**Birzeit University**
**Mathematics Department**
# Math234
**Short Exam (KEY)**

**Instructor: Dr. Ala Talahmeh** **First Semester 2021/2022**
**Name:**............................................... **Number:**...............
**Section: 2** **Date: 10/11/2021**

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**Exercise#1 [15 marks]. True or False?**

1. (True) If $\det(A) = 1$, then $A^{-1} = \text{adj}(A)$.
2. (False) If $AB$ is equal to the identity matrix, then $A$ must be invertible matrix.
3. (True) If $B = A^T A$, then $3B$ is symmetric.
4. (False) Three elementary row operations do not change the determinant of a square matrix.
5. (True) If $A$ is row equivalent to $B$, then $|A|$ and $|B|$ are either both zero or both nonzero.
6. (False) Let $A$ be a square matrix without zero rows and columns. Then $A$ must be row equivalent to the identity matrix of the same size.
7. (True) Let $A$ be an $n \times n$ matrix. If the system $Ax = b$ has a unique solution for a given nonzero $b \in \mathbb{R}^n$, then $|A| \neq 0$.
8. (False) Let $A$ be a square and nonsingular $n \times n$ matrix. If $|\text{adj}(A)| = |A|$, then $A$ is $2 \times 2$ matrix.
9. (True) If $A$ is an $m \times n$ matrix, then the diagonal entries of $AA^T$ are nonnegative.
10. (True) If $A$ is a $3 \times 3$ nonsingular matrix with $\det(A) = 3$, then $\det(3A^{-1}) = 9$.
11. (True) If $A$ and $B$ are $n \times n$ symmetric matrices, then the matrix $AB + BA$ is symmetric.
12. (False) If $E_1$ and $E_2$ are elementary $n \times n$ matrices, then $E_1 E_2$ is elementary.
13. (False) Cramer's Rule can be used to solve any system of linear equations.
14. (False)) If $A$ is a nonsingular skew-symmetric matrix, then $A^{-1}$ is symmetric.
15. (True) If $A$ and $B$ are $n \times n$ invertible matrices, then $\text{adj}(AB) = \text{adj}(B)\text{adj}(A)$.

**Exercise 2 [12 marks]. Circle the correct answer.**

(1) If the system of linear equations whose augmented matrix is $[A|b] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 3 & 4 & 3 \\ 1 & -k & 2 \end{array}\right]$ is consistent, then the value of the constant $k$ is

**(a)** $\frac{1}{3}$

**(b)** $\frac{2}{3}$

**(c)** 1

**(d)** $-2$

**(e)** $-1$

(2) If the matrix $A = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & -2 & 2 \end{array}\right]$ is the augmented matrix of a linear system whose solution is $(a, b, c, d)$, then $a + b + c + d =$

**(a)** 0

**(b)** $-4$

**(c)** 4

**(d)** 3

**(e)** $-3$

(3) If $A$ is a $3 \times 3$ square matrix with $|A| = 4$, and the matrix $B$ is obtained from the matrix $A$ by interchanging the first and the last rows, then the value of $|2A| + 8|B^{-1}|$ is equal to

**(a)** 30

**(b)** 10

**(c)** 34

**(d)** 6

**(e)** 68

(4) If $a_{ij} = i + j$, then $A = (a_{ij})_{3\times4}$ is:

**(a)** $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{bmatrix}$

**(b)** $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

**(c)** $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{bmatrix}$

**(d)** $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

**(e)** None

(5) If $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{p\times q}$ and $AB = BA$, then

**(a)** $n = p$

**(b)** $n = p$, $m = q$

**(c)** $m = n = p = q$

**(d)** $m = q$

**(e)** None

(6) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then the value of $A^5$ is

**(a)** $5A$

**(b)** $16A$

**(c)** $10A$

**(d)** $32A$

**(e)** None

**Good Luck**

**Birzeit University**
**Mathematics Department**
# Math234
**Short Exam Form A (KEY)**

**Instructor: Dr. Ala Talahmeh** **First Semester 2021/2022**
**Name:**............................................... **Number:**...............
**Section: 5** **Date: 11/11/2021**

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**Exercise#1 [15 marks]. True or False?**

1. (False) The product of two triangular matrices is triangular.
2. (False) The system $x = y,\ y = z,\ x = z$ is inconsistent.
3. (True) If a system of linear equations has two solutions, then it has infinitely many solutions.
4. (False) A row reduced matrix always has a 1 in the second column of the second row.
5. (False) If $A^2 \neq O$, then $A$ is invertible.
6. (True) If $A$ is a square matrix and $A^2 + 8A - I = O$, then $A$ is invertible and $A^{-1} = A + 8I$.
7. (False) If $A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 0 & 3 \\ 4 & 0 & 2 \end{bmatrix}$, then the entry $(3, 2)$ of $\operatorname{adj}(A)$ equals $-8$.
8. (True) If $A^T A = A$, then $A^2 = A$.
9. (True) Let $A$ be an $m \times n$ matrix such that $n \geq 3$. Suppose that $b = a_2 - a_3 = a_1 + 2a_2$. Then the system $Ax = b$ has infinitely many solutions.
10. (True) If $u$ and $v$ are both solutions to $Ax = 0$, then $w = (2021)u + (1443)v$ is a solution to $Ax = 0$.
11. (True) Let $A$ and $B$ be $3 \times 3$ matrices with $\det(A) = x$ and $\det(B) = y$. Let $E$ be a $3 \times 3$ elementary matrix of type I. Then $\det(EAB^T) = -xy$.
12. (True) Every diagonal matrix with nonzero diagonal entries is invertible.
13. (True) If $A$ and $B$ are square matrices. If $AB = I$, then $BA = I$. Hence, $A$ is nonsingular.
14. (False) If $A$ is row equivalent to $B$, then $A = B$.
15. (True) If $A$ is a $3 \times 3$ skew-symmetric matrix, then $A$ is singular.

**Exercise 2 [12 marks]. Circle the correct answer.**

(1) The value of $\alpha$ for which the system with augmented matrix $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & \alpha+1 & \alpha^2+3\alpha-4 \end{array}\right]$ has infinitely many solution is:

**(a)** $\alpha = -4$

**(b)** $\alpha = -1$

**(c)** $\alpha = 1$

**(d)** $\alpha = 0$

**(e)** does not exist

(2) The sum of all elements of the inverse matrix of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

**(a)** $\frac{4}{3}$

**(b)** $\frac{2}{3}$

**(c)** $\frac{5}{3}$

**(d)** $-\frac{2}{3}$

**(e)** $-\frac{1}{3}$

(3) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = 3$, then $\begin{vmatrix} 2d & 2e & 2f \\ -2a & -2b & -2c \\ 2g+2a & 2h+2b & 2k+2c \end{vmatrix} =$

**(a)** 24

**(b)** 48

**(c)** $-6$

**(d)** 12

**(e)** $-12$

(4) If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then $(a,b) =$

**(a)** $(2,2)$, $(1,1)$

**(b)** $(2,4)$, $(4,2)$

**(c)** $(3,3)$, $(3,4)$

**(d)** $(2,3)$, $(1,4)$

**(e)** None

(5) If $A = \begin{bmatrix} \alpha\beta & \beta^2 \\ -\alpha^2 & -\alpha\beta \end{bmatrix}$, then the value of $A^{2021}$ is

**(a)** $-I$

**(b)** $I$

**(c)** $O$

**(d)** $(2021)I$

**(e)** None

(6) If the system of linear equations whose augmented matrix is $[A|b] = \left[\begin{array}{cc|c} 1 & h & 3 \\ 5 & -10 & k \end{array}\right]$ is inconsistent, then the value of the constants $h$ and $k$ must be

**(a)** $h = -2,\ k = 15$

**(b)** $h \neq -2,\ k \neq 15$

**(c)** $h \neq -2,\ k = 15$

**(d)** $h = -2,\ k \neq 15$

**(e)** $h = 2,\ k = 15$

**Good Luck**

**Birzeit University**
**Mathematics Department**
# Math234
**Short Exam Form B (KEY)**

**Instructor: Dr. Ala Talahmeh** **First Semester 2021/2022**
**Name:**............................................... **Number:**...............
**Section: 5** **Date: 11/11/2021**

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**Exercise#1 [15 marks]. True or False?**

1. (True) If a system of linear equations has two solutions, then it has infinitely many solutions.

2. (True) The system $x = y,\ y = z,\ x = z$ is consistent.

3. (False) The product of two triangular matrices is triangular.

4. (False) A row reduced matrix always has a 1 in the second column of the second row.

5. (False) If $A^T A = A$, then $A^2 = I$.

6. (True) If $u$ and $v$ are both solutions to $Ax = b$, then $w = \frac{1}{4}u + \frac{3}{4}v$ is a solution to $Ax = b$.

7. (True) If $A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 0 & 3 \\ 4 & 0 & 2 \end{bmatrix}$, then the entry $(3, 2)$ of $\mathrm{adj}(A)$ equals 8.

8. (False)If $A^2 \neq O$, then $A$ is invertible.

9. (True)) Let $A$ be an $m \times n$ matrix such that $n \geq 3$. Suppose that $b = a_2 - a_3 = a_1 + 2a_2$. Then the system $Ax = b$ has infinitely many solutions.

10. (False) If $A$ is row equivalent to $B$, then $A = B$.

11. (False) Let $A$ and $B$ be $3 \times 3$ matrices with $\det(A) = x$ and $\det(B) = y$. Let $E$ be a $3 \times 3$ elementary matrix of type III. Then $\det(EAB^T) = -xy$.

12. (False) Every diagonal matrix with nonzero diagonal entries is singular.

13. (True) If $A$ and $B$ are square matrices. If $AB = I$, then $BA = I$. Hence, $A$ is nonsingular.

14. (True) If $A$ is a square matrix and $A^2 + 8A - I = O$, then $A$ is invertible and $A^{-1} = A + 8I$.

15. (False) If $A$ is a $3 \times 3$ skew-symmetric matrix, then $A$ is nonsingular.

**Exercise 2 [12 marks]. Circle the correct answer.**

(1) If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then $(a, b) =$

- **(a)** $(2, 2)$, $(1, 1)$
- **(b)** $(2, 3)$, $(1, 4)$
- **(c)** $(3, 3)$, $(3, 4)$
- **(d)** $(2, 4)$, $(4, 2)$
- **(e)** None

(2) If $A = \begin{bmatrix} \alpha\beta & \beta^2 \\ -\alpha^2 & -\alpha\beta \end{bmatrix}$, then the value of $A^{2021} + I$ is

- **(a)** $-I$
- **(b)** $I$
- **(c)** $O$
- **(d)** $(2021)I$
- **(e)** None

(3) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = 6$, then $\begin{vmatrix} 2d & 2e & 2f \\ -2a & -2b & -2c \\ 2g+2a & 2h+2b & 2k+2c \end{vmatrix} =$

**(a)** 24

**(b)** 48

**(c)** $-6$

**(d)** 12

**(e)** $-12$

(4) The value of $\alpha$ for which the system with augmented matrix $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & \alpha+1 & \alpha^2+3\alpha-4 \end{array}\right]$ has no solution is:

**(a)** $\alpha = -4$

**(b)** $\alpha = 0$

**(c)** $\alpha = 1$

**(d)** $\alpha = -1$

**(e)** does not exist

(5) The sum of all elements of the inverse matrix of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

**(a)** $\frac{4}{3}$

**(b)** $\frac{2}{3}$

**(c)** $\frac{5}{3}$

**(d)** $-\frac{2}{3}$

**(e)** $-\frac{1}{3}$

(6) If the system of linear equations whose augmented matrix is $[A|b] = \left[\begin{array}{cc|c} 1 & h & 3 \\ 5 & -10 & k \end{array}\right]$ has infinitely many solutions, then the value of the constants $h$ and $k$ must be

**(a)** $h = -2,\ k = 15$

**(b)** $h \neq -2,\ k \neq 15$

**(c)** $h \neq -2,\ k = 15$

**(d)** $h = -2,\ k \neq 15$

**(e)** $h = 2,\ k = 15$

**Good Luck**