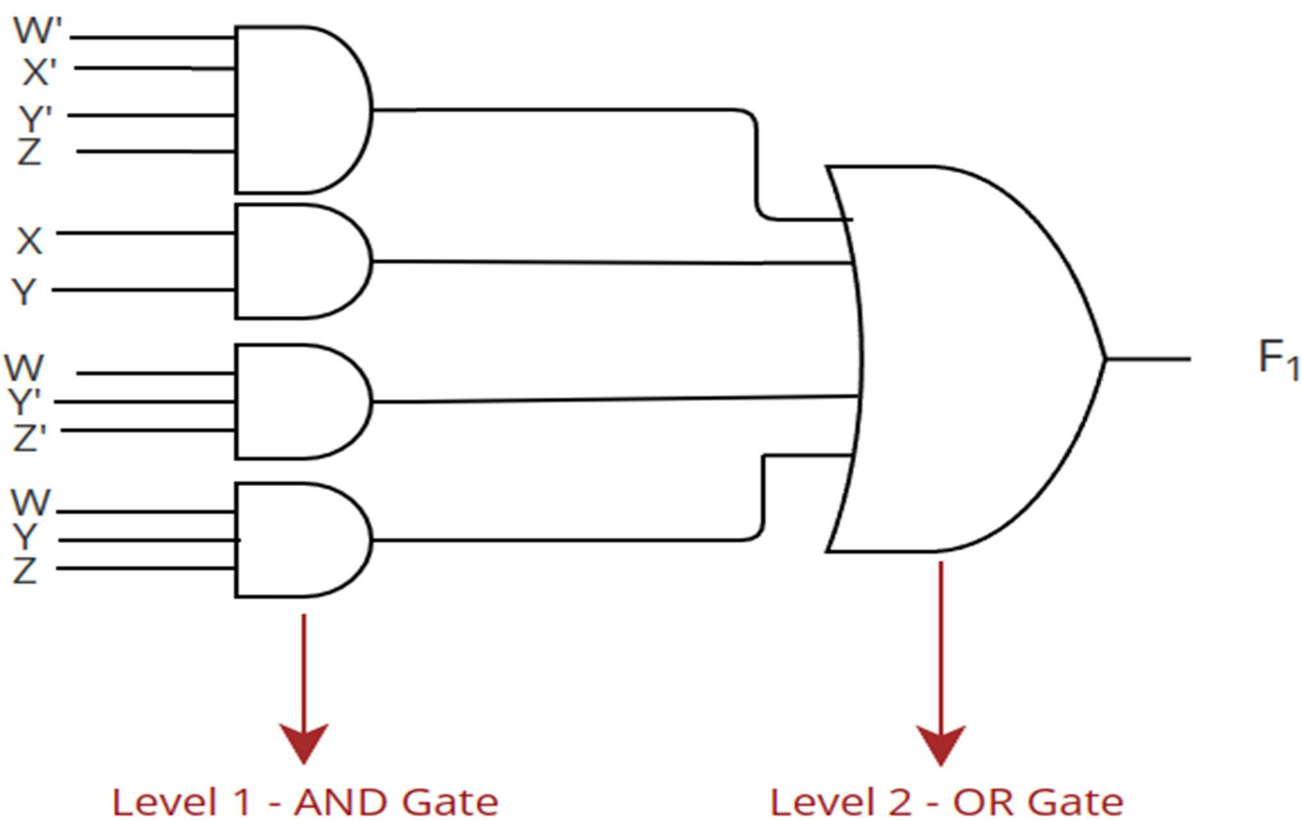
= W'X'Y'Z + W'XYZ' + W'XYZ + WX'Y'Z' + WX'YZ + WXY'Z' + WXYZ' + WXYZ

= W'X'Y'Z + W'XY(Z+Z') + WY'Z'(X+X') + WYZ(X'+X) + WXY(Z'+Z)

= W'X'Y'Z + W'XY + WY'Z' + WYZ + WXY

= W'X'Y'Z + XY(W'+W) + WY'Z' + WYZ

= W'X'Y'Z + XY + WY'Z' + WYZ



b) to draw a 2 level OR – AND circuit that implements F_2, we should find the expression of F_2 in POM / POS :

F_2 = Product of Maxterm entries that evaluate to 0

$F_2(W, X, Y, Z) = \prod(1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 15)$

$= M_1 . M_2 . M_3 . M_4 . M_5 . M_8 . M_9 . M_{10} . M_{11} . M_{12} . M_{13} . M_{15}$

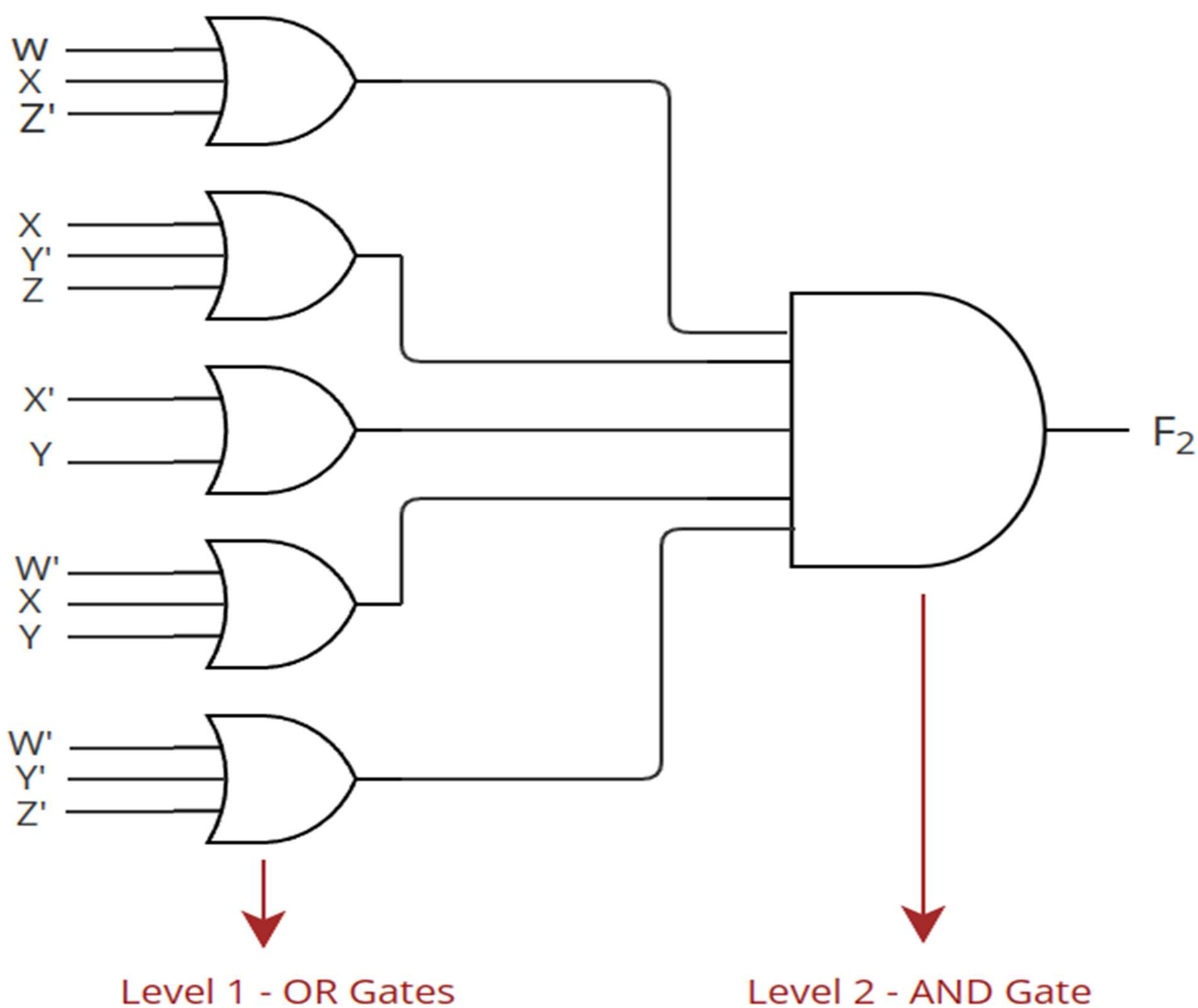
= (W+X+Y+Z’)(W+X+Y’+Z)(W+X+Y’+Z’)(W+X’+Y+Z)(W+X’+Y+Z’)(W’+X+Y+Z)(W’+X+Y+Z’)(W’+X+Y’+Z)(W’+X+Y’+Z’)(W’+X’+Y+Z)(W’+X’+Y+Z’)(W’+X’+Y’+Z’)

= (W+X+Z’+Y.Y’)(X+Y’+Z+W.W’)(W+X’+Y+Z.Z’)(W’+X+Y+Z.Z’)(W’+Y’+Z’+X.X’)(W’+X’+Y+Z.Z’)

= (W+X+Z’)(X+Y’+Z)(W+X’+Y)(W’+X+Y)(W’+Y’+Z’)(W’+X’+Y)

= (W+X+Z’)(X+Y’+Z)(X’+Y+W.W’)(W’+X+Y)(W’+Y’+Z’)

= (W+X+Z’)(X+Y’+Z)(X’+Y)(W’+X+Y)(W’+Y’+Z’)



c) The algebraic expression for the dual of function F_1

F_1 = W'X'Y'Z + W'XYZ' + W'XYZ + WX'Y'Z' + WX'YZ + WXY'Z' + WXYZ' + WXYZ

Dual of (F_1) =
(W'+X'+Y'+Z)(W'+X+Y+Z')(W'+X+Y+Z)(W+X'+Y'+Z')(W+X'+Y+Z)(W+X+Y'+Z')(W+X+Y+Z')(W+X+Y+Z)

After simplification : F_1 = W'X'Y'Z + XY + WY'Z' + WYZ , Then

Dual of (F_1) = (W'+X'+Y'+Z)(X+Y)(W+Y'+Z')(W+Y+Z)

Question 4 : Given the following Boolean functions f and g:

$f(x, y, z) = \prod(1, 2, 5, 6, 7)$ $g(x, y, z) = \sum(0, 2, 4, 6, 7)$

a) Write an algebraic expression for f as a *sum-of-minterms*.

$f(x,y,z) = \prod(1,2,5,6,7) = \sum(0,3,4) = m_0 + m_3 + m_4$ = X'Y'Z' + X'YZ + XY'Z'

b) Express the function $f + g$ as a sum-of-minterms, $f + g = \sum(\ldots)$.

$$(f+g)(x,y,z) = \sum(0,3,4) + \sum(0,2,4,6,7) = \sum(0,2,3,4,6,7)$$

c) Write an algebraic expression for $(f' \cdot g)$ as a *product-of-maxterms.*

$$(f'.g)(x,y,z) = (f+g')' = (\sum(0,3,4) + \sum(1,3,5))' = (\sum(0,1,3,4,5))'$$

$$= \prod(0,1,3,4,5) = M_0.M_1.M_3.M_4.M_5$$

$$= (X+Y+Z)(X+Y+Z')(X+Y'+Z')(X'+Y+Z)(X'+Y+Z')$$

Question 5 : Find the value of X that satisfies the following equations (You should clearly show steps about how you get the answer):

a) $(C372)_{16} - (395E)_{16} = (X)_{16}$

$15's\ comp\ of\ (395E) = (C6A1)\ , then\ 16's\ comp = 15'scomp + 1 = (26A2)$

$(C372)_{16} - (395E)_{16} = (C372)_{16} + (26A2)_{16} = (8A14)_{16}$

1 1

$C372$

$+\ C6A2$

~~1~~ $8A14$

b) $(0010\ 1000\ 0000\ 0111)_{BCD} + (0001\ 1001\ 1001\ 0101)_{BCD} = (X)_{BCD}$

$$
\begin{array}{rrrrr}
 & ^{1\,1\,1} & ^{1\,1} & ^{1\,1} & ^{1\,1\,1} \\
 & 0010 & 1000 & 0000 & 0111 \\
+ & 0001 & 1001 & 1001 & 0101 \\
\hline
 & 0100 & 10010 & 1010 & 1100 \\
 & 0000 & 0110 & 0110 & 0110 \\
\hline
 & (0100 & 1000 & 0000 & 0010)_{BCD}
\end{array}
$$

when the sum > 9 it's invalid in BCD

to make it valid, we add 6 (0110) for it

c) $(35)_X + (18)_X = (51)_X$

$[X^0(5) + X^1(3)] + [X^0(8) + X^1(1)] = [X^0(1) + X^1(5)]$

$5 + 3X + 8 + X = 1 + 5X$

$4X + 13 = 1 + 5X \quad \rightarrow \quad -X = -12 \quad \rightarrow \quad \therefore\ base\ X = 12$

d) $(10110.11)_5 = (X)_{15}$

$(10110.11)_5 = \left((5^0 \times 0) + (5^1 \times 1) + (5^2 \times 1) + (5^3 \times 0) + (5^4 \times 1) + (5^{-1} \times 1) + (5^{-2} \times 1)\right)_{10}$

$= (655.24)_{10}$

Division	Quotient	Remainder
655/15	43	A
43/15	2	D
2/15	0	2

Multiplication	New Fraction	Bit
0.24 X 15 = 3.6	0.6	3
0.6 X 15 = 9.0	0.0	9

$then, (10110.11)_5 = (2DA.39)_{15}$

e) $(2404)_{10} = (C3A)_X$

$X^0(A) + X^1(3) + X^2(C) = 2404$

$10 + 3X + 12X^2 = 2404$

$12X^2 + 3X + 10 = 2404$

$12X^2 + 3X - 2394 = 0$

$4X^2 + X - 798 = 0$

$X = \frac{-1 \pm \sqrt{1+(4)(4)(798)}}{2(4)} = 14 \quad or \quad -14.25 \qquad then, the\ base\ X = 14$

f) $X = the\ 15's\ complement\ of\ (2B070)_{15}$

$14's\ Complement\ of\ (2B070)_{15} = C3E7E$ $(By\ Comp.each\ bit\ to\ 14)$

$15's\ Complement\ of\ (2B070)_{15} = 14's\ Comp. + 1$

$C3E7E$

$+ \quad 1$ $then,\ X = 15'sComplement\ of\ (2B070)_{15} = C3E80$

$C3E80$

g) $X = the\ Gray\ code\ for\ the\ binary\ value\ (101100)_2$

$let\ the\ most\ significant\ bit\ as\ it\ is\ 1\ , then\ compare\ the\ left\ bit\ with\ the\ next\ one\ by$

$exclusive\ or\ (XOR):$ 1 $(1 \oplus 0)$ $(0 \oplus 1)$ $(1 \oplus 1)$ $(1 \oplus 0)$ $(0 \oplus 0)$

$then,$ $X = (111010)_{GrayCode}$

Question 6 : **A new university has 3,500 students. It is required to give each student a unique binary code (inside the registration software):**

a) How many bits would we need?

Bits we need of unique binary code for 3500 students = $\lceil \log_2 3500 \rceil = \lceil 11.77 \rceil = 12\ bits$

b) If it is anticipated that the number of students will double every 5 years, how many bits would we need after 20 years?

After 20 years number of students will double 4 times,

number of students after n years = $2^{\frac{n}{5}} \times 3500$ **,**

then after 20 years number of students = $2^{\frac{20}{5}} \times 3500 = 2^4 \times 3500 = 56000\ students$

Bits we need of unique binary code for 56000 students = $\lceil \log_2 56000 \rceil = \lceil 15.77 \rceil = 16\ bits$

c) For the sake of documentation, how many Hex digits would we need to represent these codes (now and after 20 years)? Comment on the number of Hex digits needed as compared to the binary bits needed.

- Digits we need of unique hex code for 3500 students (now)

$= \lceil \log_{16} 3500 \rceil = \lceil 2.94 \rceil = 3\ digits$

- Digits we need of unique Hex code for 56000 students (after 20 years)

$= \lceil \log_{16} 56000 \rceil = \lceil 3.94 \rceil = 4\ digits$

- **As we see for binary code we need a large number of bits (12 for 3500 students and 16 for 56000 students) due to the number of unique bits in binary only 2 unique bits (0,1).**
- **On the other hand, for Hex code we need a small number of digits (only 3 digits for 3500 students and 4 digits of 56000 students) due to the number of unique digits in Hex, 16 unique digits (0,1,….,9,A,B,….,F)**
- **So, for Hex code we need a much less number of digits than the binary code and the cause of this difference is the difference between number of unique digits for each numbering system.**

Question 7 : **If 8-bit registers are used, perform the following signed 2's complement arithmetic operations on the provided signed 2's complement binary numbers. For each case, state whether the result is correct, or an overflow has occurred.**

a) 11001101 + 01101011

```
 1   1 1  1 1
  11001101  (−51)
+ 01101011  (107)
-1 00111000  (56)
```

The result is correct,

No overflow (overflow = 0)

b) 01110010 – 10010111

2's complement of (10010111) = (01101001)

```
 1 1
  01110010  (114)
+ 01101001  (105)
  11011011  (37)
```

The result is wrong,

There is an overflow (overflow = 1)

c) 11111011 – 10000

2's complement of(00010000) = (11110000)

```
   1 1  1
   11111011  (−5)          The result is correct,
+  11110000  (-16)         No overflow (overflow = 0)
-----------
1  11101011  (−21)
```

d) 01101 – 11101101

2's complement of (11101101) = (00010011)

```
       1 1  1 1  1
   00001101  (13)          The result is correct,
+  00010011  (19)          No overflow (overflow = 0)
-----------
   00100000  (32)
```

e) 010011 – 01101

2's Complement of (00001101) = (11110011)

```
   1 1  1      1 1
   00010011  (19)          The result is correct,
+  11110011  (-13)         No overflow (overflow = 0)
-----------
1  00000110  (6)
```

f) 10011 + 101101

```
     1 1  1 1 1  1
   00010011  (19)          The result is correct,
+  00101101  (45)          No overflow (overflow = 0)
-----------
   01000000  (64)
```

Question 8 : Determine the decimal value of the 7-bit binary numbers A = (1011001) and B = (0111010) when interpreted as:

a) Unsigned numbers.

$$A = (1011001)_2 = ((2^0 \times 1) + (2^1 \times 0) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 0) + (2^6 \times 1))_{10}$$
$$= (1 + 8 + 16 + 64)_{10} = (89)_{10}$$

$$B = (0111010)_2 = ((2^0 \times 0) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 1) + (2^6 \times 0))_{10}$$
$$= (2 + 8 + 16 + 32)_{10} = (58)_{10}$$

b) Signed-magnitude numbers.

Leftmost bit express the sign only

$$A = (1011001)_2 = ((2^0 \times 1) + (2^1 \times 0) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 0))_{10}$$
$$= (1 + 8 + 16)_{10} = (-25)_{10}\text{, the leftmost bit} = 1\text{, then the sign is negative}$$

$$B = (0111010)_2 = ((2^0 \times 0) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 1))_{10}$$
$$= (2 + 8 + 16 + 32)_{10} = (58)_{10}\text{, the leftmost bit} = 0\text{, then the sign is positive}$$

c) Signed 1's complement numbers.

$-$ A as 1's complement $=$ (1011001)

Then A $=$ 1's complement of (1011001) $\rightarrow$ A = 0100110

$$A = (0100110)_2 = ((2^0 \times 0) + (2^1 \times 1) + (2^2 \times 1) + (2^3 \times 0) + (2^4 \times 0) + (2^5 \times 1) + (2^6 \times 0))_{10}$$
$$= (2 + 4 + 32)_{10} = (-38)_{10}$$

$-$ B as 1's complement $=$ (0111010)

$$B = (0111010)_2 = ((2^0 \times 0) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 1) + (2^6 \times 0))_{10}$$
$$= (2 + 8 + 16 + 32)_{10} = (58)_{10}$$

d) Signed 2's complement numbers.

$-$ A as 2's complement $=$ (1011001)

Then A $=$ 2's complement of (1011001) $\rightarrow$ A = 0100111

$$A = (0100111)_2 = ((2^0 \times 1) + (2^1 \times 1) + (2^2 \times 1) + (2^3 \times 0) + (2^4 \times 0) + (2^5 \times 1) + (2^6 \times 0))_{10}$$
$$= (1 + 2 + 4 + 32)_{10} = (-39)_{10}$$

$-$ B as 2's complement $=$ (0111010)

$$B = (0111010)_2 = ((2^0 \times 0) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 1) + (2^6 \times 0))_{10}$$
$$= (2 + 8 + 16 + 32)_{10} = (58)_{10}$$

Question 9: **Express the following Boolean functions in sum-of-minterms and product-of-maxterms canonical forms:**

a) F(W, X, Y, Z) = WX'Y' + WXZ' + W'XZ + YZ'

F(W,X,Y,Z) = WX'Y'(Z+Z') + WXZ'(Y+Y') + W'XZ(Y+Y') + YZ'(W+W')(X+X')

= WX'Y'Z + WX'Y'Z' + WXYZ' + WXY'Z' + W'XYZ + W'XY'Z + WXYZ' + WX'YZ' + W'XYZ' + W'X'YZ'

= WX'Y'Z + WX'Y'Z' + WXYZ' + WXY'Z'+ W'XYZ + W'XY'Z + WX'YZ' + W'XYZ' + W'X'YZ'

$= m_2 + m_5 + m_6 + m_7 + m_8 + m_9 + m_{10} + m_{12} + m_{14}$

$= \sum(2, 5, 6, 7, 8, 9, 10, 12, 14)$ ***in sum-of-minterms***

Then, F(W,X,Y,Z) = $\prod(0, 1, 3, 4, 11, 13, 15) = M_0 . M_1 . M_3 . M_4 . M_{11} . M_{13} . M_{15}$

$= (W + X + Y + Z).(W + X + Y + Z').(W + X + Y' + Z').(W + X' + Y + Z).(W' + X + Y' + Z')$

$.(W' + X' + Y + Z').(W' + X' + Y' + Z')$ ***in product-of-maxterms***

b) F(A, B, C, D) = D(A' + B) + B'D

F(A,B,C,D) = A'D + BD + B'D = A'D(B+B')(C+C') + BD(A+A')(C+C') + B'D(A+A')(C+C')

= A'BCD + A'BC'D + A'B'CD + A'B'C'D + ABCD + ABC'D + A'BCD + A'BC'D + AB'CD + AB'C'D + A'B'CD + A'B'C'D = A'BCD + A'BC'D + A'B'CD + A'B'C'D + ABCD + ABC'D + AB'CD + AB'C'D

$= m_1 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$

$= \sum(1, 3, 5, 7, 9, 11, 13, 15)$ ***in sum-of-minterms***

Then, F(A,B,C,D) = $\prod(0, 2, 4, 6, 8, 10, 12, 14) = M_0 . M_2 . M_4 . M_6 . M_8 . M_{10} . M_{12} . M_{14}$

$= (A + B + C + D).(A + B + C' + D).(A + B' + C + D).(A + B' + C' + D).(A' + B + C + D)$

$.(A' + B + C' + D).(A' + B' + C + D).(A' + B' + C' + D)$ ***in product-of-maxterms***

c) F(A, B, C, D) = (A + B' + C)(A + B')(A + C' + D')(A + B + C + D')(B + C' + D')

F(A,B,C,D) = (A+B'+C+DD')(A+B'+CC'+DD')(A+C'+D'+BB')(A+B+C+D')(B+C'+D'+AA')

= (A+B'+C+D)(A+B'+C+D')(A+B'+C+D)(A+B'+C+D')(A+B'+C'+D)(A+B'+C'+D')(A+B+C'+D')(A+B'+C'+D')(A+B+C+D')(A+B+C'+D')(A'+B+C'+D')

= (A+B'+C+D)(A+B'+C+D')(A+B'+C'+D)(A+B'+C'+D')(A+B+C'+D')(A+B+C+D')(A'+B+C'+D')

$= M_1 . M_3 . M_4 . M_5 . M_6 . M_7 . M_{11}$

$= \prod(1, 3, 4, 5, 6, 7, 11)$ ***in product-of-maxterms***

Then, F(A,B,C,D) = $\sum(0, 2, 8, 9, 10, 12, 13, 14, 15)$

$= m_0 + m_2 + m_8 + m_9 + m_{10} + m_{12} + m_{13} + m_{14} + m_{15}$

= A'B'C'D' + A'B'CD' + AB'C'D' + AB'C'D +AB'CD' + ABC'D' + ABC'D + ABCD' + ABCD

in sum-of-minterms