Exercises:

Q17: The following data are believed to have come from a Normal distribution. Use a goodness of fit test and $\boxed{\alpha = 0.05}$ to test this claim.

17	23	22	24	19	23	18	22	20	13	11	21	18	20	21
21	18	15	24	23	23	43	29	27	26	30	28	33	23	29

$n = 30$, $\bar{X} = 22.8$, $S = 6.27$, $K = \frac{n}{5} = 6$, $df = K - 3 = 3$

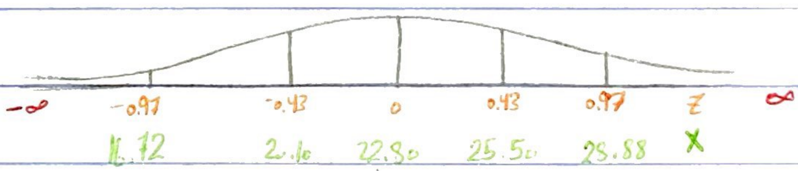
→ Find Z and X :

Area left to Z	closest Area	Z	$X = \bar{X} + SZ$
$\frac{5}{6} = 0.8333$	0.8340	0.97	28.88
$\frac{4}{6} = 0.6667$	0.6664	0.43	25.50
$\frac{3}{6} = 0.5$	–	0	22.80
$\frac{2}{6} = 0.3333$	–	-0.43	20.10
$\frac{1}{6} = 0.1667$	–	-0.97	16.72

By symmetric

→ Find category and f_i :

category	f_i	e_i	$(f_i-e_i)^2/e_i$
$-\infty$ – 16.71	3	5	0.8
16.72 – 20.10	7	5	0.8
20.10 – 22.80	5	5	0
22.80 – 25.50	7	5	0.8
25.50 – 28.88	2	5	1.8
28.88 – ∞	6	5	0.2
	30	30	4.4



critical approach : $\chi^2_{0.05} = 7.815 \rightarrow \chi^2 < \chi^2_\alpha$

p-value approach : p-value $\in (0.1, 0.9) \rightarrow$ p-value $> \alpha$

So we Don't Reject H_0 $(\alpha = 0.05)$.

⇒ The pop. has a Normal distribution.

→ Test statistic : $\chi^2 = \sum \frac{(f_i - e_i)^2}{5} = \underline{\underline{4.4}}$

Q19: -- have a poisson distribution. use $\alpha = 0.10$ and the following data to test the assumption..

$K = 7$

x_i	f_i	$x_i f_i$	$e_i = \frac{\mu^{x_i} e^{-\mu}}{x_i!} \cdot n$	$\frac{(f_i - e_i)^2}{e_i}$
0	15	0	13.53	0.16
1	31	31	27.07	0.57
2	20	40	27.07	1.85
3	15	45	18.04	0.51
4	13	52	9.02	1.76
5	4	20	3.61	0.042
6	2	12	$100 - (\Sigma \uparrow) = 1.66$ 98.34	0.069
	$\boxed{n = 100}$	200		

$$\mu = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{200}{100} = 2$$

$$\chi^2 = \sum_{i=1}^{K} \frac{(f_i - e_i)^2}{e_i} = 4.961 \quad \text{with } df = K - 2 = 5$$

★ critical value approach

$$\chi^2_{0.10} = 9.236$$

df \ α	0.10
5	9.236

$\Rightarrow \chi^2 \leq \chi^2_\alpha$

★ p-value approach:

p-value

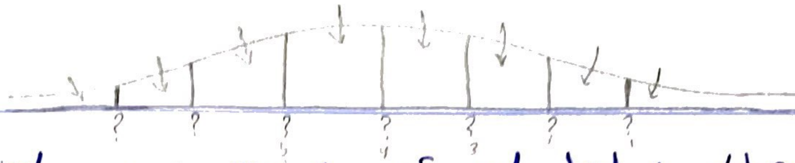
df \ α	0.90	0.10
5	1.610	9.236

p-value $\in (0.1, 0.9)$

p-value $\geq \alpha$

So we don't Reject H_0 $(\alpha = 0.10)$

So The pop. has a poisson dist.



Q21: A Random sample of final ..., use $\alpha = 0.05$ and test to determine whether a Normal distribution should be rejected as being ~~...~~ of pop. ...

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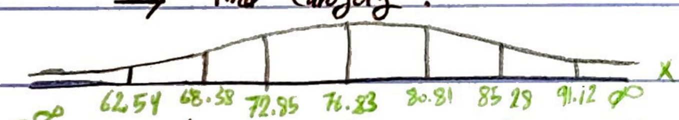
$n = 40$ $\quad \bar{X} = 76.83$ $\quad S = 12.43$ $\quad K = \frac{40}{5} = 8$ $\quad df = 5$

→ Find Z and X :

Area left to Z	Closest Area	Z	$X = \bar{X} + SZ$
$\frac{7}{8} = 0.8750$	0.8749	1.15	91.12
$\frac{6}{8} = 0.7500$	0.7517	0.68	85.28
$\frac{5}{8} = 0.6250$	0.6255	0.32	80.81
$\frac{4}{8} = 0.5$	–	0	76.83
$\frac{3}{8} = 0.375$	–	−0.32	72.85
$\frac{2}{8} = 0.25$	–	−0.68	68.38
$\frac{1}{8} = 0.125$	–	−1.15	62.54

→ Find category :



Category	f_i	e_i	$(f_i - e_i)^2 / e_i$
−∞ – 62.53	5	5	0
62.54 – 68.38	3	5	0.8
68.38 – 72.85	6	5	0.2
72.85 – 76.83	5	5	0
76.83 – 80.81	5	5	0
80.81 – 85.28	7	5	0.8
85.28 – 91.12	4	5	0.2
91.12 – ∞	5	5	0
	40	40	

→ Test statistic :

$$\chi^2 = \sum \frac{(f_i - e_i)^2}{e_i} = 2 .$$

critical approach : $\chi^2_{0.05} = 2.015$ → $\chi^2 < \chi^2_\alpha$

P-value approach : P-value $\in (0.05, 0.1)$ → P-value > α

So we Don't Reject H_0 ($\alpha = 0.05$).

So The pop. has Normal distribution.

Q22 : The number of car accidents have a poisson dist. A sample of 80 days during the past year gives the following data, Do these data support the belief that the number of accident per day has a poisson dist. $\alpha = 0.05$.

x_i	f_i	$e_i = \frac{\mu^{x_i} e^{-\mu}}{x!} \cdot n$	$\frac{(f_i - e_i)^2}{e_i}$	
0	34	29.43	0.71	
1	25	29.43	0.67	
2	11	14.72	0.94	
3	7	4.91	0.89	
4	3	$80-(\Sigma\uparrow) = 1.51$	1.47	
	80			

$$\rightarrow \mu = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{0 + 25 + 22 + 21 + 12}{80} = \frac{80}{80} = 1$$

$$\rightarrow \chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i} = 4.68 \quad \text{with } df = 5-2 = 3$$

* critical approach :

$\chi^2_{0.05} = 7.815$

$df \backslash \alpha$	0.05
3	7.815

$\chi^2 \leq \chi^2_\alpha$

* p-value approach :

$df \backslash \alpha$	0.9	0.1
3	0.584	6.251

p-value

p-value $\geq \alpha$

so we don't Reject H_0 $(\alpha = 0.05)$

So The population has a poisson dist. $(\alpha = 0.05)$.

Reem