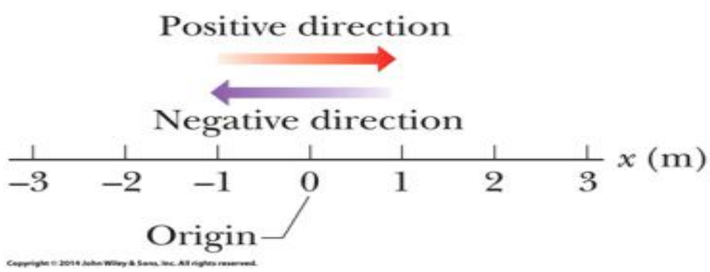
Chapter 2: Motion along a Straight Line

Mechanics: concerned with the motions of physical objects, more specifically the relationships among force, matter, and motion. (Kinematics and Dynamics)

→→ If all parts of an object move in the same direction at the same rate, we can treat it as a (point-like) **particle.**

- Position is measured relative to a reference point (the origin, or zero point, of an axis) [meter]



- Distance is a scalar quantity, length of the path that the object took in travelling from one place to another [scalar quantity, meter](always positive)

- Displacement = final position – initial position

→→ DISPLACEMENT is a vector quantity [meter]
→→ can be Zero or negative or positive

$$\Delta x = x_{final} - x_{initial}$$

Example: A particle moves . . .

- From x = 5 m to x = 12 m: Δx = 7 m (positive direction)
- From x = 5 m to x = 1 m: Δx = -4 m (negative direction)
- From x = 5 m to x = 200 m to x = 5 m: Δx = 0 m

- Average speed: (always positive)[scalar quantity, m/s]

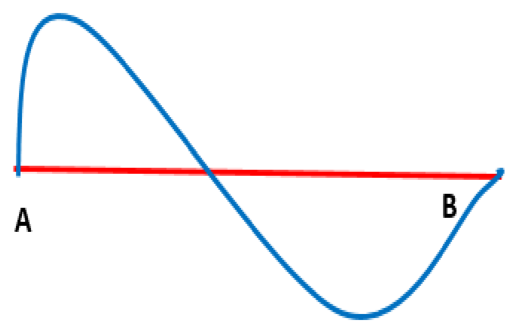
$$S_{avg} = \frac{total\ distance}{\Delta\ t}$$

- Average velocity: (time interval) [vector quantity, m/s]

$$v_{avg} = \frac{\Delta\ x}{\Delta\ t}$$

Note: The average speed is not the magnitude of the average velocity. For example, a runner ends at her starting point. Her displacement is zero so her average velocity is zero. However, the distance travelled is not zero, so the speed is not zero.

Example: Cars on both paths have the same average velocity (same displacement and time interval). The car on blue path will have a greater average speed since the distance it travelled is larger while the time is kept constant for both.



- Instantaneous velocity: (at single moment in time) [vector quantity, m/s]

$$v_{Ins} = \frac{dx}{dt}$$

Example: A particle moves from x = 3 m to x = -3 m in 2 seconds.

- Average velocity = -3 m/s; average speed = 3 m/s

Checkpoint 2

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

Answers:

(a) Situations 1 and 4 (zero), (b) Situations 2 and 3

- Speed is the magnitude of instantaneous velocity.

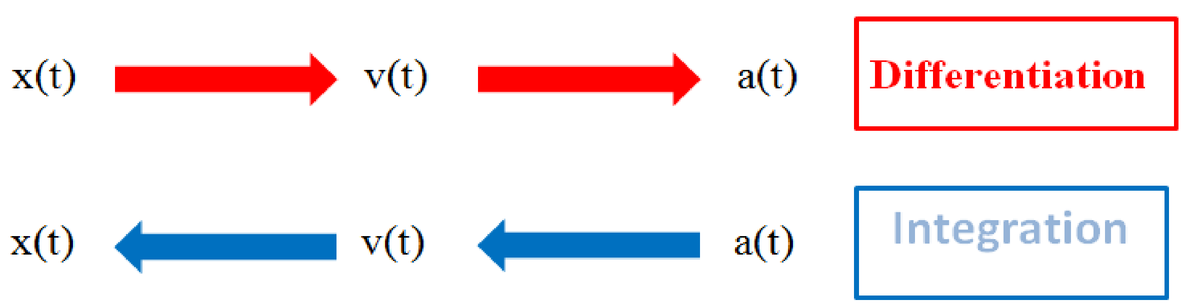
- Average acceleration: The rate at which the velocity change over time [vector quantity, m/s^2]

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

- An object accelerates if its speed, direction, or both change.

- Instantaneous acceleration: [vector quantity, m/s^2]

$$a_{Ins} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

Example: If a car with velocity *v* = -25 m/s is braked to a stop in 5.0 s, then *a* = + 5.0 m/s^2. Acceleration is positive, but speed has decreased.

Initial velocity (v_i)	Acceleration (a)	Motion
+	+	Speeding up
−	−	Speeding up
+	−	Slowing down
−	+	Slowing down
+ or −	0	Constant velocity
0	+ or −	Speeding up from rest
0	0	Remaining at rest (stationary)

Checkpoint 3

A wombat moves along an *x* axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Answers: (a) + (b) - (c) - (d) +

- Graphical Interpretation of velocity:

  $\Delta\, x, v\ and\ a$ are physical quantities that describe the motion and they are changing with time.

  - Average velocity = slope of the line joining the initial and final position.

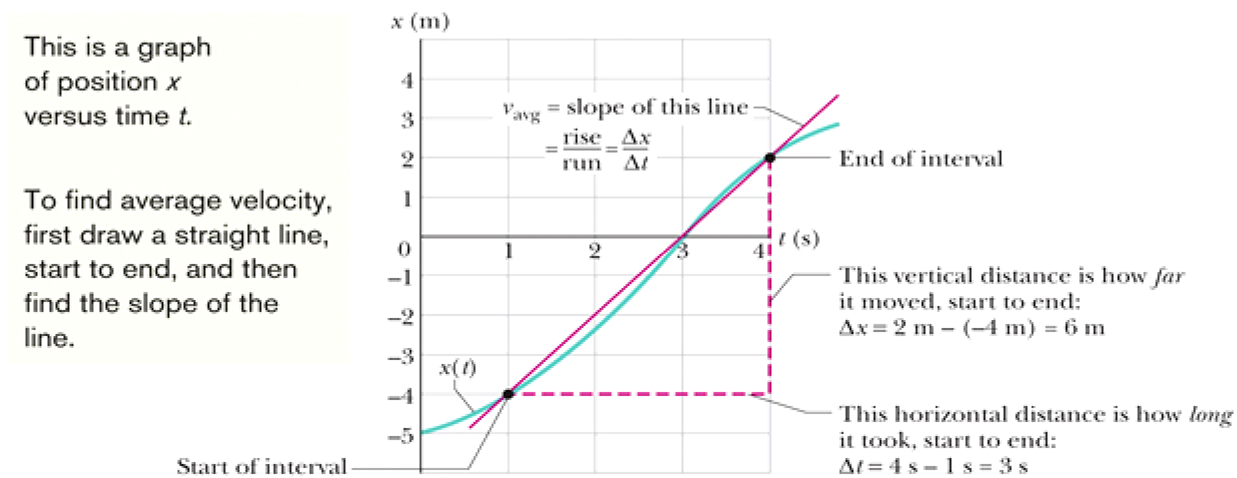
$$v_{avg} = \frac{\Delta\, x}{\Delta\, t}$$

  - Instantaneous velocity = slope of the tangent line to the position versus time graph at a certain point (time).

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

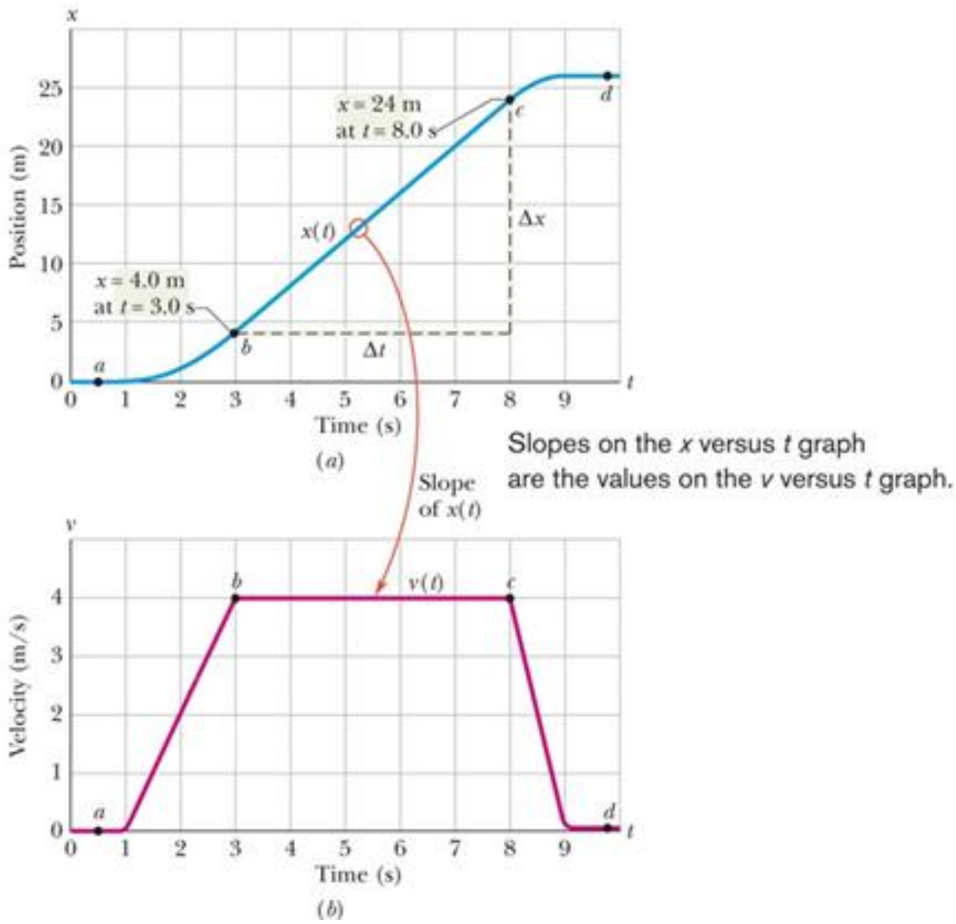
Position versus time graph:

- On a graph of *x* vs. *t*, the average velocity is the slope of the straight line that connects two points
- Average velocity is therefore a vector quantity
  - Positive slope means positive average velocity
  - Negative slope means negative average velocity



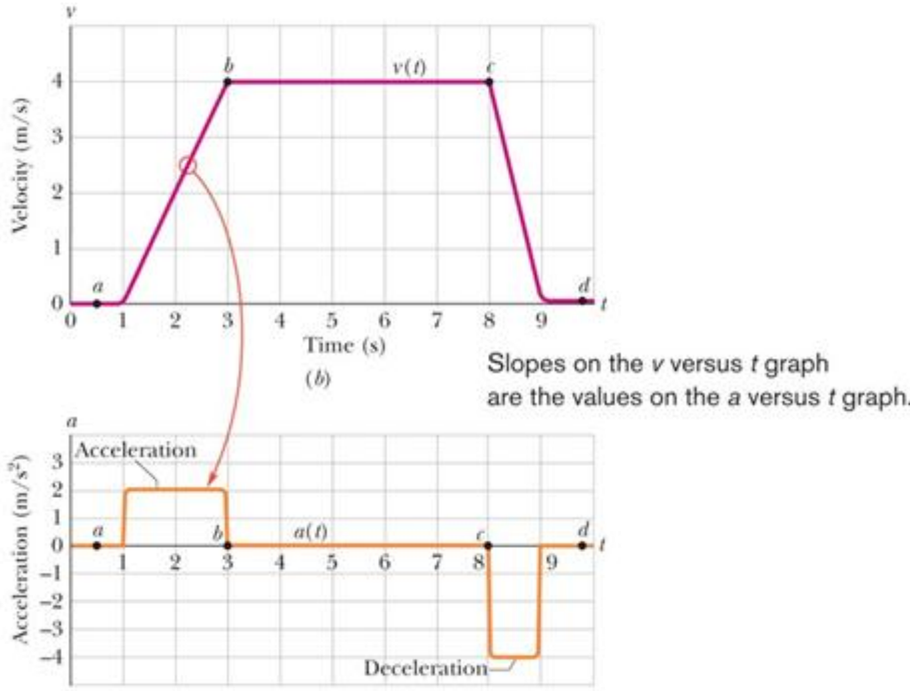
The sign of any vector quantity represents the direction

Example: The graph shows the position and velocity of an elevator cab over time.



- The slope of $x(t)$, and so also the velocity v, is zero from 0 to 1 s, and from 9s on.
- During the interval *bc*, the slope is constant and nonzero, so the cab moves with constant velocity (4 m/s).

Example: The graph shows the velocity and acceleration of an elevator cab over time.



- ✓ When acceleration is 0 (e.g. interval *bc*) velocity is constant.
- ✓ When acceleration is positive (*ab)* upward velocity increases.
- ✓ When acceleration is negative (*cd)* upward velocity decreases.

For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in *x* is 20 m.

$$\Delta x = \left(4.0\frac{\text{m}}{\text{s}}\right)(8.0\text{ s} - 3.0\text{ s}) = 20\text{ m}$$

- Constant Acceleration:

- In many cases acceleration is constant, or nearly so.
- For these cases, **5 special equations** can be used.
- Note that constant acceleration means a velocity with a constant slope, and a position with varying slope (unless $a = 0$).

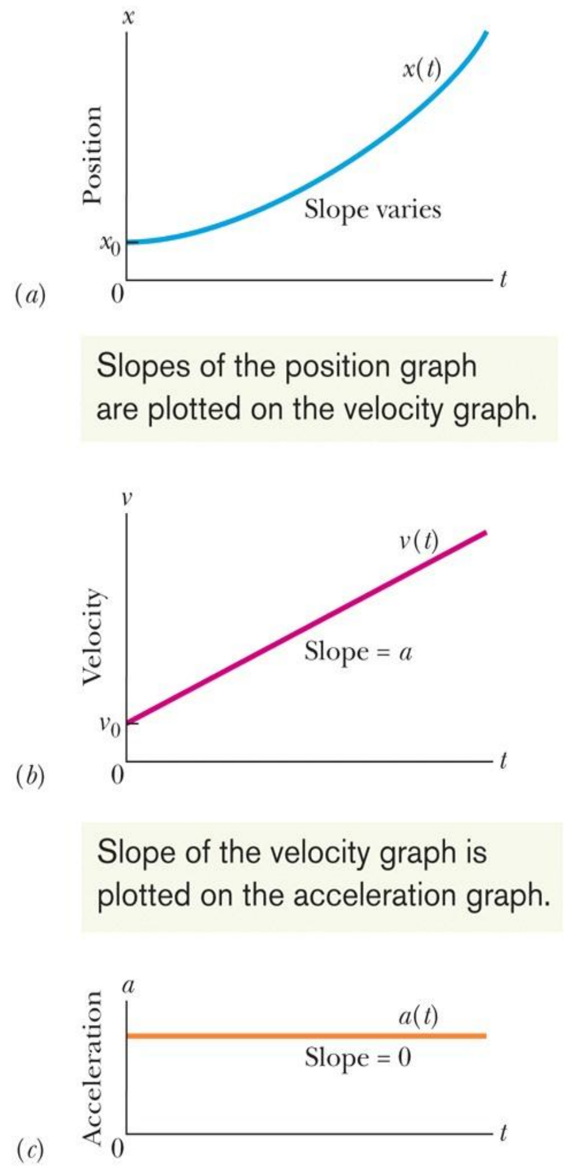


Table 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Note: starting from rest $(v_0 = 0)$

- **First basic equation**
  - When the acceleration is constant, the average and instantaneous accelerations are equal

$$a = a_{avg} = \frac{v - v_0}{t - 0} \longrightarrow v = v_0 + at$$

- **Second basic equation**

$$v_{avg} = \frac{x - x_0}{t - 0} \longrightarrow x = x_0 + v_{avg} t$$

Average = ((*initial*) + (*final*)) / 2

$$v_{avg} = \frac{1}{2}(v_0 + v) \longrightarrow v_{avg} = v_0 + \frac{1}{2}at$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

These two equations can be obtained by integrating a constant acceleration.

Checkpoint 4

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

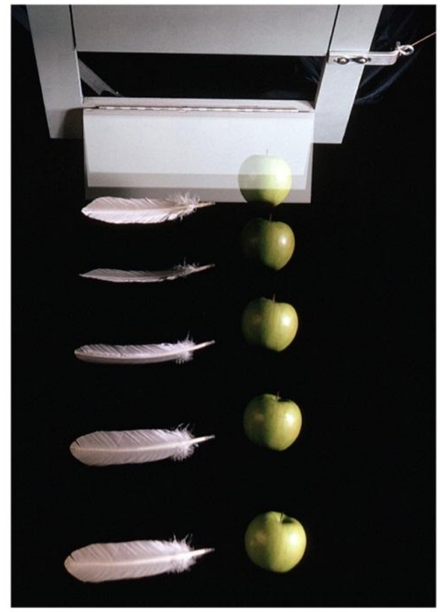
Answer: Situations 1 (a = 0) and 4.

- Free- Fall Acceleration: Straight-Line motion with constant acceleration

An object rising or falling freely near Earth's surface. (Air resistance can be neglected)

+ y →→ vertically upward

$a = -g = -9.8\ m/s^2$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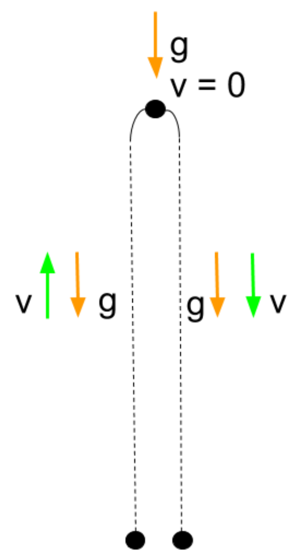
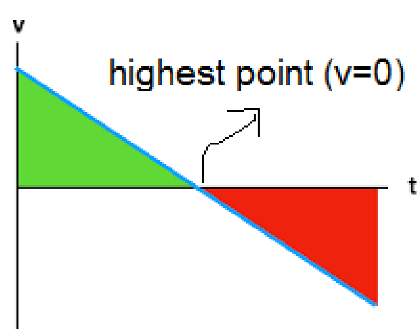
Note: Free- fall acceleration is the same for all objects regardless of its mass, density, shape …. [Galileo]

Example: Consider a ball thrown up into the air.



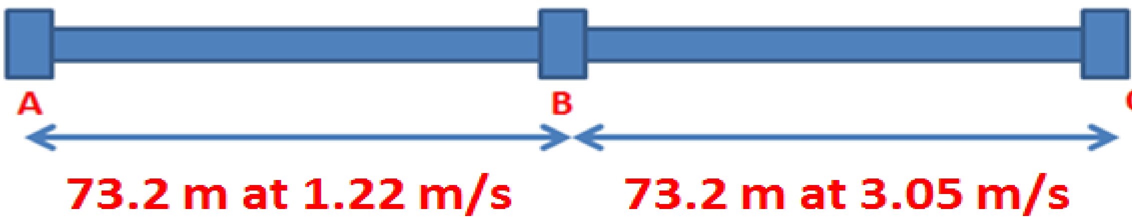
Direction of velocity and acceleration for a ball thrown up in the air. Acceleration from gravity is always constant and downward, but the direction and magnitude of velocity change.

- Moving upward (During Ascent): speed decreases (velocity becomes less positive)
- Top of the path (highest point): $v = 0$
- Moving downward (During descent): speed increases (velocity becomes more negative)



•2 Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph x versus t for both cases and indicate how the average velocity is found on the graph.

a) Trip: A → B, then B → C

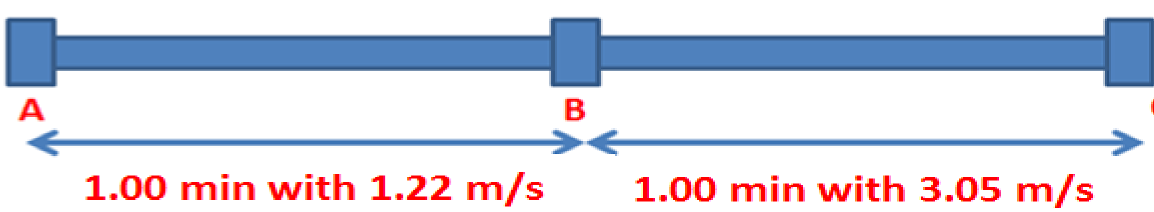


$$t_{\,A\to B} = \frac{distance}{velocity} = \frac{73.2m}{1.22\ m/s} = 60.0\ s$$

$$t_{\,B\to C} = \frac{distance}{velocity} = \frac{73.2\ m}{3.05\ m/s} = 24.0\ s$$

$$v_{avg,\ A\to C} = \frac{\Delta x}{\Delta t} = \frac{(x_{A\to C} - 0)}{((t_{\,A\to B} + t_{\,B\to C}) - 0)} = \frac{(146.4\ m - 0)}{(84.0\ s - 0)} = +1.74\ m/s$$

b) Trip: A → B, then B → C

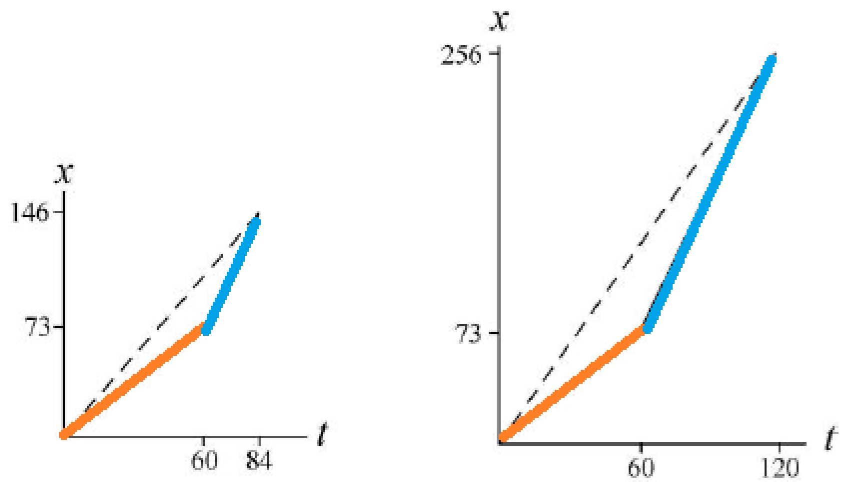


$$d_{\,A\to B} = vt = (1.22\,m/s)(1\ min)\left(\frac{60\ s}{1\ min}\right) = 73.2\ m$$

$$d_{\,B\to C} = vt = (3.05\,m/s)(1\ min)\left(\frac{60\ s}{1\ min}\right) = 183\ m$$

$$v_{avg,\ A\to C} = \frac{\Delta x}{\Delta t} = \frac{(x_{A\to C} - 0)}{((t_{\,A\to B} + t_{\,B\to C}) - 0)} = \frac{73.2\ m + 183\ m}{(2)(60)\ s} = +2.14\ m/s$$

c)



The slope of the dashed line drawn from the origin to the final point $(\Delta t, \Delta x)$ represents the average velocity.

$$slope\ of\ the\ dashed\ line = \frac{\Delta\, x}{\Delta\, t}$$

The slope of the first line segment is 1.22 m/s and the slope of the second one is 3.05 m/s in both graphs.

•14 GO An electron moving along the x axis has a position given by $x = 16te^{-t}$ m, where t is in seconds. How far is the electron from the origin when it momentarily stops?

$$x = 16\, te^{-t}$$

Momentarily stops →→ $v = 0$

$$v(t) = \frac{dx}{dt} = 16\, t\, (-e^{-t}) + 16\, e^{-t} = 16\, e^{-t}\, (1 - t)$$

$$v = 0 \;\rightarrow\rightarrow\; 16\, e^{-t}\, (1 - t) = 0$$

$(1 - t) = 0$; e^{-t} cannot be zero

$$t = 1\, s$$

$$\boldsymbol{x(t = 1\, s) = 16\, (1\, s)e^{-(1\, s)} = 5.88\, m}$$

•20 (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when, if ever, is the particle's velocity zero? (b) When is its acceleration a zero? (c) For what time range (positive or negative) is a negative? (d) Positive? (e) Graph $x(t)$, $v(t)$, and $a(t)$.

$$\boldsymbol{x = 20\,t - 5\,t^3}$$

$$\boldsymbol{v(t) = \frac{dx}{dt} = 20 - 15\,t^2}$$

$$\boldsymbol{a(t) = \frac{dv}{dt} = -30\,t}$$

a) $v(t) = 0$

$$20 - 15\,t^2 = 0$$

$$t = \sqrt{\frac{20}{15}} = \pm 1.15\ s$$

The particle momentarily stopped at t= 1.2 s.

b)

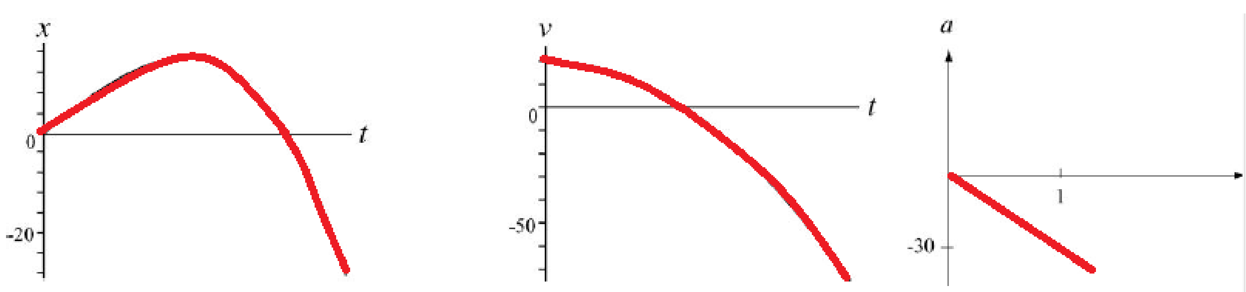
$$\boldsymbol{a(t) = 0}$$

$$\boldsymbol{-30\,t = 0\,, \qquad t = 0\ s}$$

C) $\boldsymbol{a(t) < 0\ for\ t > 0}$

d) $\boldsymbol{a(t) > 0\ for\ t < 0}$

e)



••41 GO As two trains move along a track, their conductors suddenly notice that they are headed toward each other. Figure 2-31 gives their velocities v as functions of time t as the conductors slow the trains. The figure's vertical scaling is set by v_s = 40.0 m/s. The slowing processes begin when the trains are 200 m apart. What is their separation when both trains have stopped?

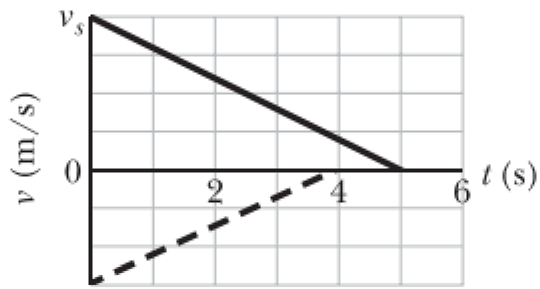
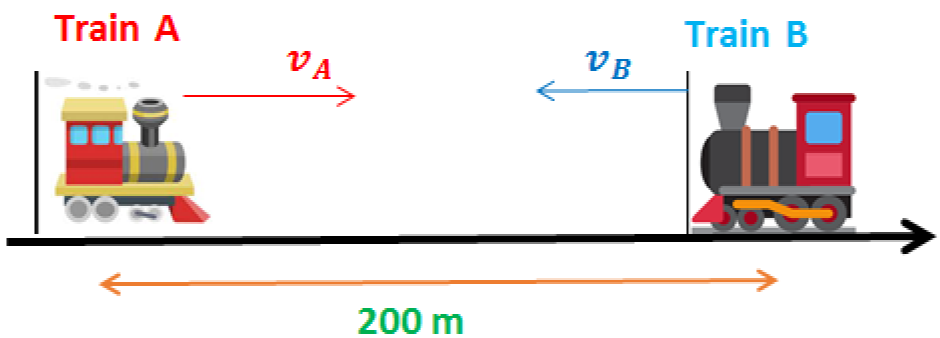


Figure 2-31 Problem 41.



The displacement = the area under the velocity vs. time curve

$$\Delta x\ (train\ A) = \left(\frac{1}{2}\right)(5\ s)(40.0\ m/s) = 100\ m$$

$$\Delta x\ (train\ B) = \left(\frac{1}{2}\right)(4\ s)(30.0\ m/s) = -60\ m$$

The separation between the two trains when they stopped is 40 m (200 m – 160 m).

••51 As a runaway scientific balloon ascends at 19.6 m/s, one of its instrument packages breaks free of a harness and free-falls. Figure 2-34 gives the vertical velocity of the package versus time, from before it breaks free to when it reaches the ground. (a) What maximum height above the break-free point does it rise? (b) How high is the break-free point above the ground?

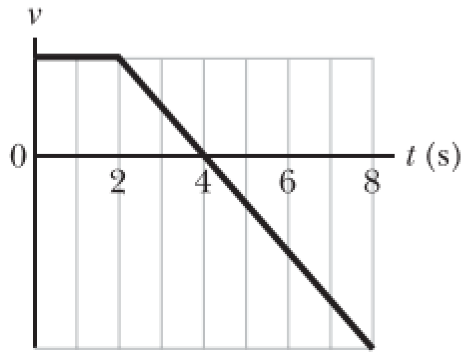
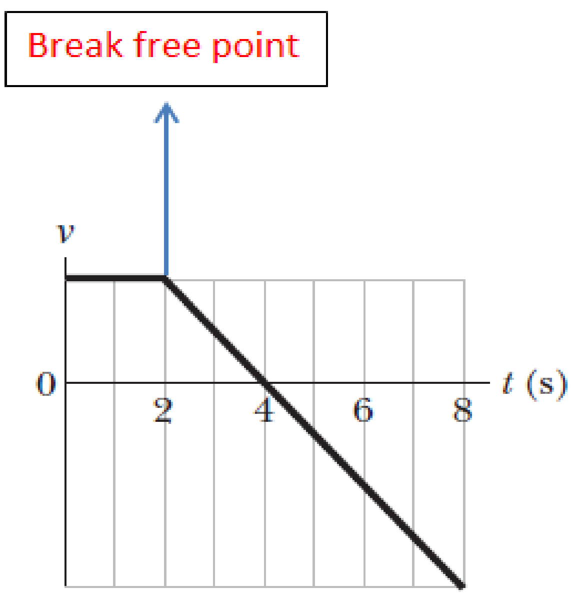
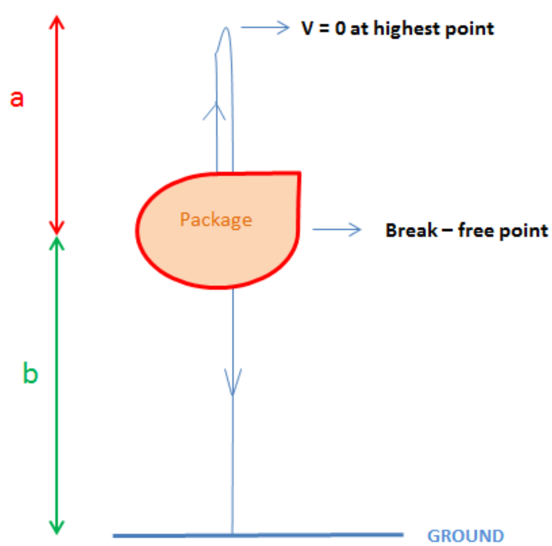


Figure 2-34 Problem 51.





From the figure:

- At t = 2 sec, the package breaks free of the balloon.(its velocity starts decreasing)
- at t =4.0 s the package reaches its maximum height (v = 0) before it free – fall and it falls for 4.0 s until it reaches the ground.

At the maximum height: $v = 0$ *and it needs* 2.0 *s to reach it.*

$$v = v_0 + a\,t \;\rightarrow\rightarrow 0 = v_0 - (9.8\ m/s^2)\ 2.0\ s$$

$v_0 = 19.6\ m/s$, the package velocity when it breaks from the balloon

The maximum height above the break – free point :

$$\Delta y = v_0\,t + \frac{1}{2}\,g\,t^2$$

$$\Delta y = (19.6\ m/s)(2.0\ s) + \frac{1}{2}(-\,9.8\ m/s^2)(2.0\ s)^2 = 19.6\ m$$

The package is at its highest point and then falls for 4.0 s until it reaches the ground.

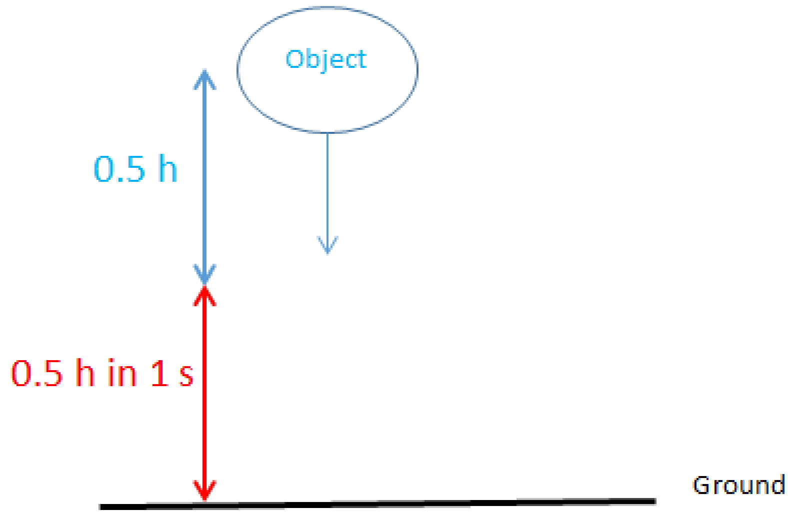
$$\Delta y = v_0\,t + \frac{1}{2}\,g\,t^2, v_0 = 0\ (maximum\ height)$$

$$\Delta y = \frac{1}{2}\,g\,t^2 = \frac{1}{2}(\,9.8\ m/s^2)(4.0\ s)^2 = 78.4\ m$$

78.4 m is the total distance between the maximum point and the ground.

The high of the break-free point above the ground is 78.4 m – 19.6 m = 58.8 m

••58 An object falls a distance h from rest. If it travels $0.50h$ in the last 1.00 s, find (a) the time and (b) the height of its fall. (c) Explain the physically unacceptable solution of the quadratic equation in t that you obtain.



Free falling:

In the first half of motion:

$$v^2 = v_0^2 + 2\,g\,\Delta y$$
$$v^2 = 0 + 2\,g\,(0.5\,h)$$
$$v^2 = gh \text{ .... } (1)$$

In the second half of motion:

$$\Delta y = \; v_0\,t + \frac{1}{2}\,g\,t^2$$
$$0.5\,h = \; v\,(1.0\,s) + \frac{1}{2}\,g\,(1.0\,s)^2$$
$$0.5\,h = \; v + 4.9 \text{ .... } (2)$$

$$\rightarrow\rightarrow\rightarrow \; from\;(2) \quad h = 2\,v + g \text{ .... } (3)$$
$$\textbf{Plug (3) in (1)} \rightarrow\rightarrow\rightarrow \; v^2 = (g)(2v + g) = 19.6\,v + 96.04$$
$$v^2 - 19.6\,v - 96.04 = 0$$

$$v^2 - 19.6\, v - 96.04 = 0$$

$$v = \frac{- -19.6 \pm \sqrt{(19.6)^2 - 4(-96.04)}}{2}$$

$$v = +23.6 \frac{m}{s}$$

$$or - 4.06 \frac{m}{s} \text{ Ignore it (no physical meaning)}$$

$$h = 2\, v + g = 2\,(23.6) + 9.8 = 57\, m$$

To find the time of the flight:

In the first half of motion:

$$\Delta y = v_0\, t + \frac{1}{2}\, g\, t^2$$

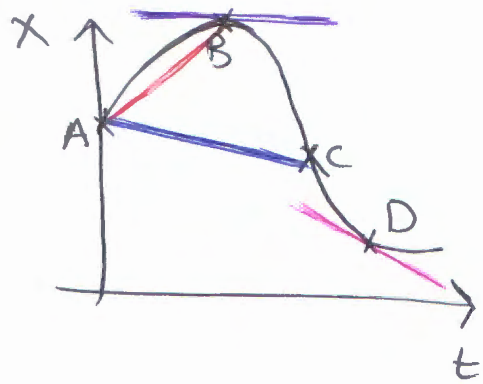
$$0.5\, h = 0 + \frac{1}{2}\, g\, t^2$$

$$0.5\, h = 4.9 t^2$$

$$t = \sqrt{\frac{h}{g}} = \sqrt{\frac{57\, m}{9.8\, m/s^2}} = 2.43\, s$$

$$\text{The total time of the flight is } 2.43\, s + 1.0\, s = 3.43\, s$$

3 Non-Constant velocity



$v_{avg\,(A\to B)}$ = slope of Red line

$v_{avg}\,(A\to C)$ = slope of blue line

$v_{at\ point\ B}$ = zero
slope of purple line

v_D = slope of pink line (negative)

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example

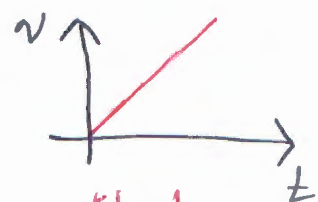
v → → → →

Constant velocity (zero acceleration)

example

Speeding up

v → → → → →



velocity is increasing and the Acceleration is constant
and v and a are in the same direction

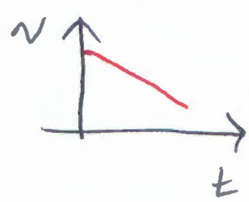
a → → → → →

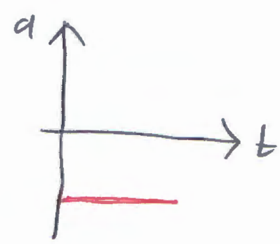
example

Slowing down

v → → → →

a ← ← ← ←



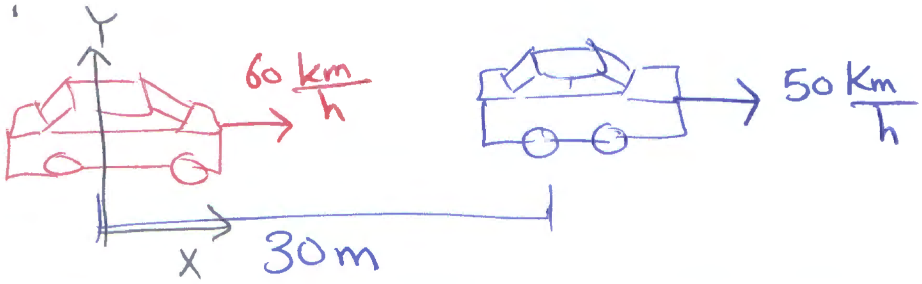


velocity is decreasing , acceleration is constant
a and v are in the opposite direction

Problem 1.4 A driver in a blue car travelling at 50 km h^{-1} sees a red car approaching in his rear-view mirror. The red car is travelling at 60 kmh^{-1} and is 30 m behind the blue car when first spotted.

(a) How many seconds from the time the driver of the blue car first first noticed it until the red car passes the blue car?

(b) How much farther down the road will the blue car travel in this time?



(a) They pass each other (the two cars meet at the same position)

$X_{f,red} = X_{f,blue}$ $\Delta X = X_f - X_i = v_i t + \frac{1}{2} a t^2$

$v_{blue} = 50 \frac{km}{h} = 13.89\ m/s$, $v_{red} = 60 \frac{km}{h} = 16.67\ m/s$

No acceleration for both cars $(a = 0)$

$X_{f,red} = X_{i,red} + v_{i,red}\, t$ // $X_{f,blue} = X_{i,blue} + v_{i,blue}\, t$

$X_{f,red} = 0 + (16.67 \frac{m}{s}) t$ // $X_{f,blue} = 30\ m + (13.89 \frac{m}{s}) t$

$\Rightarrow$ Red car passes the blue one $X_{f,red} = X_{f,blue}$

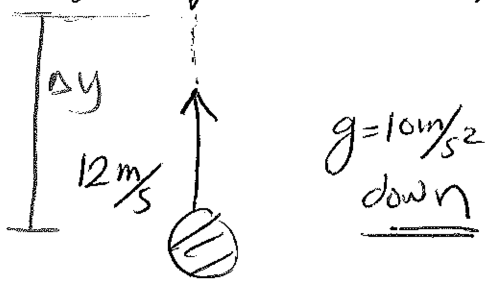
$$16.67\, t = 30 + 13.89\, t$$

$$\boxed{t = 10.8\ sec}$$

(b) Blue car 13.89 m/s for 10.8 sec

$$\Delta X = v_{i,blue}\, t + \cancel{\tfrac{1}{2} a t^2}^{\,0} = (13.89 \tfrac{m}{s})(10.8 sec) = \underline{150\ m}$$

Example 1.2 If you throw a cricket ball straight up at $12\,ms^{-1}$, How high will go?



$v_i = 12$ m/s up , at highest point $v_f = 0$

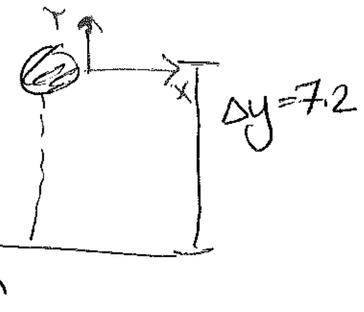
$g = 10 m/s^2$ down

$$v_f^2 = v_i^2 + 2g\,\Delta y$$

$$0 = (12)^2 + 2(-10)\,\Delta y$$

$$\boxed{\Delta y = 7.2m}$$

How long does it take the ball to fall to its origional position from its maximum height?



$$\Delta y = v_i t + \frac{1}{2} g t^2$$

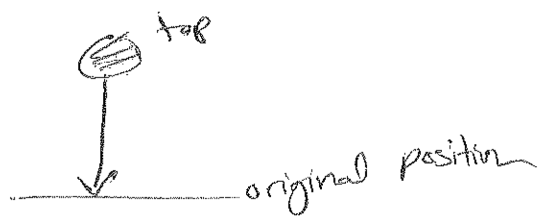
$$-7.2 = 0 + \frac{1}{2}(-10)\,t^2$$

$$t = 1.2\ s$$

$t_{down} = 1.2$ s and we can conclude that $t_{up} = 1.2$s

زمن الصعود = زمن الهبوط (اذا عادت الكرة لنفس نقطة الاطلاق)

v_{ball} when it hits the ground?



$$v_f = v_i + g\,t_{down}$$

$$= 0 - 10\,t_{down}$$

$$v_f = -10(1.2) = -12\,m/s$$

negative sign indicates the direction of the velocity

Problem 1.3 A jogger starting their morning run accelerates from a standstill to their steady jogging pace of 8.0 km/h. They reach a speed of 8 km/h, 5 s after starting. How long does it take the jogger to reach the end of their 20m driveway? Answer: 11.49 sec

- $8 \frac{km}{h} = 8 \frac{km}{h} \times \left[\frac{1000m}{1km}\right] \times \left[\frac{1h}{3600s}\right] = 2.22 m/s$

v: $0 \rightarrow 2.22$ m/s in 5 seconds

$v_f = v_i + at$

$2.22 \frac{m}{s} = 0 + a(5 sec) \Rightarrow \boxed{a = 0.44 m/s^2}$

$v_f^2 = v_i^2 + 2a\Delta X$

$(2.22 \frac{m}{s})^2 = 0 + 2(0.44 \frac{m}{s^2})\Delta X \Rightarrow \boxed{\Delta X = 5.6m}$

خلال 5 ثوانٍ قطع 5.6 متر بتسارع 0.44 م/ث²

$20 - 5.6 = 14.4$ m the remaining distance

after 5 sec, He travelled 5.6m with 0.44 m/s^2 and he reach his steady pace of 8.0 $\frac{km}{h}$ ($2.22 \frac{m}{s}$) and continue his trip with constant velocity (2.22 m/s)

14.4m with constant velocity 2.22 m/s | $a = 0$

$\Delta X = v_i t + \frac{1}{2} a t^2$

$14.4m = 2.22 \frac{m}{s} t \Rightarrow t = 6.49$ sec

He needs **11.49 sec** (5sec + 6.49sec) to reach 20m driveway