Started on	Tuesday, 19 December 2023, 11:30 AM
State	Finished
Completed on	Tuesday, 19 December 2023, 12:00 PM
Time taken	29 mins 50 secs
Grade	**12.00** out of 12.00 (**100**%)

Question **1**

Correct

Mark 2.00 out of 2.00

The **largest interval** in which a solution of the IVP

$$(t-2)(\ln t)y' + y = \tan t, \quad y\left(\tfrac{\pi}{4}\right) = 1$$

is certain to exist is

Select one:

- $(0, \frac{\pi}{2})$
- $(0, \pi)$
- $(1, \infty)$
- $(0, 1)$ ✔

The correct answer is: $(0, 1)$

Question **2**

Correct

Mark 2.00 out of 2.00

Using the substitution $v = y^{-5}$, the Bernoulli differential equation

$$y' + P(x)y = Q(x)y^6, x > 0$$

can be written as

Select one:

- $v' + (-5)P(x)v = (7)Q(x)$
- $v' + (-5)P(x)v = (-5)Q(x)$ ✔
- $v' + (7)P(x)v = (-5)Q(x)$
- $v' + (7)P(x)v = (7)Q(x)$

The correct answer is: $v' + (-5)P(x)v = (-5)Q(x)$

Question **3**

Correct

Mark 2.00 out of 2.00

A tank contains originally (at $t = 0$) $3L$ of fresh water. Water containing $3g$ of salt per liter is entering at a rate of $2L/min$ and the well-stirred mixture leaves the tank at a rate of $2L/min$. The I.V.P. for the quantity of salt in the tank $Q(t)$ is

Select one:

- $\frac{dQ}{dt} = 6 - \frac{2Q(t)}{3+(0)t}, Q(0) = 0$ ✔
- $\frac{dQ}{dt} = 7 - \frac{2Q(t)}{3+(0)t}, Q(0) = 0$
- $\frac{dQ}{dt} = 7 - \frac{2Q(t)}{3+(1)t}, Q(0) = 3$
- $\frac{dQ}{dt} = 6 - \frac{2Q(t)}{3+(1)t}, Q(0) = 0$

The correct answer is: $\frac{dQ}{dt} = 6 - \frac{2Q(t)}{3+(0)t}, Q(0) = 0$

Question **4**

Correct

Mark 2.00 out of 2.00

Consider the IVP $\frac{dy}{dx} = xy^{1/3}, y(1) = -1$. The largest rectangle in which the conditions of the **existence and uniqueness theorem** are satisfied is

Select one:

- $x > 1, y > 0$
- $x \in \mathbb{R}, y$
- $x > 0, y > 0$
- $x \in \mathbb{R}, y > 0$ ✖

The correct answer is: $x \in \mathbb{R}, y$

Question **5**

Correct

Mark 2.00 out of 2.00

According to the **Existence and Uniqueness Theorem**, the IVP

$$\frac{dy}{dx} = \frac{\sqrt{y^2-4}}{x}, \ y(\alpha) = \beta$$

has a unique solution if

Select one:

- $\alpha = 1, \beta = -1$
- $\alpha = 0, \beta = 0$
- $\alpha = 0, \beta = 1$
- $\alpha = 1, \beta = 3$ ✔

The correct answer is: $\alpha = 1, \beta = 3$

Question **6**

Correct

Mark 2.00 out of 2.00

A small metal bar is dropped into a large container of water with temperature $96^\circ C$. At $t = 2$ second, the temperature of the bar is measured to be $4^\circ C$. At $t = 3$ second, the temperature of the bar increases to $24^\circ C$. The I.V.P. for the the temperature of the bar $T(t)$ is

Select one:

- $\frac{dT}{dt} = k(T - (96)), T(4) = 2, T(24) = 3$
- $\frac{dT}{dt} = k(T - (96)), T(2) = 4, T(3) = 24$ ✔
- $\frac{dT}{dt} = k(T + (96)), T(2) = 4, T(3) = 24$
- $\frac{dT}{dt} = k(T + (96)), T(4) = 2, T(24) = 3$

The correct answer is: $\frac{dT}{dt} = k(T - (96)), T(2) = 4, T(3) = 24$