Chapter 14
Introduction to Frequency Selective Circuits

14.1 Some preliminaries

14.2 Low-pass filters

14.3 High-pass filters

14.4 Bandpass filters

14.5 Bandreject filters

Overview

- We have seen that the response of a circuit depends on the types of elements, the way the elements are connected, and the impedance of the elements that varies with frequency.

- In this chapter, we analyze the effect of varying source frequency on circuit voltages and currents. In particular, the circuits made of passive elements (R, L, C) that pass only a finite range of input frequencies.

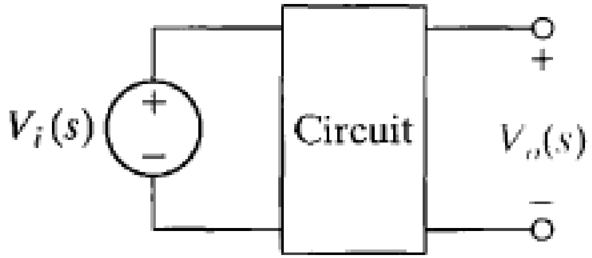
Frequency response plot / **Bode Plot**

- The steady-state response due to a sinusoidal input $A\cos\omega t$ is determined by sampling the transfer function $H(s)$ along the imaginary axis, i.e. $H(j\omega)$.

- A frequency response plot consists of two parts: (1) magnitude plot $|H(j\omega)|$, (2) phase angle plot $\theta(j\omega)$

Filters:

Circuits that pass signals at certain frequencies and block (Reject or attenuate) signals of other frequencies



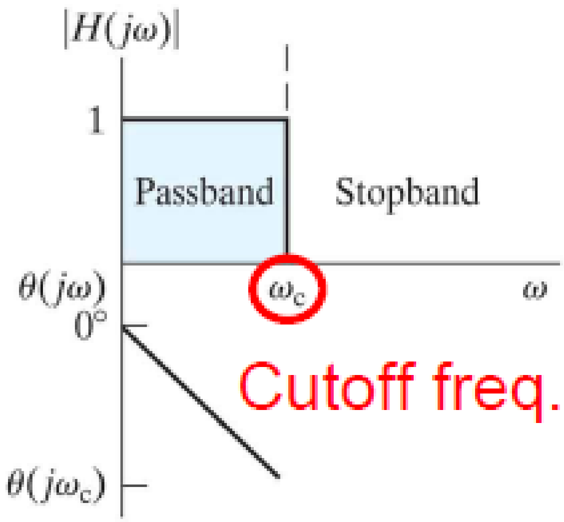
If

$$V_i(t) = V_m Sin(\omega t + \phi)$$

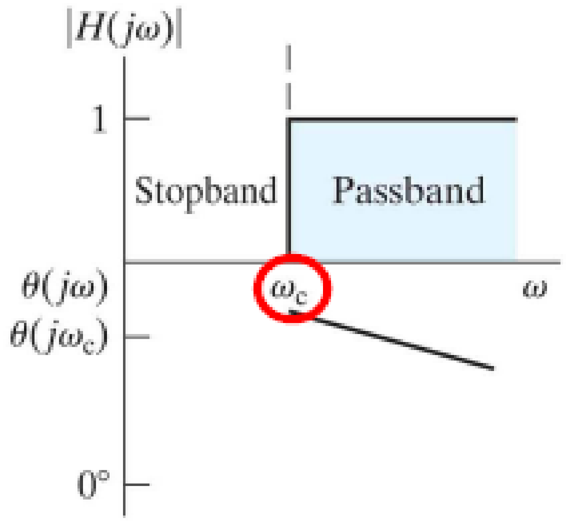
$$V_o(t) = V_m \left|H(j\omega)\right| Sin(\omega t + \phi + \theta_H)$$

Four types of ideal filters

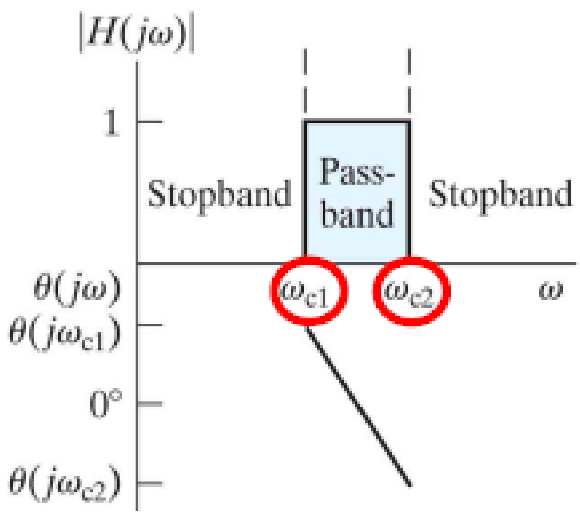
LPF



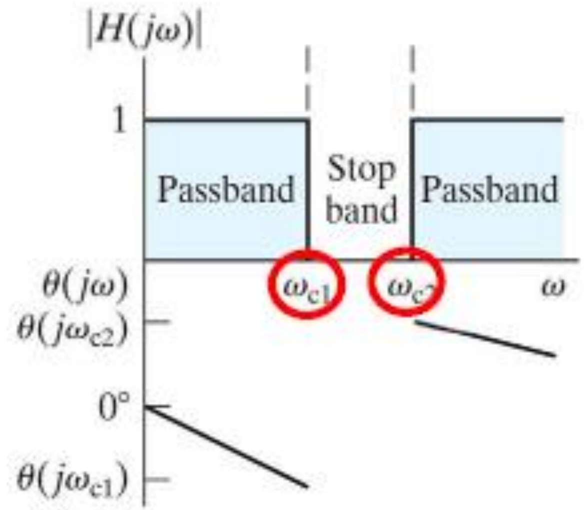
HPF



BPF



BRF

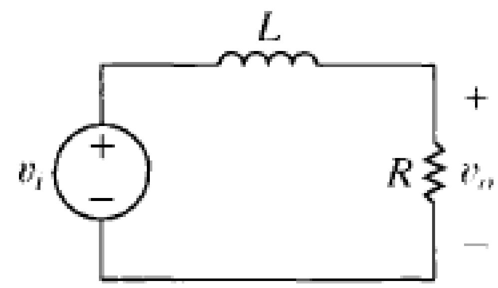


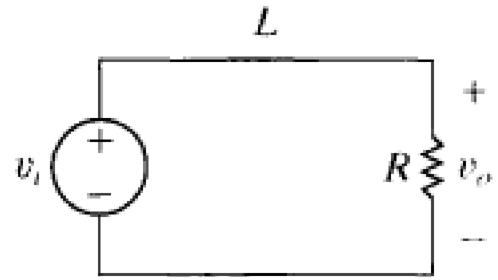
Low Pass Filter (LPF)

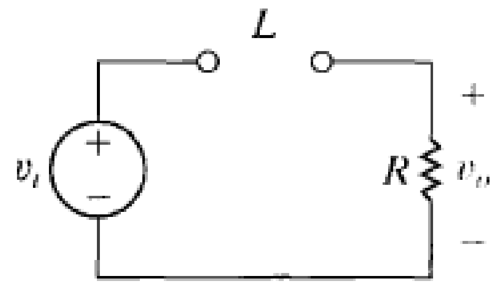
- Series RL Filter
- Qualitative analysis:

$$\text{at } \omega = 0 \ \text{(DC)} \ , X_L = \omega L = 2\pi f L = 0$$
$$\rightarrow \text{L short}$$
$$\text{Vo} = \text{Vi}$$

$$\text{at } \omega = \infty \ , X_L = \omega L = 2\pi f L = \infty$$
$$\rightarrow \text{L open}$$
$$\rightarrow \text{Vo} = 0$$

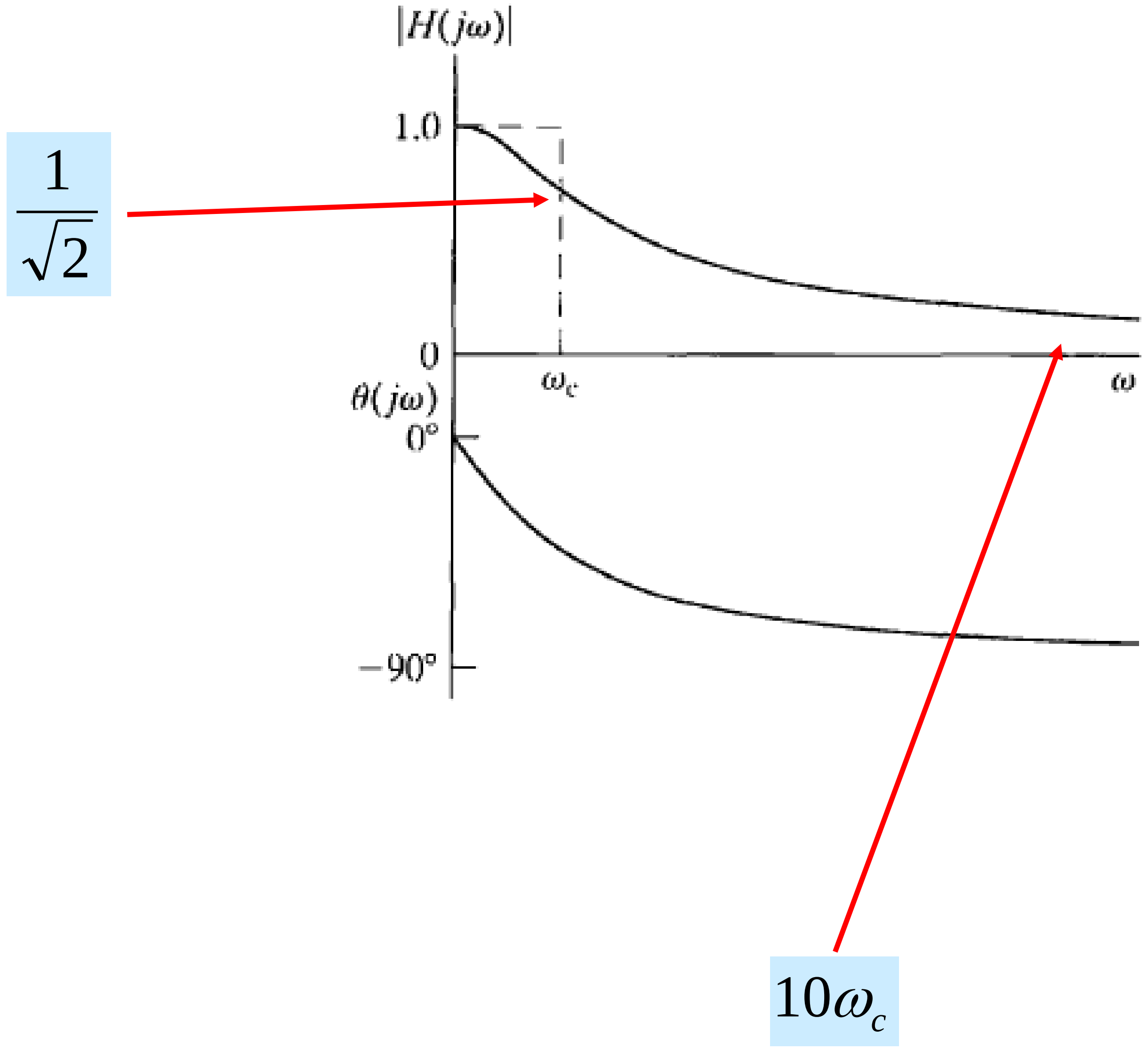






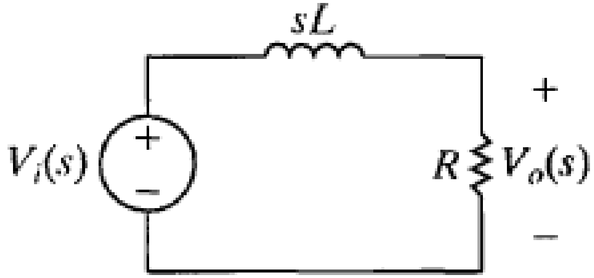
Cut-off frequency

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max},$$



Series RL Filter (LPF)

- Quantitative analysis:



Voltage Transfer Function

$$H(s) = \frac{R/L}{s + R/L};$$

$$s = j\omega$$

$$H(j\omega) = \frac{R/L}{j\omega + R/L}$$

$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}}$$

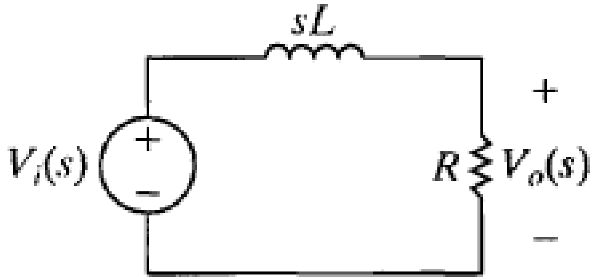
$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

When $\omega = 0$, the denominator and the numerator are equal and $|H(j0)| = 1$. This means that at $\omega = 0$, the input voltage is passed to the output terminals without a change in the voltage magnitude.

$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Series RL Filter (LPF)



$$\omega_C = \frac{1}{\tau} = \frac{1}{L/R} = R/L$$

$$H(s) = \frac{\omega_C}{j\omega + \omega_C} = \frac{1}{1 + j\dfrac{\omega}{\omega_C}};$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_C}\right)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

Series RL Filter (LPF)

$$\text{For } \omega = 0 \rightarrow \left|H(j\omega)\right| = \frac{1}{\sqrt{1 + \left(\frac{0}{\omega_C}\right)^2}} = 1 \qquad \text{(max)}$$

$$\text{For } \omega = \omega_C \rightarrow \left|H(j\omega_C)\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega_C}{\omega_C}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\text{For } \omega = 10\omega_C \rightarrow \left|H(j\omega_C)\right| = \frac{1}{\sqrt{1 + \left(\frac{10\omega_C}{\omega_C}\right)^2}} = \frac{1}{10}$$

$$\text{For } \omega = \infty \rightarrow \left|H(j\infty)\right| = \frac{1}{\sqrt{1 + \left(\frac{\infty}{\omega_C}\right)^2}} = 0 \qquad \text{(min)}$$

Example

- Choose values for *R* and *L* in the circuit of the RL filter such that the resulting circuit could be used in an electrocardiograph to filter out any noise above 10 Hz and pass the electric signals from the heart at or near 1 Hz.
- Then compute the magnitude of *Vo* at 1 Hz, 10 Hz, and 60 Hz to see how well the filter performs.

$$\omega_C = \frac{1}{\tau} = \frac{1}{L/R} = R/L$$

$$H(s) = \frac{\omega_C}{j\omega + \omega_C} = \frac{1}{1 + j\dfrac{\omega}{\omega_C}};$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_C}\right)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

Example

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_C}\right)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

$$\omega_C = 2\pi f_C = \frac{1}{\tau} = \frac{1}{L/R} = R/L$$

$$f_C = 10\ \text{Hz}$$

$$\text{choose}\ \ L = 100\ \text{mH}$$

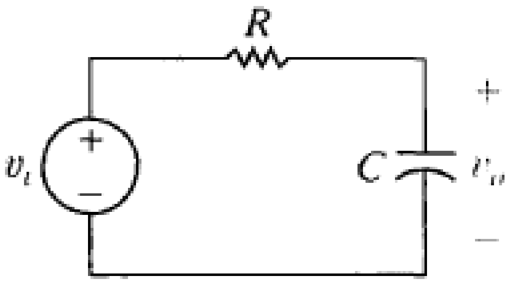
$$\therefore\ \ R = 2\pi f_C L = 2\pi x 10 x 100\ \text{mH} = 6.28\ \Omega$$

$$\left|V_o\right| = \left|H(j\omega)\right| . \left|V_i\right|$$

$$\left|V_o(\omega)\right| = \frac{2\pi(10)}{\sqrt{\omega^2 + (20\pi)^2}} . \left|V_i\right|$$

Vi [V]	f [Hz]	Vo [V]
1	1	0.995
1	10	0.707
1	60	0.164

Series RC Filter (LPF)

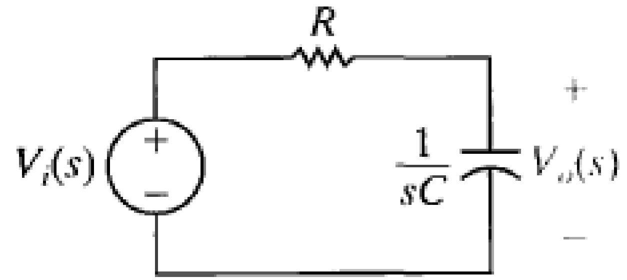


- Qualitative analysis:

$$\text{at } \omega = 0 \ \text{(DC)} \ , X_C = \frac{1}{\omega C} = \infty$$
$$\rightarrow \text{C open}$$
$$\text{Vo} = \text{Vi}$$

$$\text{at } \omega = \infty \ , X_C = \frac{1}{\omega C} = 0$$
$$\rightarrow \text{C short}$$
$$\text{Vo} = 0$$

- Quantitative analysis:



Voltage Transfer Function

$$H(s) = \frac{1/sC}{1/sC + R}$$

$$H(s) = \frac{1/RC}{s + 1/RC};$$

$$s = j\omega$$

$$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}$$

$$\omega_C = 1/\tau = 1/RC$$

$$H(j\omega) = \frac{\omega_C}{j\omega + \omega_C} = \frac{1}{1 + j\dfrac{\omega}{\omega_C}}$$

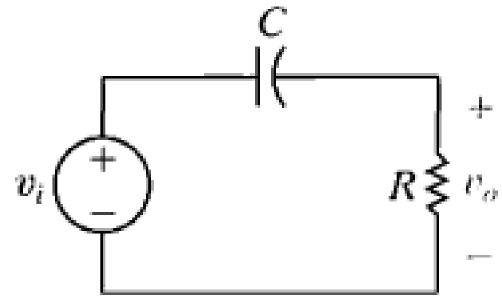
$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_C}\right)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

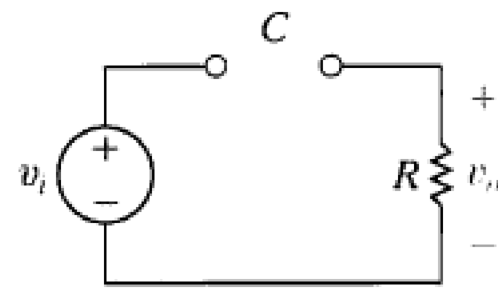
Same as RL circuit

High Pass Filter (HPF)

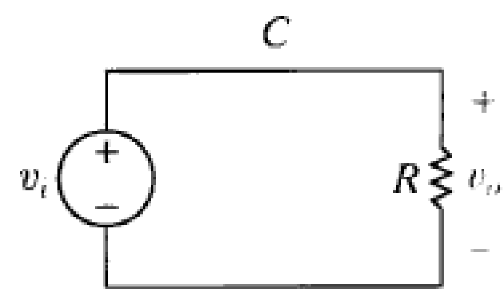
- Series RC Filter
- Qualitative analysis:



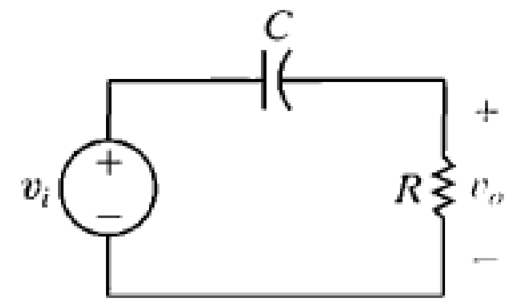
$$\text{at } \omega = 0 \ \text{(DC)} \ , X_C = 1/\omega C = \infty$$
$$\rightarrow \text{C open}$$
$$Vo = 0$$



$$\text{at } \omega = \infty \ , X_C = 1/\omega C = 0$$
$$\rightarrow \text{C short}$$
$$Vo = Vi$$



- Quantitative analysis:



Voltage Transfer Function

$$H(s)=\frac{R}{R+1/sC}$$

$$=\frac{s}{s+1/RC};$$

$$s=j\omega$$

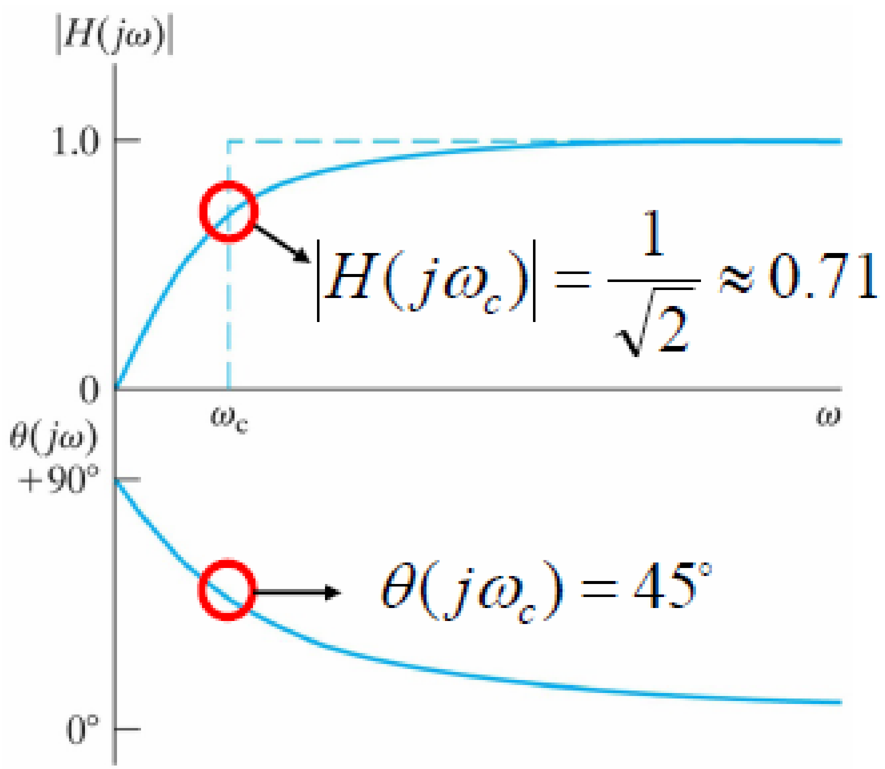
$$H(j\omega)=\frac{j\omega}{j\omega+\omega_C}$$

$$\omega_C=1/\tau=1/RC$$

$$H(j\omega)=\frac{j\omega}{j\omega+\omega_C}=\frac{1}{1+\frac{\omega_C}{j\omega}}$$

$$|H(j\omega)|=\frac{1}{\sqrt{1+\left(\frac{\omega_C}{\omega}\right)^2}}$$

$$\theta(j\omega)=90-\tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$



|H(jω)|
1.0
0
ω_c
ω
θ(jω)
+90°
0°
$|H(j\omega_c)| = \frac{1}{\sqrt{2}} \approx 0.71$
$\theta(j\omega_c) = 45^\circ$

Series RL Filter (HPF)

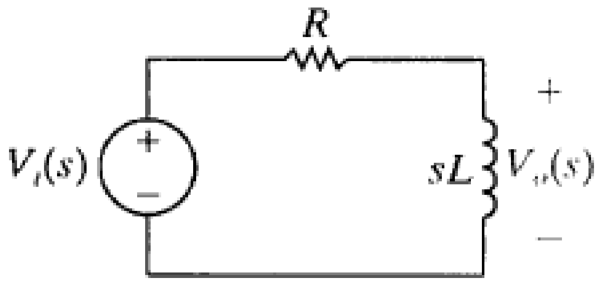
- Quantitative analysis:

Voltage Transfer Function

$$H(s) = \frac{sL}{sL + R} = \frac{s}{s + R/L};$$

$$s = j\omega$$

$$H(j\omega) = \frac{j\omega}{j\omega + \omega_C}$$



$$\omega_C = 1/\tau = R/L$$

$$H(j\omega) = \frac{j\omega}{j\omega + \omega_C} = \frac{1}{1 + \dfrac{\omega_C}{j\omega}}$$

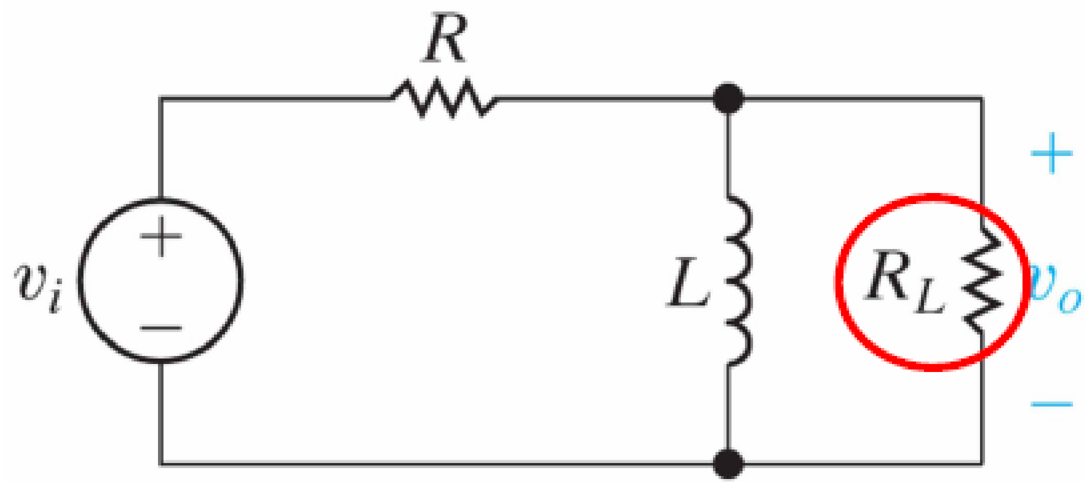
$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\dfrac{\omega_C}{\omega}\right)^2}}$$

$$\theta(j\omega) = 90 - \tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

Same as RC circuit

Example 14.4: RL circuit with load (1)

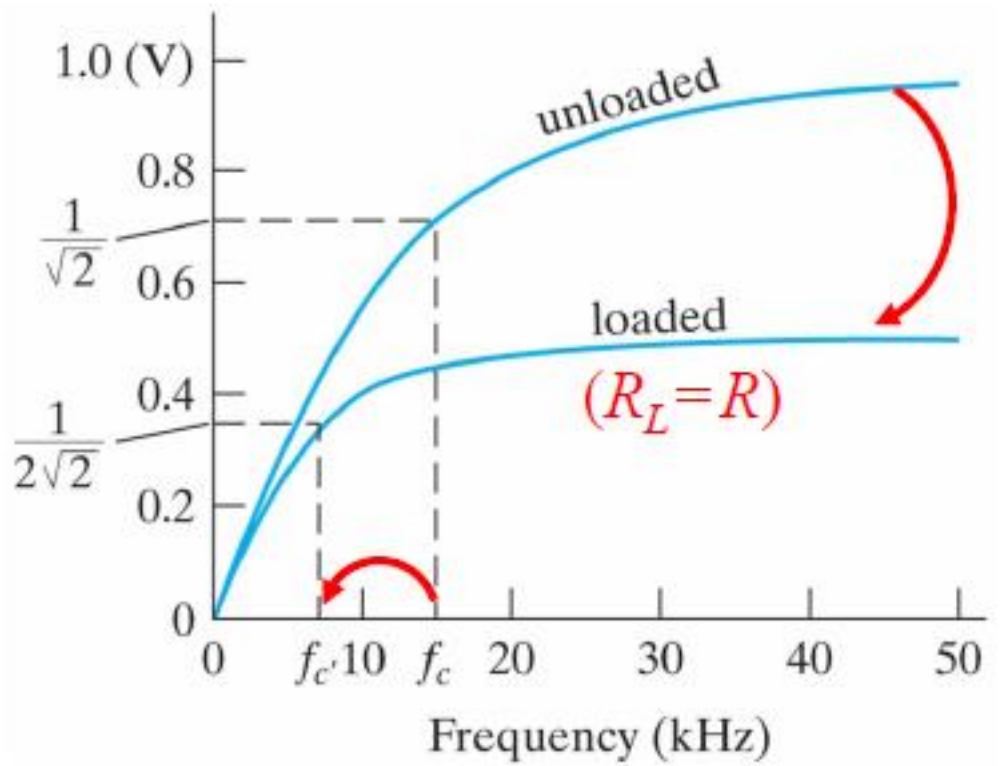
- An RL circuit can be an HPF if $v_o \equiv v_L$.
- When a load resistor R_L is in parallel with the output inductor L:



$$\Rightarrow H(s) = \frac{Ks}{s + K\omega_c}, \text{ where } \omega_c = \frac{R}{L}, \ K = \frac{1}{1 + (R/R_L)} < 1.$$

Example 14.4: RL circuit with load (2)

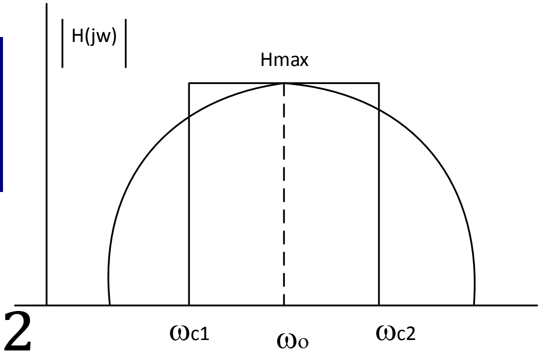
- The effects of R_L are: (1) reducing the passband magnitude by a factor of K, (2) lowering the cutoff frequency by the same factor of K.



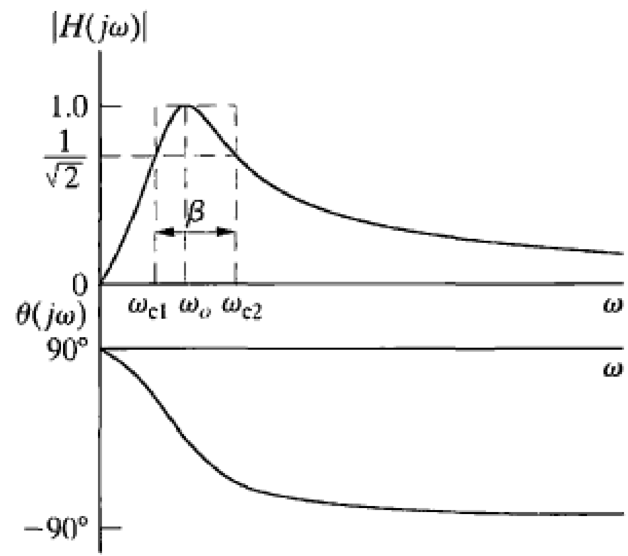
- Problem: the response varies with the load.

  Solution: use active filters

Bandpass Filters (BPFs)



- Two cut-off frequencies : ω_{C1} & ω_{C2}
- Center frequency (Resonant Frequency): ω_o
- ω_o is the center of pass-band
- $\omega_o = \sqrt{\omega_{c1}\omega_{c2}}$
- $H_{max} = |H(j\omega_o)|$
- Bandwidth $\beta = \omega_{c2} - \omega_{c1}$
- Quality factor $Q = \frac{\omega_o}{\beta}$

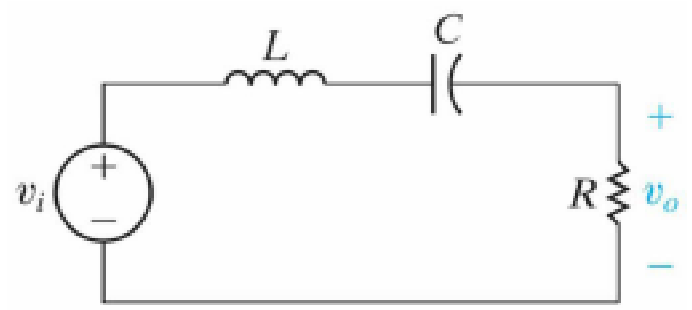


- Only two of these parameters can be specified independently, others are calculated

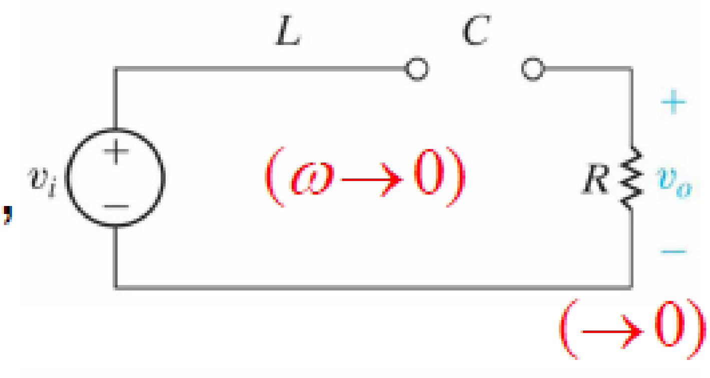
Bandpass Filters (BPFs)

Series RLC circuit

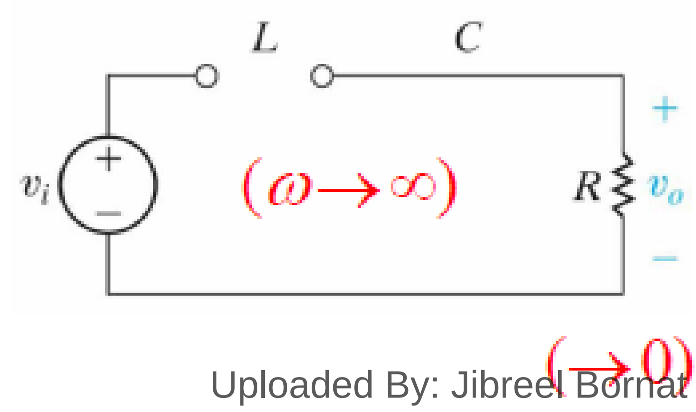
- $Z_c = 1/(j\omega C)$, $Z_L = j\omega L$.



- $Z_c \to \infty$ as $\omega \to 0$, $\Rightarrow v_o \to 0$.
- $Z_L \to \infty$ as $\omega \to \infty$, $\Rightarrow v_o \to 0$.



- At some frequency $\omega_0 \in (0, \infty)$, $Z_c + Z_L = 0$, $v_o = v_i$.
- The input voltage can pass through the circuit if the source frequency is near ω_0.



Voltage Transfer Function

$$H(s) = \frac{R}{R + sL + 1/sC}$$

$$= \frac{R}{\frac{sCR + s^2 LC + 1}{sC}}$$

$$H(s) = \frac{sCR/LC}{sCR/LC + s^2 LC/LC + 1/LC}$$

$$= \frac{s\left(\frac{R}{L}\right)}{s^2 + s\left(\frac{R}{L}\right) + 1/LC}$$

$$s = j\omega$$

$$\left|H(j\omega)\right| = \frac{\omega\left(\frac{R}{L}\right)}{\sqrt{\left[\frac{1}{LC} - \omega^2\right]^2 + \left[\omega\left(\frac{R}{L}\right)\right]^2}}$$

In General for BPF

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}$$

$$at\ \omega = \omega_0$$

$$j\omega_o L + \frac{1}{j\omega_o C} = 0$$

$$\Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$\therefore at\ \omega = \omega_c$$

$$|\mathrm{H}(\mathrm{j}\omega_\mathrm{C})| = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{\omega_C\left(\frac{R}{L}\right)}{\sqrt{\left[1/LC - \omega_C^2\right]^2 + \left[\omega_C\left(R/L\right)\right]^2}}$$

$$= \frac{1}{\sqrt{\left[\omega_C\left(L/R\right) - \left(1/\omega_C RC\right)\right]^2 + 1}}$$

$$\pm 1 = \omega_C\left(L/R\right) - \left(1/\omega_C RC\right)$$

4 possible values of ω_C , only two are positive and have physical meaning

$$\omega_{C1} = -\frac{\mathrm{R}}{2\mathrm{L}} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C2} = \frac{\mathrm{R}}{2\mathrm{L}} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_{C1}\omega_{C2}} = \frac{1}{\sqrt{LC}}$$

$$\beta = \omega_{C1} - \omega_{C2} = \frac{R}{L}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\frac{1}{\sqrt{LC}}}{\frac{R}{L}} = \sqrt{\frac{L}{CR^2}}$$

$$\omega_{C1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o{}^2}$$

$$\omega_{C2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o{}^2}$$

or

$$\omega_{C1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{C2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

Example 14.5

Using the series *RLC* circuit , choose values for *R, L,* and C that yield a bandpass circuit able to select inputs within the 1-10 kHz frequency band. Such a circuit might be used in a graphic equalizer to select this frequency band from the larger audio band (generally 0-20 kHz) prior to amplification.

Solution

We need to compute values for *R*, *L*, and C that produce a bandpass filter with cutoff frequencies of 1 kHz and 10 kHz.

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}, \quad (14.29)$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}. \quad (14.30)$$

Complex

Simple

$$\omega_o = \sqrt{\omega_{C1}\omega_{C2}} = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\frac{1}{\sqrt{LC}}}{\frac{R}{L}} = \sqrt{\frac{L}{CR^2}}$$

Any approach we choose will provide only two equations—insufficient to solve for the three unknowns—because of the dependencies among the bandpass filter characteristics. Thus, we need to select a value for either *R*, L, or C and use the two equations we've chosen to calculate the remaining component values. Here, we choose 1 μF as the capacitor value, because there are stricter limitations on commercially available capacitors than on inductors or resistors.

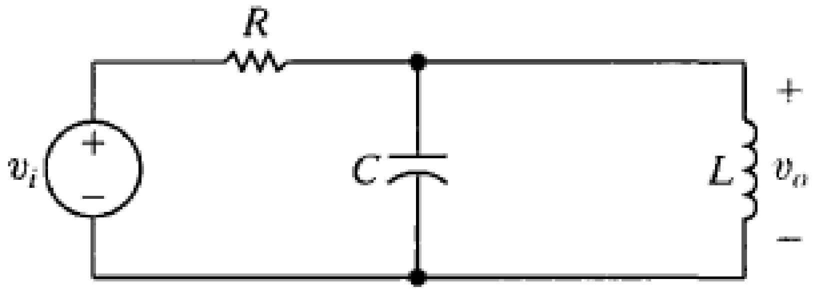
$$f_o = \sqrt{f_{C1} f_{C2}} = \frac{1}{2\pi\sqrt{LC}} = 3162.28\,\text{Hz}.$$

$$\text{L} = \frac{1}{(2\pi f_o)^2 C} = 2.533\,\text{mH}$$

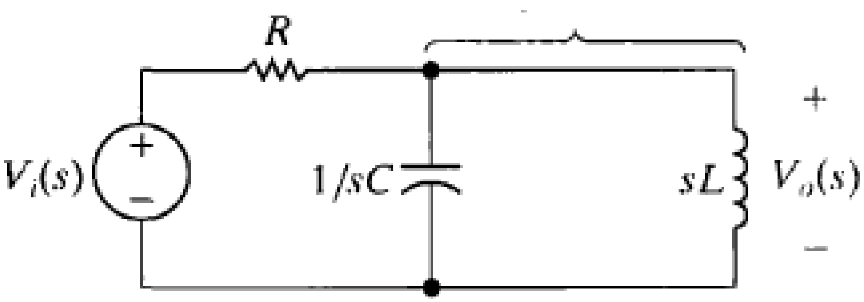
$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{\omega_{C2} - \omega_{C1}} = \frac{f_o}{f_{C2} - f_{C1}} = \sqrt{\frac{L}{CR^2}} = \frac{3126.8}{10000 - 1000} = 0.3514.$$

$$\rightarrow R = \sqrt{\frac{L}{CQ^2}} = 143.2411\,\Omega$$

Parallel RLC



a) Show that the *RLC* circuit is also a bandpass filter by deriving an expression for the transfer function *H(s).*
b) Compute the center frequency,
c) Calculate the cutoff frequencies, the bandwidth, and the quality factor, *Q.*
d) Compute values for *R* and L to yield a bandpass filter with a center frequency of 5 kHz and a bandwidth of 200 Hz, using a 5 *uF* capacitor



Voltage Transfer Function

$$H(s) = \frac{Zeq(s)}{Zeq(s) + R};$$

$$Zeq(s) = \frac{L/C}{\frac{1}{sC} + sL}$$

$$H(s) = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$at\ \omega = \omega_0$$

$$j\omega_o L + \frac{1}{j\omega_o C} = 0$$

$$\Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$\therefore at\ \omega = \omega_c$$

$$\left|H(j\omega_C)\right| = \frac{1}{\sqrt{2}}$$

$$\pm 1 = \omega_C (RC) - \left(1 \Big/ \omega_C \frac{L}{R} \right)$$

4 possible values of ω_C, only two are positive and have physical meaning

$$\omega_{C1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_{C1}\omega_{C2}} = \frac{1}{\sqrt{LC}}$$

$$\beta = \omega_{C2} - \omega_{C1} = \frac{1}{RC}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\frac{1}{\sqrt{LC}}}{\frac{1}{RC}} = \sqrt{\frac{R^2 C}{L}}$$

$$\omega_{C1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^{\ 2}}$$

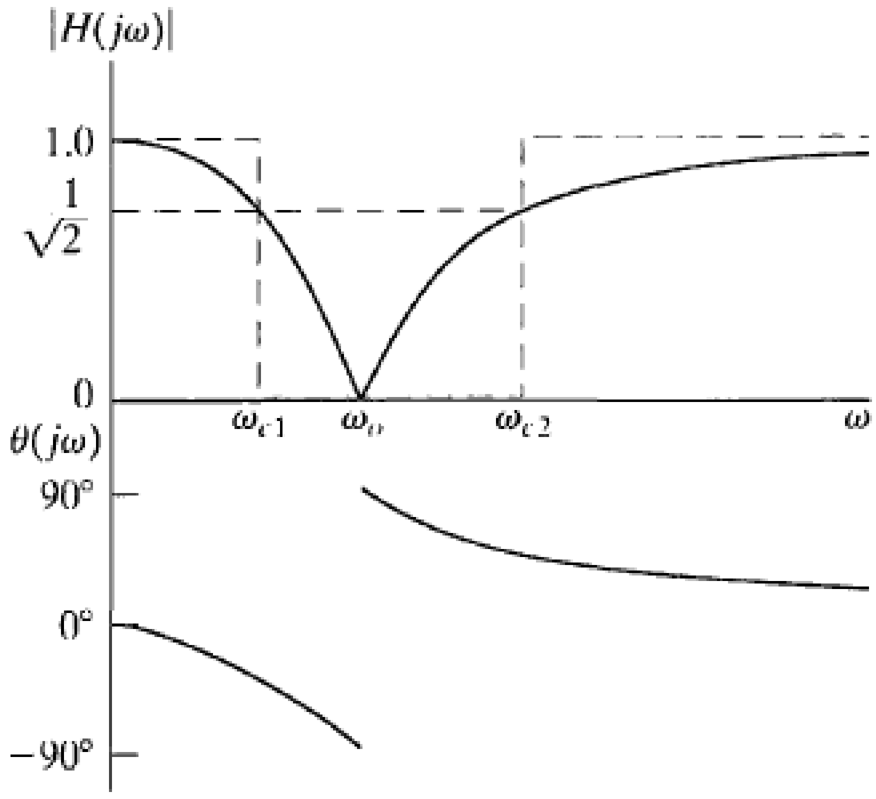
$$\omega_{C2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^{\ 2}}$$

$$Assume\ \mathrm{C} = 5\mu F$$

$$\rightarrow \mathrm{R} = \frac{1}{\beta\,\mathrm{C}} = \frac{1}{(2\pi.200)*5*10^{-6}} = 159.15\,\Omega$$

$$\rightarrow \mathrm{L} = \frac{1}{\omega_o^2 C} = \frac{1}{(2\pi*5000)^2*5*10^{-6}} = 202.64\ \mu H$$

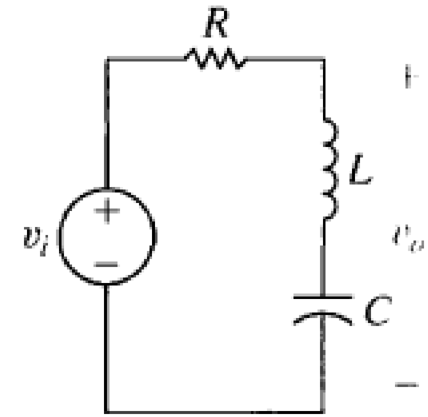
Band Reject Filter

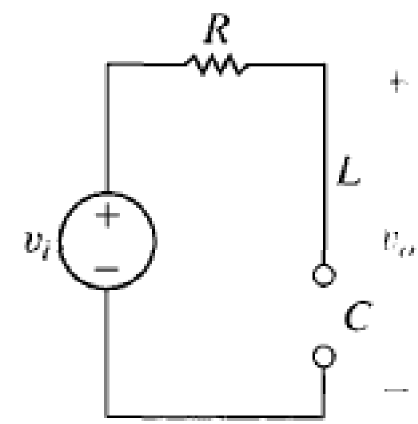


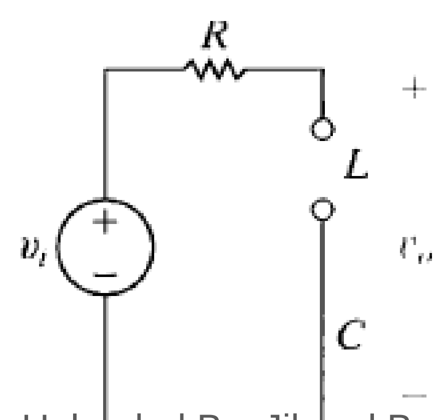
Band Reject Filter

Series RLC circuit

- $Z_c = 1/(j\omega C)$, $Z_L = j\omega L$.
- $Z_c \to \infty$ as $\omega \to 0$, $\Rightarrow$ $v_o \to Vi$
- $Z_L \to \infty$ as $\omega \to \infty$, $\Rightarrow$ $v_o \to Vi$
- At some frequency $\omega_0 \in (0,\infty)$,
  $Z_c + Z_L = 0$, $v_o \to 0$







Voltage Transfer Function

$$H(s) = \frac{sL + 1/sC}{R + sL + 1/sC}$$

$$= \frac{s^2 + \left(\frac{1}{L}\right)}{s^2 + s\left(\frac{R}{L}\right) + 1/LC}$$

In general for BRF

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}.$$

$$s = j\omega$$

$$\left|H(j\omega)\right| = \frac{\left|\frac{1}{LC} - \omega^2\right|}{\sqrt{\left[\frac{1}{LC} - \omega^2\right]^2 + \left[\omega\left(\frac{R}{L}\right)\right]^2}}$$

$$at\ \omega = \omega_0$$

$$j\omega_o L + \frac{1}{j\omega_o C} = 0$$

$$\Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$\therefore at\ \omega = \omega_c$$

$$\left|H(j\omega_C)\right| = \frac{1}{\sqrt{2}}$$

Same as series BPF

4 possible values of ω_C, only two are positive and have physical meaning

$$\omega_{C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_{C1}\omega_{C2}} = \frac{1}{\sqrt{LC}}$$

$$\beta = \omega_{C1} - \omega_{C2} = \frac{R}{L}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\frac{1}{\sqrt{LC}}}{\frac{R}{L}} = \sqrt{\frac{L}{CR^2}}$$

$$\omega_{C1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^{\ 2}}$$

$$\omega_{C2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^{\ 2}}$$

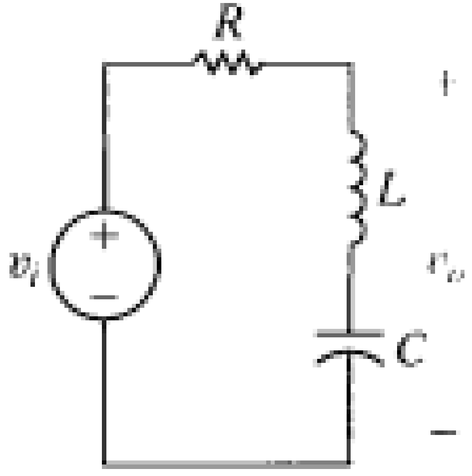
or

$$\omega_{C1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{C2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

Example 14.8 Series RLC BRF

- Using the series *RLC* circuit, compute the component values that yield a bandreject filter with a bandwidth of 250 Hz and a center frequency of 750 Hz. Use a 100 nF capacitor. Compute values for *R*, *L*, ω_{C1}, ω_{C2} and *Q*.



$$Q = \omega_o/\beta = 3.$$

$$L = \frac{1}{\omega_o^2 C}$$

$$= \frac{1}{[2\pi(750)]^2(100 \times 10^{-9})}$$

$$= 450 \text{ mH}.$$

$$R = \beta L$$

$$= 2\pi(250)(450 \times 10^{-3})$$

$$= 707\ \Omega.$$

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$= 3992.0 \text{ rad/s},$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$= 5562.8 \text{ rad/s}$$

The cutoff frequencies are at 635.3 Hz and 885.3 Hz. Their difference is $885.3 - 635.3 = 250$ Hz, confirming the specified bandwidth. The geometric mean is $\sqrt{(635.3)(885.3)} = 750$ Hz, confirming the specified center frequency.

Parallel RLC BRF

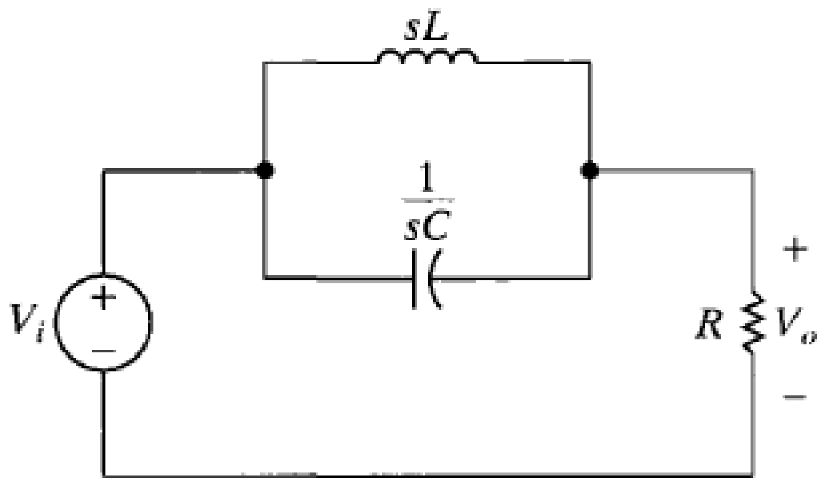
Voltage Transfer Function

$$H(s) = \frac{R}{R + sL // 1/sC}$$

$$= \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + s\left(\frac{1}{RC}\right) + 1/LC}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\beta = \frac{1}{RC}$$



$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

$$\omega_{C1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^{\ 2}}$$

$$\omega_{C2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^{\ 2}}$$

14.2 Use a 1 mH inductor to design a low-pass, *RL*, passive filter with a cutoff frequency of 5 kHz.

DESIGN PROBLEM PSPICE MULTISIM

a) Specify the value of the resistor.

b) A load having a resistance of 68 Ω is connected across the output terminals of the filter. What is the corner, or cutoff, frequency of the loaded filter in hertz?

c) If you must use a single resistor from Appendix H for part (a), what resistor should you use? What is the resulting cutoff frequency of the filter?

[a] $$\frac{R}{L} = 10{,}000\pi \text{ rad/s}$$

$$R = (0.001)(10{,}000)(\pi) = 31.42\,\Omega$$

[b] $$R_e = 31.42\|68 = 21.49\,\Omega$$

$$\omega_{\text{loaded}} = \frac{R_e}{L} = 21{,}488.34 \text{ rad/s}$$

$$\therefore \quad f_{\text{loaded}} = 3419.98\,\text{Hz}$$

[c] The 33 Ω resistor in Appendix H is closest to the desired value of 31.42 Ω. Therefore,

$$\omega_c = 33\,\text{krad/s} \quad \text{so} \quad f_c = 5252.11\,\text{Hz}$$

14.5 Use a 500 nF capacitor to design a low-pass passive filter with a cutoff frequency of 50 krad/s.

DESIGN PROBLEM
PSPICE
MULTISIM

a) Specify the cutoff frequency in hertz.

b) Specify the value of the filter resistor.

c) Assume the cutoff frequency cannot increase by more than 5%. What is the smallest value of load resistance that can be connected across the output terminals of the filter?

d) If the resistor found in (c) is connected across the output terminals, what is the magnitude of $H(j\omega)$ when $\omega = 0$?

[a] $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{50{,}000}{2\pi} = \dfrac{50}{2\pi} \times 10^3 = 7957.75\,\text{Hz}$

[b] $\dfrac{1}{RC} = 50 \times 10^3$

$$R = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-6})} = 40\,\Omega$$

[c] With a load resistor added in parallel with the capacitor the transfer function becomes

$$H(s) = \frac{R_L\|(1/sC)}{R + R_L\|(1/sC)} = \frac{R_L/sC}{R[R_L + (1/sC)] + R_L/sC}$$

$$= \frac{R_L}{RR_L sC + R + R_L} = \frac{1/RC}{s + [(R + R_L)/RR_L C]}$$

This transfer function is in the form of a low-pass filter, with a cutoff frequency equal to the quantity added to s in the denominator. Therefore,

$$\omega_c = \frac{R + R_L}{RR_L C} = \frac{1}{RC}\left(1 + \frac{R}{R_L}\right)$$

$$\therefore \quad \frac{R}{R_L} = 0.05 \qquad \therefore \quad R_L = 20R = 800\,\Omega$$

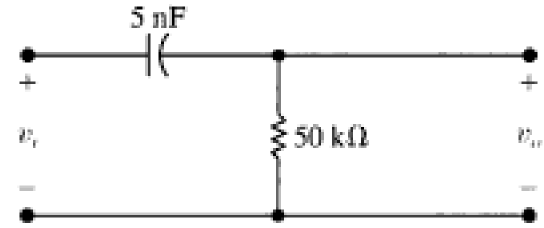
[d] $H(j0) = \frac{R_L}{R + R_L} = \frac{800}{840} = 0.9524$

14.10 a) Find the cutoff frequency (in hertz) for the high-pass filter shown in Fig. P14.10.

b) Find $H(j\omega)$ at ω_c, $0.2\omega_c$, and $5\omega_c$.

c) If $v_i = 500\cos\omega t$ mV, write the steady-state expression for v_o when $\omega = \omega_c$, $\omega = 0.2\omega_c$, and $\omega = 5\omega_c$.

Figure P14.10



[a] $$\frac{1}{RC} = \frac{1}{(50\times 10^3)(5\times 10^{-9})} = 4000\,\text{rad/s}$$

$$f_c = \frac{4000}{2\pi} = 636.62\,\text{Hz}$$

[b] $$H(s) = \frac{s}{s+\omega_c} \quad \therefore \quad H(j\omega) = \frac{j\omega}{4000+j\omega}$$

$$H(j\omega_c) = H(j4000) = \frac{j4000}{4000+j4000} = 0.7071\underline{/45^\circ}$$

$$H(j0.2\omega_c) = H(j800) = \frac{j800}{4000+j800} = 0.1961\underline{/78.69^\circ}$$

$$H(j5\omega_c) = H(j20\omega_c) = \frac{j20{,}000}{4000+j20{,}000} = 0.9806\underline{/11.31^\circ}$$

[c] $v_o(t)|_{\omega_c} = (0.7071)(500)\cos(4000t + 45^\circ)$

$$= 353.55\cos(4000t + 45^\circ)\,\text{mV}$$

$$v_o(t)|_{0.2\omega_c} = (0.1961)(500)\cos(800t + 78.60^\circ)$$

$$= 98.06\cos(800t + 78.69^\circ)\,\text{mV}$$

$$v_o(t)|_{5\omega_c} = (0.9806)(500)\cos(20{,}000t + 11.31^\circ)$$

$$= 490.29\cos(20{,}000t + 11.31^\circ)\,\text{mV}$$

14.15 Consider the circuit shown in Fig. P14.15.

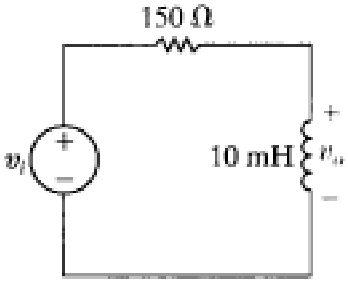
a) With the input and output voltages shown in the figure, this circuit behaves like what type of filter?

b) What is the transfer function, $H(s) = V_o(s)/V_i(s)$, of this filter?

c) What is the cutoff frequency of this filter?

d) What is the magnitude of the filter's transfer function at $s = j\omega_c$?

Figure P14.15



[a] For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.
For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.
Thus, the circuit is a high-pass filter.

[b] $$H(s) = \frac{sL}{R + sL} = \frac{s}{s + R/L} = \frac{s}{s + 15{,}000}$$

[c] $$\omega_c = \frac{R}{L} = 15{,}000 \text{ rad/s}$$

[d] $$|H(jR/L)| = \left|\frac{jR/L}{jR/L + R/L}\right| = \left|\frac{j}{j+1}\right| = \frac{1}{\sqrt{2}}$$

14.16 Suppose a 150 Ω load resistor is attached to the filter in Fig. P14.15.

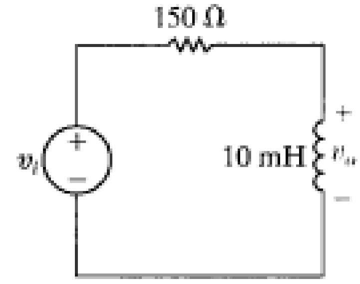
a) What is the transfer function, $H(s) = V_o(s)/V_i(s)$, of this filter?

b) What is the cutoff frequency of this filter?

c) How does the cutoff frequency of the loaded filter compare with the cutoff frequency of the unloaded filter in Fig. P14.15?

d) What else is different for these two filters?

Figure P14.15



$$\text{[a]}\quad H(s) = \frac{V_o}{V_i} = \frac{R_L \| sL}{R + R_L \| sL} = \frac{s\left(\dfrac{R_L}{R+R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)}$$

$$= \frac{\frac{1}{2}s}{s + \frac{1}{2}(15{,}000)}$$

$$\text{[b]}\quad \omega_c = \frac{R}{L}\left(\frac{R_L}{R+R_L}\right) = \frac{1}{2}(15{,}000) = 7500 \text{ rad/s}$$

[c] $\omega_{c(L)} = \frac{1}{2}\omega_{c(UL)}$

[d] The gain in the passband is also reduced by a factor of 1/2 for the loaded filter.

14.18 Calculate the center frequency, the bandwidth, and the quality factor of a bandpass filter that has an upper cutoff frequency of 121 krad/s and a lower cutoff frequency of 100 krad/s.

$$\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(121)(100)} = 110\,\text{krad/s}$$

$$f_o = \frac{\omega_o}{2\pi} = 17.51\,\text{kHz}$$

$$\beta = 121 - 100 = 21\,\text{krad/s} \quad \text{or} \quad 2.79\,\text{kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{110}{21} = 5.24$$

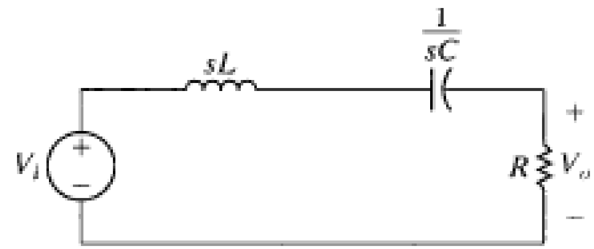
14.20 DESIGN PROBLEM PSPICE MULTISIM Use a 5 nF capacitor to design a series *RLC* bandpass filter, as shown at the top of Fig. 14.27. The center frequency of the filter is 8 kHz, and the quality factor is 2.

a) Specify the values of *R* and *L*.

b) What is the lower cutoff frequency in kilohertz?

c) What is the upper cutoff frequency in kilohertz?

d) What is the bandwidth of the filter in kilohertz?



[a] $\omega_o^2 = \frac{1}{LC}$ so $L = \frac{1}{[8000(2\pi)]^2(5 \times 10^{-9})} = 79.16\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16 \times 10^{-3})}{2} = 1.99\,\text{k}\Omega$$

[b] $f_{c1} = 8000\left[-\frac{1}{4} + \sqrt{1 + \frac{1}{16}}\right] = 6.25\,\text{kHz}$

[c] $f_{c2} = 8000\left[\frac{1}{4} + \sqrt{1 + \frac{1}{16}}\right] = 10.25\,\text{kHz}$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4\,\text{kHz}$$

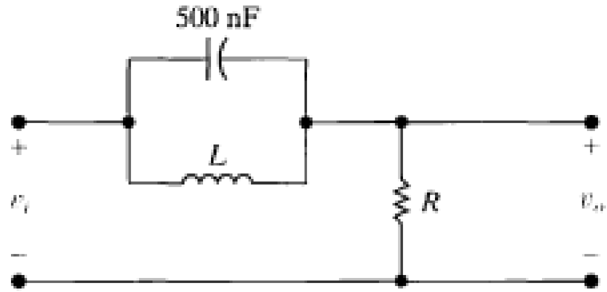
14.36 DESIGN PROBLEM PSPICE MULTISIM Use a 500 nF capacitor to design a bandreject filter, as shown in Fig. P14.36. The filter has a center frequency of 4 kHz and a quality factor of 5.

a) Specify the numerical values of R and L.

b) Calculate the upper and lower corner, or cutoff, frequencies in kilohertz.

c) Calculate the filter bandwidth in kilohertz.

Figure P14.36



[a] $\omega_o = 2\pi f_o = 8\pi\ \text{krad/s}$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(8000\pi)^2(0.5\times 10^{-6})} = 3.17\,\text{mH}$$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(8000\pi)(0.5\times 10^{-6})} = 397.89\,\Omega$$

[b] $$f_{c2} = f_o\left[\frac{1}{2Q} + \sqrt{1+\left(\frac{1}{2Q}\right)^2}\right] = 4000\left[\frac{1}{10} + \sqrt{1+\frac{1}{100}}\right]$$

$$= 4.42\,\text{kHz}$$

$$f_{c1} = f_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 4000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$$

$$= 3.62\,\text{kHz}$$

[c] $\beta = f_{c2} - f_{c1} = 800\,\text{Hz}$

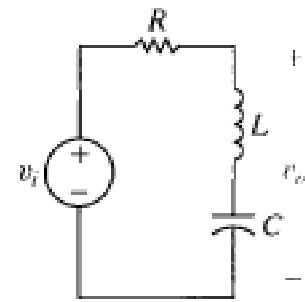
or

$$\beta = \frac{f_o}{Q} = \frac{4000}{5} = 800\,\text{Hz}$$

14.39 Design an RLC bandreject filter (see Fig. 14.28[a]) with a quality of 2.5 and a center frequency of 25 krad/s, using a 200 nF capacitor.

a) Draw your circuit, labeling the component values and output voltage.

b) For the filter in part (a), calculate the bandwidth and the values of the two cutoff frequencies.

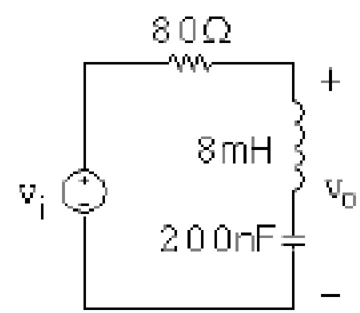


(a)

[a] $\omega_o = \sqrt{1/LC}$ so $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(25{,}000)^2(200 \times 10^{-9})} = 8\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{25{,}000}{2.5} = 10{,}000 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (8 \times 10^{-3})(10{,}000) = 80\,\Omega$$



[b] From part (a), $\beta = 10{,}000$ rad/s.

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{10{,}000}{2} + \sqrt{\left(\frac{10{,}000}{2}\right)^2 + 25{,}000^2} = \pm 5000 + 25{,}495.1$$

$\omega_{c1} = 20{,}495.1$ rad/s $\qquad \omega_{c2} = 30{,}495.1$ rad/s

Following Slides for Information Only

Application of BPFs

- Q: How to tell which button was pushed? How to tell the difference between the button tones and the normal vocal sounds (both are within 300-3000 Hz) or ringing bell tones (20 Hz)?

Dual-tone-multiple-frequency (DTMF)

- Each bottom is encoded with a unique pair of sinusoidal tones (f_L, f_H), where the relative timing and amplitudes must be close enough.

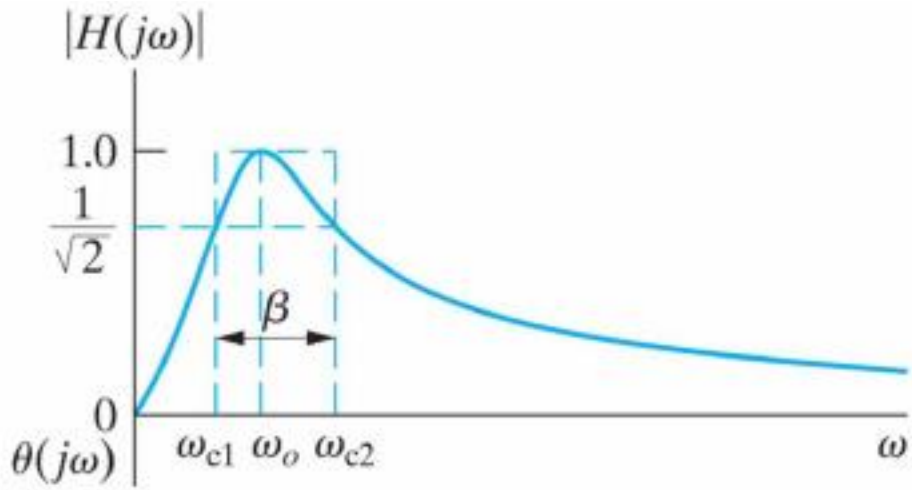
"High Group" frequencies [Hz]

"Low Group" frequencies [Hz]	1209	1336	1477	1633	
697	1	2	3	A	(Row 1)
770	4	5	6	B	(Row 2)
852	7	8	9	C	(Row 3)
941	*	0	#	D	(Row 4)
	(Column 1)	(Column 2)	(Column 3)	(Column 4)	

- BPFs: (1) identify whether both freq. groups are present, (2) select among possible tones.

BPF design for the low-frequency group

- The transmission spectrum of a series RLC is:



$$|H(j\omega)| = \frac{\beta\omega}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + (\beta\omega)^2}},$$

$$\text{where } \beta = \frac{R}{L},\ \omega_0 = \frac{1}{\sqrt{LC}}.$$

- For the low-freq. group, $\omega_{c1} = 2\pi(697) = 4379$ rad/s, $\omega_{c2} = 2\pi(941) = 5912$ rad/s, $\Rightarrow \beta = \omega_{c2} - \omega_{c1} = 1533$ rad/s.

- $\beta = R/L, \Rightarrow L = R/\beta = (600\,\Omega)/1533 = 0.39$ H.

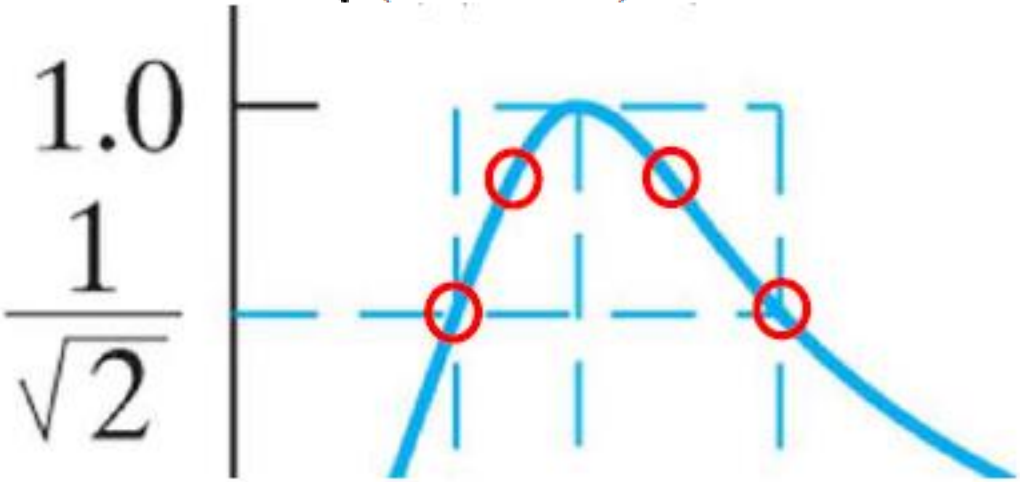
$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = 1/\sqrt{LC}, \Rightarrow C = 1/(L\omega_{c1}\omega_{c2}) = 100$ nF.

BPF performance (1)

- The transmission of the 4 frequencies in the low-frequency group are imperfect (<1):

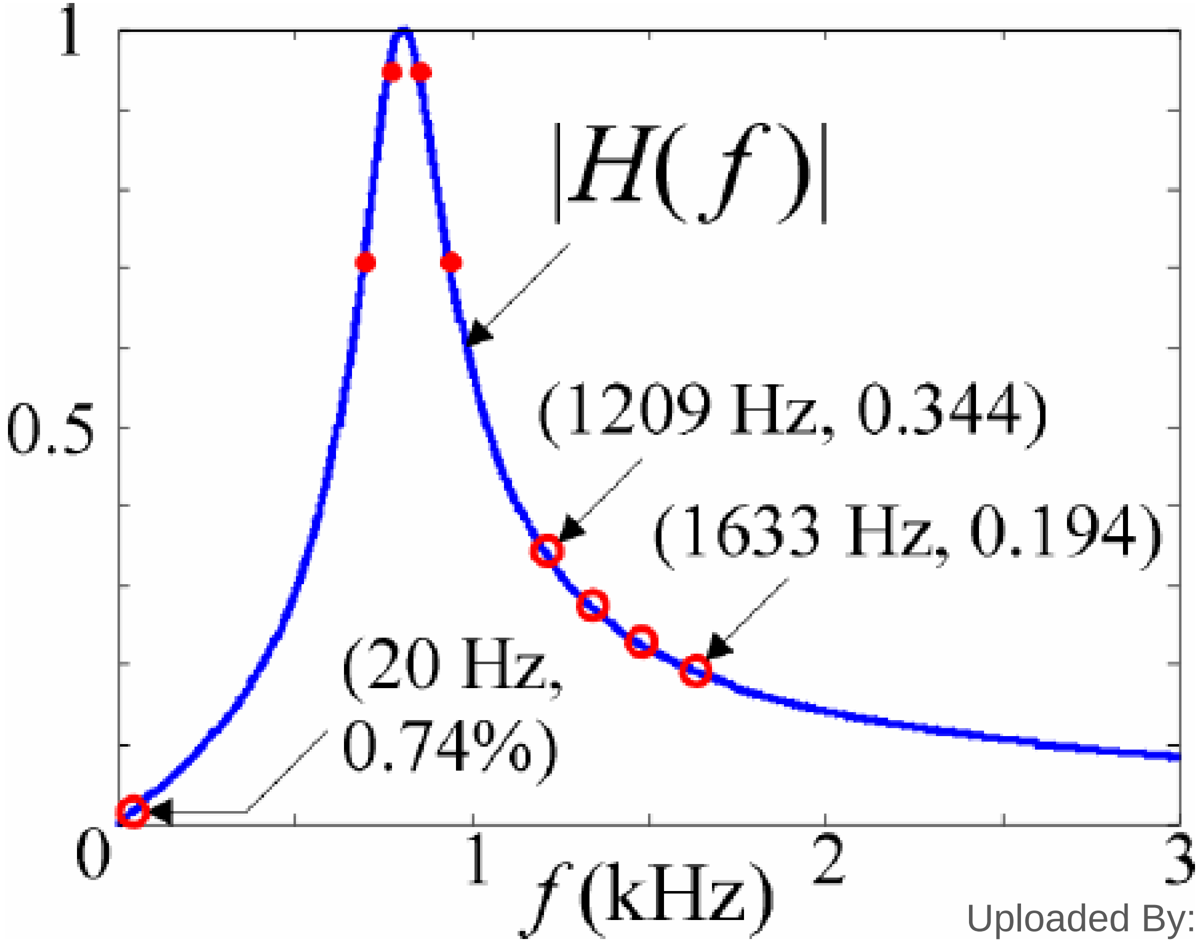
$$\left|H_{697\text{Hz}}\right| = \left|H_{941\text{Hz}}\right| = \frac{1}{\sqrt{2}} = 0.707,$$

$$\left|H_{770\text{Hz}}\right| = \left|H_{852\text{Hz}}\right| = \frac{\beta\omega}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + (\beta\omega)^2}} = 0.948.$$



BPF performance (2)

- The transmission of the high-frequency group tones and the 20 Hz ringing tone are imperfect (>0):



BPF design for the high-frequency group

- The transmission spectrum of a series RLC is:

$$|H(j\omega)| = \frac{\beta\omega}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + (\beta\omega)^2}}, \quad \beta = \frac{R}{L}, \ \omega_0 = \frac{1}{\sqrt{LC}}.$$

- For the high-freq. group, $\omega_{c1} = 2\pi(1209) = 7596$ rad/s, $\omega_{c2} = 2\pi(1633) = 10260$ rad/s, $\Rightarrow \beta = \omega_{c2} - \omega_{c1} =$ 2664 rad/s.
- Both L and C values are smaller:

$$\beta = R/L, \Rightarrow L = R/\beta = (600\ \Omega)/2664 = 0.26\ \text{H}.$$

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = 1/\sqrt{LC}, \Rightarrow C = 1/(L\omega_{c1}\omega_{c2}) = 57\ \text{nF}.$$

BPF performance

- The transmission of the low-frequency group tones and the 20 Hz ringing tone are imperfect (>0):