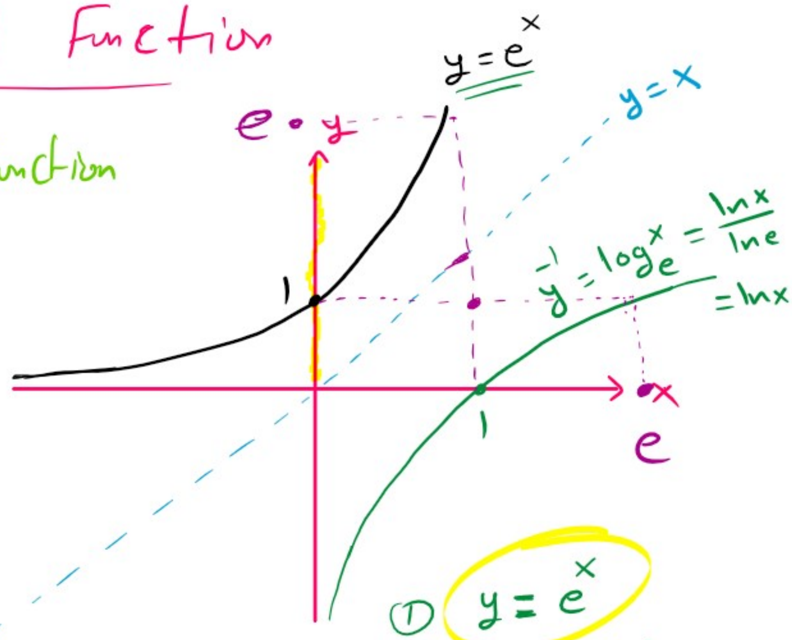
Exponential Function



$y = e^x$ is positive function

$D = (-\infty, \infty)$

$R = (0, \infty)$

$\lim_{x \to \infty} e^x = \infty$

$\lim_{x \to -\infty} e^x = 0$

x	y
$\frac{1}{2}$	$e^{\frac{1}{2}}$
0	$e^0 = 1$
1	e
2	e^2
⋮	

① $y = e^x$

② $\ln y = \ln e^x$

$\ln y = x(\ln e) \to 1$

$\ln y = x$

③ $y = \ln x$

④ $f^{-1}(x) = \ln x$

Properties for e^x

[1] $e^{\ln x} = x \quad \forall \; \underline{x > 0}$

$e^{\ln 3} = 3$, $e^{\ln \sqrt{2}} = \sqrt{2}$, $e^{\ln \frac{1}{2}} = \frac{1}{2}$

[2] $\ln e^x = x \quad \forall \; \underline{x}$

$x \ln e = x(1) = x$

$+ \Rightarrow e^{-2} = \frac{1}{e^2} > 0$

$\ln e^{3} = 3$, $\ln e^{-2} = -2$

Exp Solve for x

① $e^{\ln 2x} = 10$

$2x = 10$

$\boxed{x = 5}$

② $e^{(\ln 2)x} = 10$

$\ln e^{(\ln 2)x} = \ln 10$

$\frac{\cancel{(\ln 2)} x}{\cancel{\ln 2}} = \frac{\ln 10}{\ln 2}$

$\boxed{x = \frac{\ln 10}{\ln 2}}$

Q. How to derive exponential function

$u(x)$

A) If $y = a^{u(x)}$ where $u(x)$ diff
$a > 0$

$$y' = u'(x)\, a^{u(x)} \boxed{\ln a}$$

Exp ① $y = x^3 \Rightarrow y' = 3x^2$

$\Rightarrow$ ② $y = 3^{x^2} \Rightarrow y' = 2x\, 3^{x^2} \boxed{\ln 3}$

$$y = a^{u(x)} \Rightarrow y' = a^{u(x)}\, u'(x) \ln a$$

$$\int a^{u(x)}\, u'(x) \ln a \, dx = a^{u(x)} + c$$

Exp Find y' if ① $y = e^{x^3}$

$y' = e^{x^3} (3x^2) \ln e$

$$y' = e^{x^3}(3x^2)\ln e$$

$$= 3x^2 e^{x^3}$$

② $y = e^{3-5x^2}$ Find $y'(0)$

$$y' = e^{3-5x^2}(-10x)\ln e$$

$$= -10x\, e^{3-5x^2}$$

$$y'(0) = -10(0)\, e^{3} = 0$$

③ $y = \pi^{\sin x}$ Find $y'(\pi)$

$$y' = \pi^{\sin x}(\cos x)\ln \pi$$

$$y'(\pi) = \pi^{\sin \pi \to 0}(\cos \pi)\ln \pi$$

$$= \pi^{0}(-1)\ln \pi$$

$= 1 \quad (-1) \quad \ln \pi$

$= -\ln \pi$

$= \ln \pi^{-1}$

$= \ln \frac{1}{\pi}$

$y = e^x$

$y' = e^x \; (1)(1)$

$y' = e^x$

Exp Find ① $\int_{\ln 3}^{\ln 4} e^x \, dx$

$= e^x \Big|_{\ln 3}^{\ln 4} = e^{\ln 4} - e^{\ln 3}$

$= 4 - 3$

$= 1$

② $\int_0^1 3t^2 e^{t^3} \, dt = e^{t^3} \Big|_0^1$

$u = t^3$

$du = \boxed{3t^2 \, dt}$

$\int_0^1 e^u \, du = e^u \Big|_0^1 = e^1 - e^0$

$= e - 1$

$t = 0 \Rightarrow u = 0^3 = 0$

$t=0 \Rightarrow u=0^3=0$

$t=1 \Rightarrow u=1^3=1$

(3) $\displaystyle\int_0^{\frac{\pi}{3}} 5^{\sec\theta} \ln 5 \ \sec\theta \tan\theta \, d\theta$

$u=\sec\theta$

$du=\sec\theta\tan\theta\, d\theta$

$\theta=0 \Rightarrow u=\sec 0=1$

$\theta=\frac{\pi}{3} \Rightarrow u=\sec\frac{\pi}{3}=\frac{1}{\frac{1}{2}}=2$

$$\int_1^2 5^u \ln 5 \, du$$

$$5^u \Big|_1^2 = 5^2 - 5^1$$

$$= 25 - 5$$

$$= 20$$

(4) $\displaystyle\int 3^x \, dx = \frac{1}{\ln 3}\int 3^x \ln 3 \, dx$

$$= \frac{1}{\ln 3} 3^x + C$$

Properties

(1) $e^{x_1} e^{x_2} = e^{x_1 + x_2}$

$e^{-2} e^{5} = e^{-2+5} = e^{3}$

$e^{\frac{1}{2}} e^{3} = e^{\frac{1}{2}+3} = e^{3.5} = e^{\frac{7}{2}}$

(2) $\dfrac{e^{x_1}}{e^{x_2}} = e^{x_1 - x_2}$

$\dfrac{e^{3}}{e^{1}} = e^{3-1} = e^{2}$

$\dfrac{e^{\sqrt{5}}}{e^{2}} = e^{\sqrt{5}-2}$

(3) $e^{-x} = \dfrac{1}{}$

③ $e^{-x} = \frac{1}{e^{x}}$

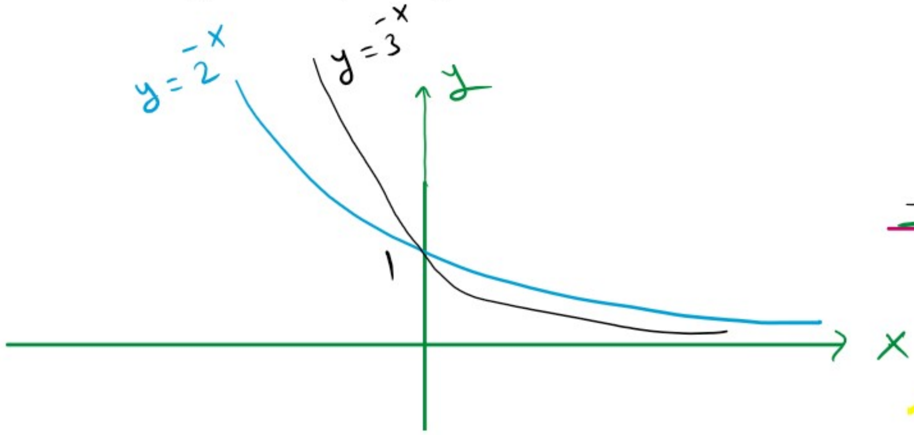
$e^{-2} = \frac{1}{e^{2}}$, $e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{e}}$

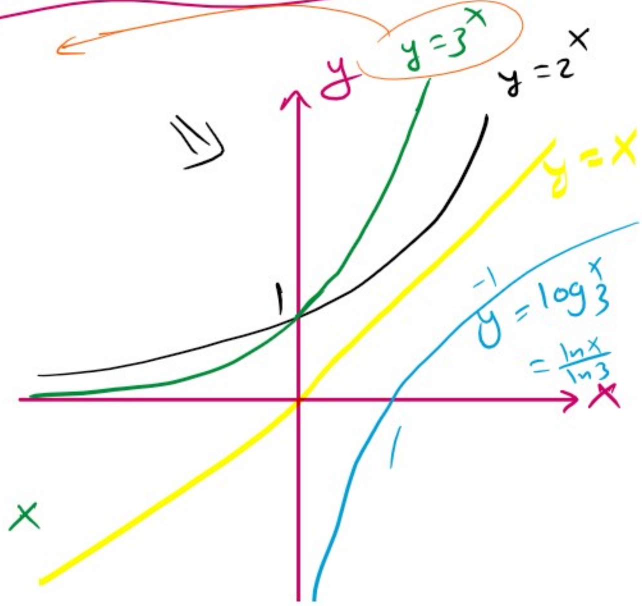
④ $\left(e^{x_1}\right)^{x_2} = e^{x_1 \cdot x_2}$

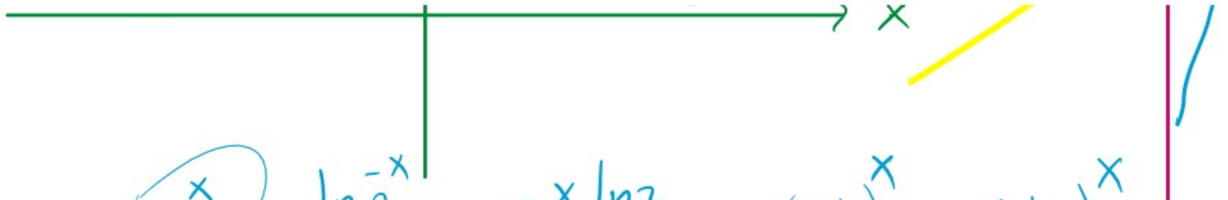
$\left(e^{2}\right)^{3} = e^{2\cdot 3} = e^{6}$

$\left(e^{-\frac{1}{2}}\right)^{14} = e^{-\frac{1}{2}\cdot 14} = e^{-7} = \frac{1}{e^{7}}$

$y = 2^{x} = e^{x\ln 2}$, $y = 3^{x} = e^{\ln 3^{x}} = e^{x\ln 3}$







$$y = 2^{-x} = e^{\ln 2^{-x}} = e^{-x \ln 2} = \left(2^{-1}\right)^{x} = \left(\frac{1}{2}\right)^{x}$$

$$y = 3^{-x} = e^{\ln 3^{-x}} = e^{-x \ln 3} = \left(3^{-1}\right)^{x} = \left(\frac{1}{3}\right)^{x}$$

Exp $y = x^{x}$ Find $y'(1)$

$$\ln y = \ln x^{x}$$

$$\ln y = \underset{\text{اترك}}{x} \; \underset{\text{اشتق}}{\ln x}$$

$$\frac{y'}{y} = (x)\left(\frac{1}{x}\right) + (\ln x)(1)$$

$$= 1 + \ln x$$

$$y' = y\left[1 + \ln x\right]$$

$$y' = x^{x}\left[1 + \ln x\right]$$

$$y'(1) = 1\left[1 + \ln 1\right.$$

$$y'(1) = 1\left[1 + \ln 1\right]$$
$$= 1\left[1 + 0\right]$$
$$= 1$$

Exp Find y' if ① $y = \log_4 x^2$

$$y = \frac{\ln x^2}{\ln 4} = \frac{1}{\ln 4} \ln x^2$$

$$y' = \frac{1}{\ln 4} \frac{2x}{x^2}$$

$$= \frac{1}{\ln 2^2} \frac{2x}{x^2} = \frac{1}{\cancel{2}\ln 2} \frac{\cancel{2}\cancel{x}}{x^{\cancel{2}}} = \frac{1}{x \ln 2}$$

② $y = \log_2 8x^{\ln 2}$

$$= \frac{\ln 8 \cdot x^{\ln 2}}{\ln 2} = \frac{\ln 8 + \ln x^{\ln 2}}{\ln 2}$$

$$= \frac{\ln 2^3 + \ln 2 \ln x}{\ln 2}$$

$$= \frac{3\ln 2 + \ln 2 \ln x}{\ln 2}$$

$$= \frac{\cancel{\ln 2}[3 + \ln x]}{\cancel{\ln 2}}$$

$$y = 3 + \ln x$$

$$y' = 0 + \frac{1}{x}$$

$$= \frac{1}{x} \checkmark$$

3 $$y = \int_0^{\log_2 x} 2\ln 2 \; 4^t \, dt$$

$$y' = 2\ln 2 \; 4^{\log_2 x} \left(\log_2 x\right)'$$

$$4^{\log_2 x} = (2^2)^{\log_2 x}$$

$$= 2^{2\log_2 x}$$

$$\log_2 x^2$$

$$= 2\ln 2 \; y^{\log_2 x} \left(\frac{\ln x}{\ln 2}\right)'$$

$$\log_2 x^2 = 2$$

$$= x^2$$

$$= 2\cancel{\ln 2} \; y^{\log_2 x} \frac{1}{\cancel{\ln 2}} \frac{1}{x}$$

$$= 2 \; \cancel{x^2} \frac{1}{\cancel{x}}$$

$$= 2x$$

$$y^{\log_2 x} = x^2$$

$$\boxed{a^{\log_a f(x)} = f(x)}$$

$$\ln a^{\log_a f(x)} \overset{??}{=} \ln f(x)$$

$$\boxed{\log_a f(x)} \ln a \overset{??}{=} \ln f(x)$$

$$\frac{\ln f(x)}{\cancel{\ln a}} \cancel{\ln a} = \ln f(x)$$

$$\ln f(x) = \ln f(x)$$

$$\ln f(x) = \ln f(x)$$

Exp 2 If $\log_2^{4x} = 8$ Find x

$$2^8 = 4x$$

$$x = \frac{2^8}{4} = 64$$

$$\frac{\ln 4x}{\ln 2} = 8 \Rightarrow \ln 4x = 8 \ln 2$$

$$\ln (4x) = \ln 2^8$$

$$4x = 2^8$$

$$4x = 2^4 \, 2^4$$

$$x = \frac{\cancel{16}^{4} \cdot 16}{\cancel{4}_{1}}$$

$$\boxed{x = 64}$$

$$\log_2^{4x} = 8$$

$$2^{\log_2^{4x}} = 2^8$$

$$4x = 2^8 \Rightarrow x = \frac{2^8}{4} = 64$$

(26) Find y' if $\ln(xy) = e^{x+y}$

$$y' = \frac{dy}{dx}$$

$$\ln x + \ln y = e^{x+y}$$

$y = \frac{du}{dx}$ $\ln x + \ln y = C$

$$\frac{1}{x} + \frac{y'}{y} = e^{x+y}(1+y')\ln e$$

$\ln e = 1$

$$\frac{1}{x} + \frac{y'}{y} = e^{x+y}(1+y')$$

$$\frac{1}{x} + \frac{y'}{y} = e^{x+y} + y'e^{x+y}$$

$$y'\left[\frac{1}{y} - e^{x+y}\right] = e^{x+y} - \frac{1}{x}$$

$$y' = \frac{e^{x+y} - \frac{1}{x}}{\frac{1}{y} - e^{x+y}}$$

42

$$\int \frac{e^{-\frac{1}{x^2}}}{x^3}\,dx$$

$u = -\frac{1}{x^2} = -x^{-2}$

$du = 2x^{-3}\,dx$

$\frac{du}{2} = \frac{dx}{x^3}$

$$\int e^{u}\,\frac{du}{2}$$

$$\int e^u \frac{du}{2}$$

$$\frac{du}{2} = \frac{\;}{x^3}$$

$$\frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{-\frac{1}{x^2}} + C$$

(57) Find $\frac{dy}{dz}$ if $y = 5^{\sqrt{z}}$

$$y' = 5^{\sqrt{z}}\frac{1}{2\sqrt{z}}\ln 5$$

$$= \frac{\ln 5}{2}\,\frac{5^{\sqrt{z}}}{\sqrt{z}}$$

(92) $\int \frac{x\,2^{x^2}}{1+2^{x^2}}\,dx$

$u = 1 + 2^{x^2}$

$$1 + 2^{x^2}$$

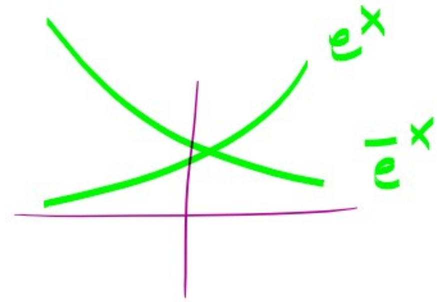
$$du = 0 + 2^{x^2}(2x)\ln 2$$

$$du = 2x\,2^{x^2}\ln 2\,dx$$

$$\frac{du}{2\ln 2} = x\,2^{x^2}\,dx$$

$$\int \frac{\frac{du}{2\ln 2}}{u}$$

$$\frac{1}{\ln 4}\int \frac{du}{u}$$



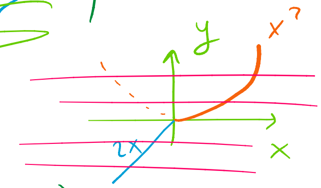
$$\frac{1}{\ln 4}\ln|u| + C$$

$$\frac{1}{\ln 4}\ln\left|1 + 2^{x^2}\right| + C$$

$$\frac{1}{\ln 4}\ln\left(1 + 2^{x^2}\right) + C$$

Exp

1s $f(x) = \begin{cases} 2x & , \quad x < 0 \\ x^2 & , \quad x \geq 0 \end{cases}$



1-1 ?? Yes

$f(x_1) = f(x_2)$

For $x_1, x_2 \in D_1(f) = (-\infty, 0)$

$\cancel{2}x_1 = \cancel{2}x_2$

$x_1 = x_2$

$x_1, x_2 \in D_2(f) = [0, \infty)$

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$\sqrt{x_1^2} = \sqrt{x_2^2}$

فترة موجبة

$|x_1| = |x_2|$

$$\boxed{x_1 = x_2}$$