8.7 Part 2

Wednesday, February 24, 2021 2:23 PM

Improper Integrals

I

$$\int_{-\infty}, \quad \int^{\infty}, \quad \int_{-\infty}^{\infty}$$

II

$$\int_{-3}^{0} \frac{dx}{x+2}$$

$-2 \in [-3, 0]$

Exp*

$$\int_{1}^{\infty} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1} & \text{if } p > 1 \\ \infty & \text{if } p \leq 1 \end{cases}$$

Exp**

$$\int_{0}^{1} \frac{dx}{x^p} = \begin{cases} \frac{1}{1-p} & \text{if } p < 1 \\ \infty & \text{if } p \geq 1 \end{cases}$$

Two Tests to check Convergence/Divergence

Direct Comparsion Test (DCT)

f, g are cont. on $[a, \infty)$ s.t

$$0 \le f(x) \le g(x) \quad \forall\, x \in [a, \infty) \quad \text{Then}$$

- If $\int_a^\infty g(x)\,dx$ converges then $\int_a^\infty f(x)\,dx$ converges
- If $\int_a^\infty f(x)\,dx$ diverges then $\int_a^\infty g(x)\,dx$ diverges

Exp Check the Convergence/Divergence

(1) $\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$ DCT

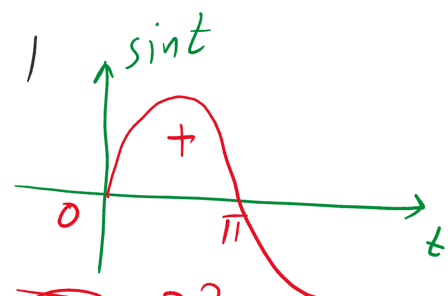
Divers

$$\le \frac{\sin^2 x}{x^2} \le \frac{1}{x^2}$$

Conves

$$\int_1^\infty \frac{dx}{x^2} = \frac{1}{2-1}$$

$\int_1^\infty \frac{\sin^2 x}{x^2}$ converges by DCT $\quad \frac{1}{2-1} = 1$



(2) $\int_0^\pi \frac{dt}{\sqrt{t} + \sin t}$ +

Div

Conv. ??

conver

II

1

1

II $\ 0$ Div

$$\le \frac{1}{\sqrt{t} + \cancel{\sin t}} \le \frac{1}{\sqrt{t}} = 2\sqrt{\pi}$$

$$\frac{1}{4} > \frac{1}{5}$$

$$\int_0^{\pi} \frac{dt}{\sqrt{t}} = \lim_{b \to 0^+} \int_b^{\pi} \frac{2\,dt}{2\sqrt{t}} = \lim_{b \to 0^+} 2\sqrt{t}\Big|_b^{\pi}$$

$$= \lim_{b \to 0^+} \left(2\sqrt{\pi} - 2\sqrt{b}\right) = 2\sqrt{\pi}$$

$$\int_0^{\pi} \frac{dx}{\sqrt{x} + \sin x}$$ converges by DCT

$$③ \int_1^{\infty} \frac{dx}{\sqrt{x^2 - 0.1}}$$ I

$x^2 - 0.1 = 0$

$x^2 = 0.1$

$x = \pm\sqrt{0.1}$

$x = \pm\sqrt{0.1} \notin [1, \infty)$

So this not II

∞ Div ??

??

∞ Div ??

$$\frac{1}{\sqrt{x^2}} \leq \frac{1}{\sqrt{x^2-0.1}} \leq \cancel{\frac{1}{\sqrt{x^2}}}$$

??

$$\frac{1}{|x|}$$

$$\frac{1}{4} \boxed{>} \frac{1}{5}$$

$$\int_1^\infty \frac{1}{x}\,dx$$

$$\cancel{\frac{1}{4-0.1}} \; \boxed{} \; \frac{1}{4-0.1}$$

Div by ∞

Exp*

Hence, $\displaystyle\int_1^\infty \frac{dt}{\sqrt{t^2-0.1}}$ div by DCT.

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4 (47) $\displaystyle\int_1^\infty \frac{dx}{x^3+1}$

Conv. by Exp*

∞

∞

$$\le \int_1 \frac{1}{x^3+1} \le \int_1^\infty \frac{1}{x^3}$$

conv. by DCT

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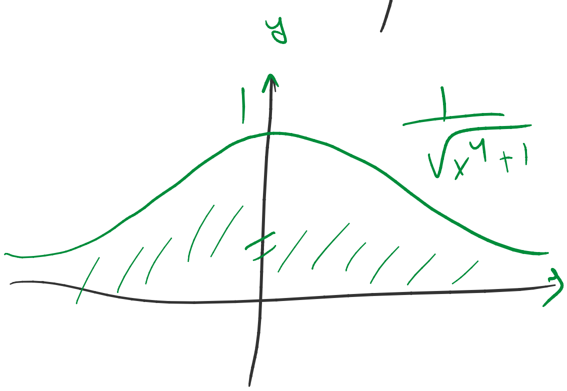
63. $$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{x^4+1}} = \int_{-\infty}^{0} \frac{dx}{\sqrt{x^4+1}} + \int^{\infty} \frac{dx}{\sqrt{x^4+1}}$$

I



$$= 2\int_0^\infty \frac{dx}{\sqrt{x^4+1}}$$

$$\frac{1}{\sqrt{x^4+1}} \le \frac{1}{\sqrt{x^4}}$$

Conve

$$\int^{\infty} \frac{1}{} = \frac{1}{}$$

$$\int_0^\infty \frac{dx}{\sqrt{x^4+1}} = \int_0^1 \frac{dx}{\sqrt{x^4+1}} + \int_1^\infty \frac{dx}{\sqrt{x^4+1}}$$

$$\int_1^\infty \frac{1}{x^2} = \frac{1}{2-1} = 1$$

Hence, $\int_{-\infty}^{\infty} \frac{dx}{\sqrt{x^4+1}}$ Conv by DCT.

LCT: limit Comparsion Test

f, g are $+$, cont. on $[a, \infty)$ and

$\lim_{x\to\infty} \frac{f}{g} = L$ where $0 < L < \infty$ Then

$\int_a^\infty f(x)\,dx$ and $\int_a^\infty g(x)\,dx$ both are diverging

or " " converging

Ex Check Conv./Divg.

Exp Check / Conv. / Divg.

① $\int_1^\infty \frac{dx}{1+x^2}$ $f = \frac{1}{1+x^2}$ $g = \frac{1}{x^2}$ Conv. ?!

$$\lim_{x\to\infty} \frac{\frac{1}{1+x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{x^2}{1+x^2} = 1$$

so $\int_1^\infty \frac{dx}{1+x^2}$ conv. by LCT

② $\int_2^\infty \frac{dx}{\sqrt{x-1}}$ $f = \frac{1}{\sqrt{x-1}}$ $D(f)$ $x>1$ + cont.

$g = \int_2^\infty \frac{1}{\sqrt{x}}$?? div

$$\int_2^\infty \frac{1}{\sqrt{x}}dx = \lim_{b\to\infty} \int_2^b \frac{2\,dx}{2\sqrt{x}} = \lim_{b\to\infty} 2\sqrt{x}\Big|_2^b$$

$\int_2$ $\sqrt{x}$ $\quad b \to \infty \quad \int_2 x \quad b \to \infty \quad 2$

$$= \lim_{b \to \infty} 2\sqrt{b} - 2\sqrt{2} = \infty$$

$$\lim_{x \to \infty} \frac{f}{g} = \lim_{x \to \infty} \frac{\frac{1}{\sqrt{x-1}}}{\frac{1}{\sqrt{x}}} = \lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x-1}} = 1$$

$$\int_2^\infty \frac{dx}{\sqrt{x-1}} \quad \text{div. by LCT}$$