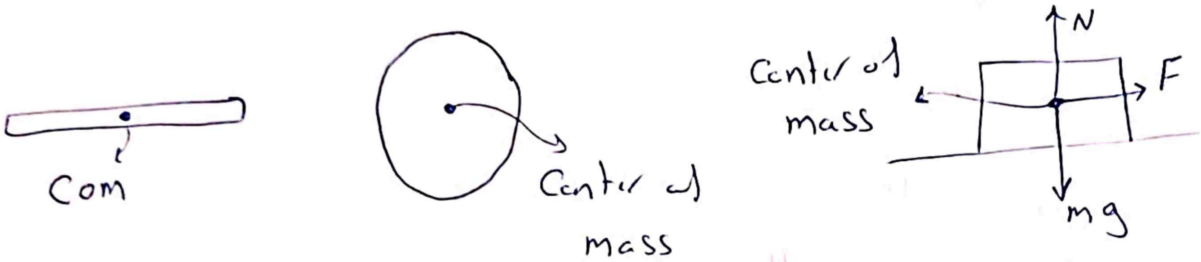
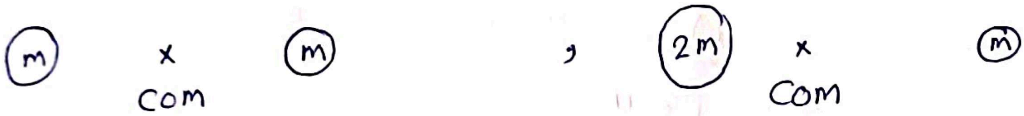
ch 9: Center of Mass & Linear Momentum.

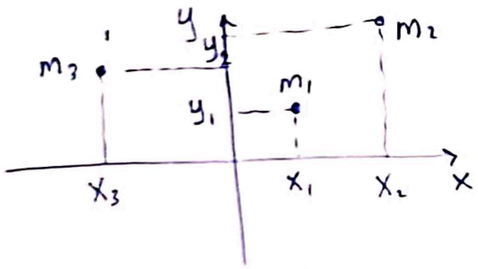
Center of mass of a system of particles: is the point that moves as though ① all the system's mass were concentrated there

② all external forces were applied there



ex:





$$X_{com} = \frac{m x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$X_{com} = \frac{\sum m_i x_i}{M} \quad , \quad y_{com} = \frac{\sum m_i y_i}{M} \quad , \quad Z_{com} = \frac{\sum m_i Z_i}{M}$$

$$M = m_1 + m_2 + \cdots$$
$$= \sum m_i$$

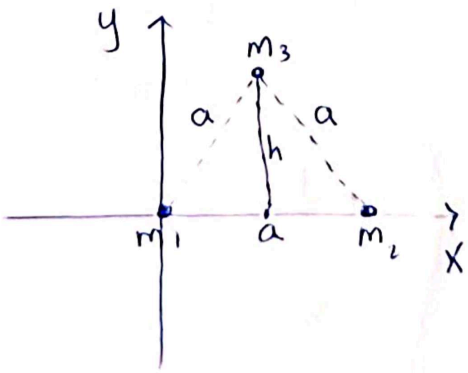
sample problem 9.01:-

$m_1 = 1.2$ kg

$m_2 = 2.5$ kg

$m_3 = 3.4$ kg

$a = 140$ cm



$\boxed{m_1\ (0,0)}$, $\boxed{m_2\ (140,0)}$, $m_3\ (\frac{a}{2}, h)$
$(a, 0)$

$h^2 = a^2 - (\frac{a}{2})^2$

$h = \sqrt{\frac{3}{4}a^2} = 121$ cm

$\boxed{m_3\ (70, 121)}$

$$X_{com} = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3}{\Sigma m}$$

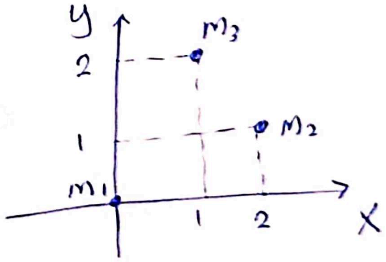
$$= \frac{1.2(0) + 2.5(140) + 3.4(70)}{1.2 + 2.5 + 3.4} = 83 \text{ cm}$$

$$Y_{com} = \frac{1.2(0) + 2.5(0) + 3.4(121)}{1.2 + 2.5 + 3.4} = 58 \text{ cm}$$

Com (83, 58) cm or $\vec{r}_{com} = 83\hat{i} + 58\hat{j}$ cm

problem 2

$m_1 = 2$ kg , $m_2 = 4$ kg
$m_3 = 8$ kg.



$m_1\ (0,0)$, $m_2\ (2,1)$, $m_3\ (1,2)$

$$X_{com} = \frac{2(0) + 4(2) + 8(1)}{2 + 4 + 8} = 1.14 \text{ m}$$

$$y_{com} = \frac{2(0) + 4(1) + 8(2)}{2 + 4 + 8} = 1.43 \text{ m}$$

Com (1.14, 1.43) m

c) if M_3 is increased $\Rightarrow$ com shift toward it.

* Newton's Second Law for a system of particles

$$\vec{r}_{com} = \frac{1}{M} \sum m_i \vec{r}_i$$

$$= \frac{1}{M} (m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots)$$

$$M \vec{r}_{com} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots$$

$$\frac{d}{dt} M \vec{r}_{com} = \frac{d}{dt} (m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots)$$

$$M \frac{d \vec{r}_{com}}{dt} = m \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + \dots$$

$$\boxed{M \vec{V}_{com} = m \vec{V}_1 + m_2 \vec{V}_2 + \dots} \quad \dots (1)$$

$$\vec{P}_{com} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \dots$$

$\vec{P} = m\vec{v}$ linear Momentum

deriv (1) $$M \frac{d\vec{V}_{com}}{dt} = m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + \dots$$

$$M \vec{a}_{com} = m \vec{a}_1 + m_2 \vec{a}_2 + \dots$$

$$\boxed{M \vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots}$$

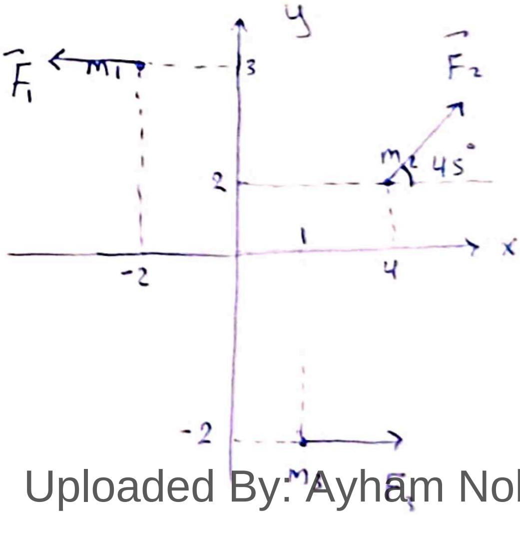
Newton's second law.

sample problem 9.3:

$m_1 = 4$ kg , $F_1 = 6N$

$m_2 = 8$ kg , $F_2 = 12N$

$m_3 = 4$ kg , $F_3 = 14N$



1) Find $\vec{a}_{com}$?

$$M\vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\boxed{(4+8+4)\,\vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3}$$

$\vec{F}_1 = -6\hat{i}$, $\vec{F}_2 = 12\cos 45\,\hat{i} + 12\sin 45\,\hat{j}$

$= 8.5\,\hat{i} + 8.5\,\hat{j}$

$\vec{F}_3 = 14\,\hat{i}$

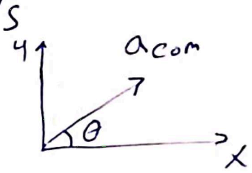
$$16\,\vec{a}_{com} = -6\hat{i} + (8.5\hat{i} + 8.5\hat{j}) + 14\hat{i}$$

$$16\,\vec{a}_{com} = 16.5\,\hat{i} + 8.5\,\hat{j}$$

$$\boxed{\vec{a}_{com} = 1.03\,\hat{i} + 0.53\,\hat{j}}$$

$$a_{com} = \sqrt{(1.03)^2 + (0.53)^2} = 1.2\ m/s^2$$

$$\theta = \tan^{-1}\left(\frac{0.53}{1.03}\right) = 27^\circ$$



2) Find $\vec{r}_{com}$ at $t=0$?

$$\vec{r}_{com} = X_{com}\,\hat{i} + Y_{com}\,\hat{j}$$

$$X_{com} = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3}{\Sigma m} = \frac{4(-2) + 8(4) + 4(1)}{16} = 1.75\ m$$

$$y_{com} = \frac{4(3) + 8(2) + 4(-2)}{16} = 1.25\ m$$

$$\vec{r}_{com} = (1.75\,\hat{i} + 1.25\,\hat{j})\,m$$

3) Find $\vec{r}_{com}$ after 10 sec ?

$$\left[\Delta\vec{r} = \vec{V}_0 t + \tfrac{1}{2}\vec{a}\,t^2\right]_{com} \quad , \quad \boxed{\vec{V}_0 = 0}$$

$$\Delta\vec{r}_{com} = 0 + \frac{1}{2}[1.03\hat{i} + 0.53\hat{j}](10)^2$$

$$\Delta\vec{r}_{com} = 51.5\hat{i} + 26.5\hat{j} \text{ m}$$

$$\vec{r}_f - \vec{r}_i = 51.5\hat{i} + 26.5\hat{j}$$

$$\vec{r}_{f\,com} - (1.75\hat{i} + 1.25\hat{j}) = 51.5\hat{i} + 26.5\hat{j}$$

$$\boxed{\vec{r}_{f\,com} = 53.25\hat{i} + 27.8\hat{j} \text{ m}}$$

3) Find $\vec{V}_{com}$ after 10 sec?

$$(\vec{V}_f = \vec{V}_i + \vec{a}t)_{com}$$

$$\vec{V}_{f\,com} = 0 + (1.03\hat{i} + 0.53\hat{j})(10)$$

$$\boxed{\vec{V}_{f\,com} = 10.3\hat{i} + 5.3\hat{j} \text{ m/s}}$$

- Linear momentum :- (كمية التحرك)

$$\vec{P} = m\vec{V}$$

Newtons Second law : $\vec{F}_{net} = m\vec{a}$

$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}} = \frac{d}{dt}(m\vec{V}) = m\frac{d\vec{V}}{dt} = m\vec{a}$$

- Impulse : (الدفع)

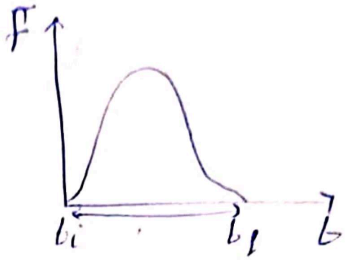
$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

$$\int_{P_i}^{P_f} d\vec{P} = \int_{t_i}^{t_f} \vec{F}_{net}\, dt \Rightarrow \Delta\vec{P} = \int_{t_i}^{t_f} F_{net}\, dt = \vec{J} \text{ (Impuls)}$$

$[J] = N.S = kg.m/s$

* If $\vec{F}$ is not Const

$\vec{J}$ = Area under the curve of $\vec{F}$ vs t from $t_i \rightarrow t_f$.

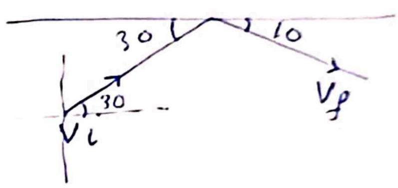


* If $\vec{F}$ is Const. $\Rightarrow$ $\vec{J} = \int_{t_i}^{t_f} F \, dt = \vec{F}_{avg} \Delta t$

sample problem 9.04 :

$v_i = 70$ m/s , $m = 80$ kg

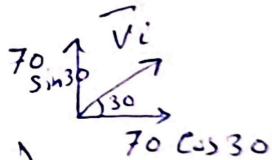
$v_f = 50$ m/s



a) Find the Impulse $\vec{J}$?

$\vec{J} = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$

$\vec{P}_i = m \vec{v}_i = m(70 \cos 30 \hat{i} + 70 \sin 30 \hat{j})$

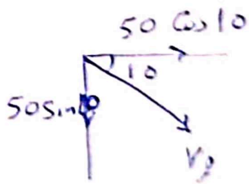


$= 80(60.6 \hat{i} + 35 \hat{j})$

$\boxed{\vec{P}_i = 4848 \hat{i} + 2800 \hat{j}}$

$\vec{P}_f = m \vec{v}_f = m(50 \cos 10 \hat{i} - 50 \sin 10 \hat{j})$

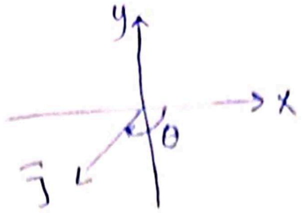
$\boxed{\vec{P}_f = 3939 \hat{i} - 694.6 \hat{j}}$



$\vec{J} = (3939 \hat{i} - 694.6 \hat{j}) - (4848 \hat{i} + 2800 \hat{j})$

$= -909 \hat{i} - 3494.6 \hat{j}$

$J = \sqrt{(909)^2 + (3494.6)^2} \simeq 3600$ N.s , $\theta = -105°$



b) Find the average force if the collision lasts for 14 ms?

$\vec{J} = \vec{F}_{avg} \Delta t$

$$F_{avg} = \frac{3600}{14 \times 10^{-3}} \simeq 2.6 \times 10^{5} \, N$$

$\Rightarrow \quad F_{avg} = m a$

$2.6 \times 10^{5} = 80 a \Rightarrow \boxed{a = 3.22 \times 10^{3} \, m/s^2}$

Ch9: Lec 2

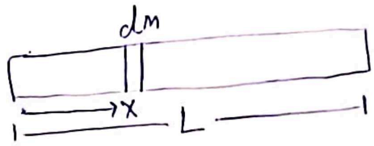
Center of mass of solid bodies:

$$x_{com} = \frac{1}{M}\sum_i m_i x_i \Rightarrow x_{com} = \frac{1}{M}\int x\,dm$$

$$y_{com} = \frac{1}{M}\sum_i m_i y_i \Rightarrow y_{com} = \frac{1}{M}\int y\,dm$$

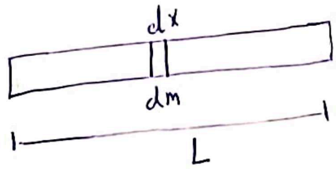
$$z_{com} = \frac{1}{M}\sum_i m_i z_i \Rightarrow z_{com} = \frac{1}{M}\int z\,dm$$

$\int x\,dm$



1-Dim

uniform linear density



$\frac{dm}{dx} = \frac{M}{L} = \lambda$ (linear density)

"the mass is distributed uniformly"

$$dm = \frac{M}{L}\,dx$$

$$x_{com} = \frac{1}{M}\int x\,dm = \frac{1}{\cancel{M}}\int_0^L x\left(\frac{\cancel{M}}{L}dx\right)$$

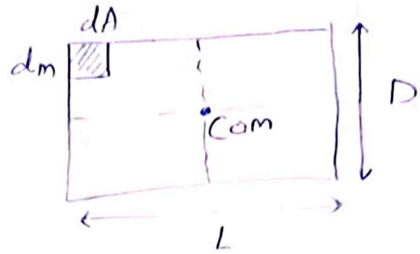
$$= \frac{1}{L}\int_0^L x\,dx = \frac{1}{L}\left(\frac{x^2}{2}\right)\Big|_0^L$$

$$= \frac{1}{L}\left(\frac{L^2}{2}\right)$$

$$= \frac{L}{2} \checkmark$$

2- dim

uniform surface density



$$\sigma = \frac{dm}{dA} = \frac{M}{A}$$

$$x_{com} = \frac{1}{M}\int x\, dm \quad ; \quad dm = \frac{M}{A}\, dA$$

$$= \frac{1}{\cancel{M}}\int x \left(\frac{\cancel{M}}{A}\, dA\right) = \frac{1}{A}\int x\, dA \qquad \begin{Bmatrix} dA = dx\,dy \\ A = LD \end{Bmatrix}$$

$$= \frac{1}{LD}\iint x\, dx\, dy$$

$$= \frac{1}{LD}\int_0^L x\, dx \int_0^D dy$$

$$= \frac{1}{LD}\left(\frac{x^2}{2}\Big|_0^L\right)\left(y\Big|_0^D\right)$$

$$= \frac{1}{\cancel{L}\cancel{D}}\left(\frac{L^{\cancel{2}}}{2}\right)\left(\cancel{D}\right)$$

$$= \frac{L}{2}$$

$$y_{com} = \frac{1}{M}\int y\, dm \qquad \text{(in the same way)}$$

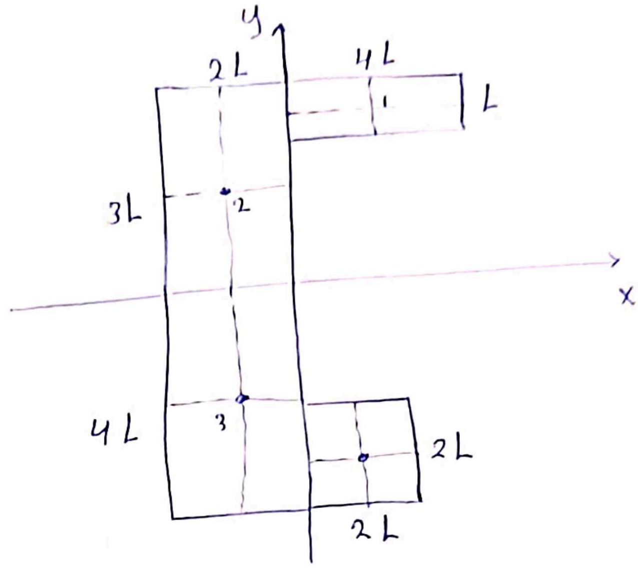
$$= \frac{D}{2}$$

3- Dim

uniform volume density

$$\rho = \frac{M}{V} = \frac{dm}{dV}$$

problem 5 :



$L = 5$ cm

uniform surface density

$1 = (2L, 2.5L) = (10, 12.5)$

$2 = (-L, 1.5L) = (-5, 7.5)$

$3 = (-L, -2L) = (-5, -10)$

$4 = (L, -3L) = (5, -15)$

now we want to find m_1, m_2, m_3, m_4?

since the plate is uniform $\Rightarrow \sigma = \frac{M}{A}$, M: total mass, A: " area

$$A = (L \times 4L) + (2L \times 3L) + (4L \times 2L) + (2L \times 2L)$$

$$= 22L^2$$

$$\sigma = \frac{M}{22L^2}$$

$$\sigma = \frac{m_1}{A_1} = \frac{M}{22L^2} \Rightarrow m_1 = \frac{M}{22L^2}(A_1) = \frac{M}{22\cancel{L^2}}(4\cancel{L^2})$$

$$\boxed{m_1 = \frac{4}{22}M}$$

$$m_2 = \frac{M}{22L^2}(A_2) = \frac{M}{22L^2}(6L^2)$$

$$\boxed{m_2 = \frac{6}{22}M}$$

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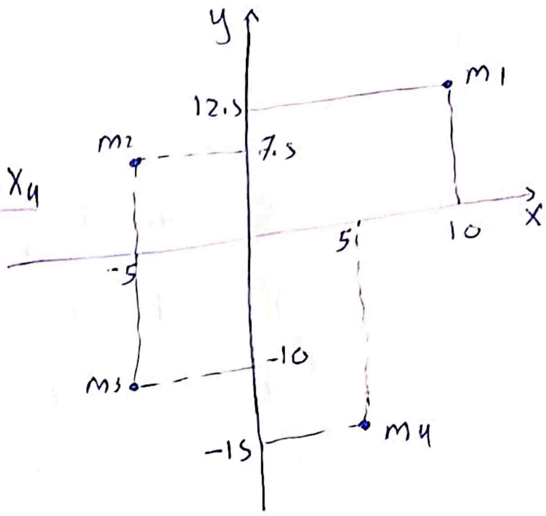
$$m_3 = \left(\frac{M}{22L^2}\right) A_3 = \left(\frac{M}{22L^2}\right)(8L^2) = \frac{8}{22} M$$

$$m_4 = \left(\frac{M}{22L^2}\right) A_4 = \left(\frac{M}{22L^2}\right)(4L^2) = \frac{4}{22} M$$



$$X_{com} = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3 + m_4 X_4}{M}$$

$$X_{com} = \frac{\frac{4}{22} M(10) + \frac{6}{22} M(-5) + \frac{8}{22} M(-5) + \frac{4}{22} M(5)}{M}$$

$$= \frac{-10}{22} \simeq -0.45 \text{ cm}$$

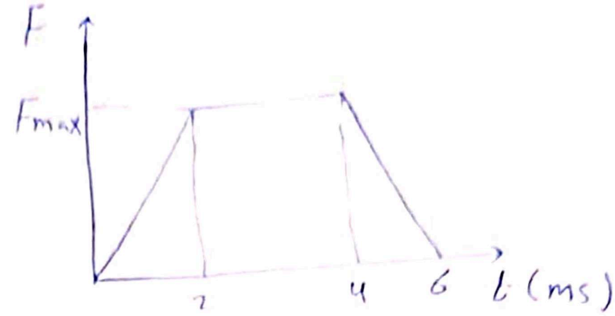
$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{M}$$

$$= \frac{\frac{4}{22} M(12.5) + \frac{6}{22} M(7.5) + \frac{8}{22} M(-10) + \frac{4}{22} M(-15)}{M}$$

$$= \frac{-45}{22} \simeq -2 \text{ cm}$$

$$\text{Com} = (-0.45, -2) \text{ cm}, \quad \vec{r}_{com} = -0.45\hat{i} - 2\hat{j} \text{ cm}$$

Problem 35



Find F_{max} on the ball from the wall during the collision?

$m = 58g = 58 \times 10^{-3}$ kg

$v_i = 34$ m/s $\rightarrow|$

$v_f = 34$ m/s $\leftarrow|$

$$\vec{J} = \Delta \vec{P} = \int_{t_i}^{t_f} F \, dt$$

$$\vec{P}_f - \vec{P}_i = \int_{t_i}^{t_f} \vec{F} \, dt = \text{Area under the curve}$$

$$m\vec{v}_f - m\vec{v}_i = \frac{1}{2}(6+2) \times 10^{-3} F_{max}$$

$$58 \times 10^{-3}(-34 - 34) = 4 \times \cancel{10^{-3}} F_{max}$$

$$-3944 = 4 F_{max}$$

$$F_{max} = 986 \text{ N} \quad (\text{to the left})$$

* Conservation of Linear Momentum

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) \quad [\text{Newton's } 2^{nd} \text{ Law}]$$

$$\text{if } \vec{F}_{net} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \boxed{\vec{P} = \text{Const}}$$

$$\Rightarrow \vec{P}_i = \vec{P}_f \quad [\text{Conservation of Linear Momentum}]$$

projectiles (x-Comp) ← → collisions
→ Explosions

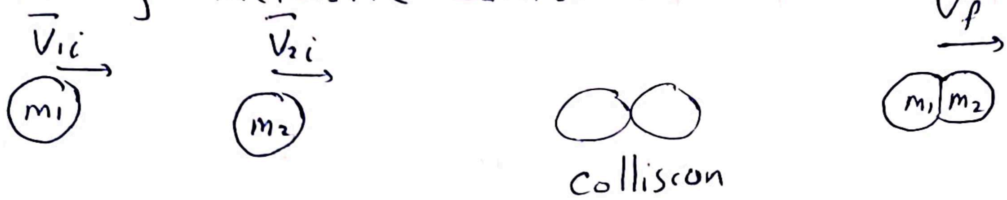
* Collisions:

$$\vec{F}_{net} = 0 \Rightarrow \vec{p} \text{ is conserved}$$

$$\vec{P}_i = \vec{P}_f$$

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f}$$

① Completely inelastic collision:



$$\vec{P}_i = \vec{P}_f$$

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = (m_1 + m_2) \vec{V}_f$$

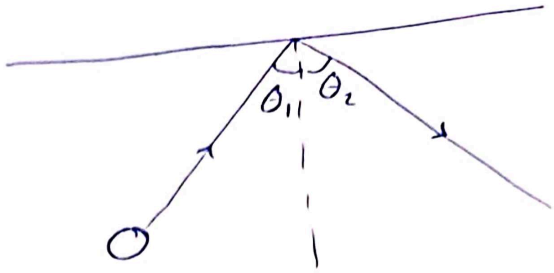
② Elastic Collision:

$$\vec{P}_i = \vec{P}_f \quad — ①$$

$$K_i = K_f \quad — ②$$

$$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

Problem 22



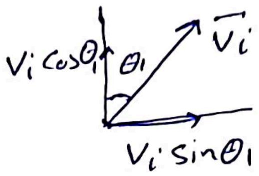
$m = 0.15$ kg

$V_i = 2$ m/s

$\theta_1 = 30^\circ$

$(V_f)_y = -(V_i)_y \quad , \quad (V_f)_x = (V_i)_x$

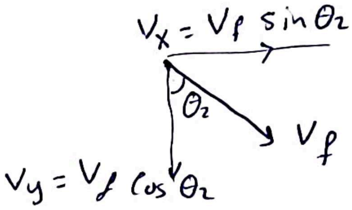
a) Find θ_2 ?



$\vec{V_i} = V_i \sin\theta_1 \hat{i} + V_i \cos\theta_1 \hat{j}$

$= 2 \sin 30 \hat{i} + 2 \cos 30 \hat{j}$

$\vec{V_i} = 1\hat{i} + 1.7\hat{j}$



$\vec{V_f} = 1\hat{i} - 1.7\hat{j}$

$\cancel{(V_f)_x = V_f \sin\theta_2 = 1}$

$\tan\theta_2 = \frac{V_{fx}}{V_{fy}} = \frac{1}{1.7} \Rightarrow \theta_2 \simeq 30^\circ$

b) $\Delta\vec{P} = \vec{P_f} - \vec{P_i}$

$= m\vec{V_f} - m\vec{V_i}$

$= 0.15[(1\hat{i} - 1.7\hat{j}) - (1\hat{i} + 1.7\hat{j})]$

$= 0.15[-1.7\hat{j} - 1.7\hat{j}]$

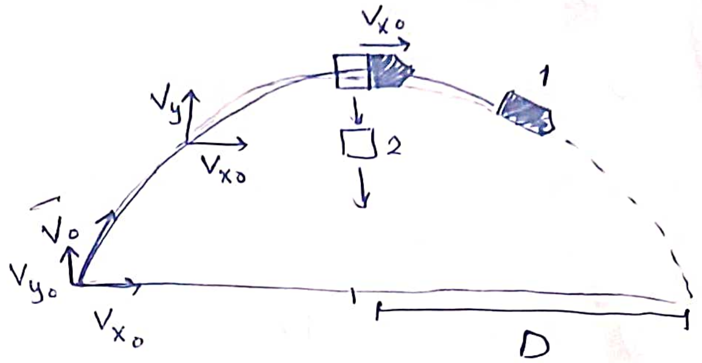
$= -0.51\hat{j}$

ch9 Lec 3

13) $V_0 = 20\ m/s$

$\theta_0 = 60^\circ$

$M = 2m$ Find D?



$\vec{V_0} = V_0 \cos 60\,\hat{i} + V_0 \sin 60\,\hat{j}$

Befor explosion:

$M = 2m$

$\vec{V_b} = V_x\,\hat{i} + V_y\,\hat{j}$, $V_x = \text{Const}$

$= V_0 \cos 60\,\hat{i} + V_y\,\hat{j} = 20 \cos 60\,\hat{i} + V_y\,\hat{j}$

$= 10\,\hat{i} + V_y\,\hat{j}$

After explosion:

$\vec{V_1} = V_{1x}\,\hat{i} - V_{1y}\,\hat{j}$

$\vec{V_2} = -V_{2y}\,\hat{j}$

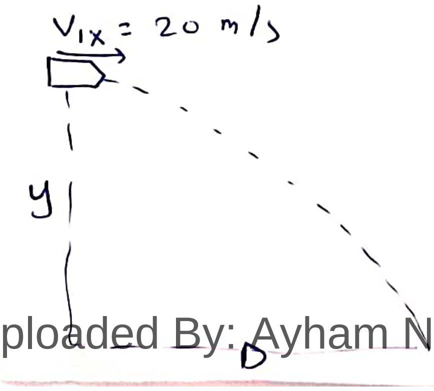
$\vec{F}_{net\,x} = 0 \Rightarrow P_{xb} = P_{xa}$

$M V_{xb} = m_1 V_{1x} + m_2 V_{2x}$

$2\cancel{m}(10) = \cancel{m} V_{1x} + m(0)$

$V_{1x} = 20\ m/s$

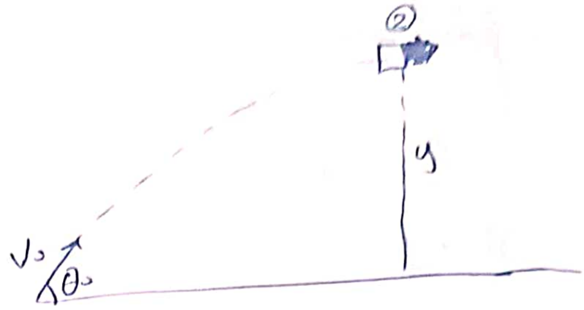
* we need to find y:



$V_{2y}^2 = V_{0y}^2 - 2gy$

$0 = (20\sin 60)^2 - 2(9.8)y$

$\boxed{y = 15.3\ m}$

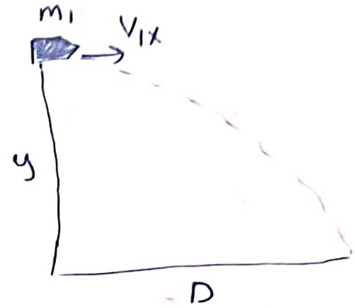


To find D we want to find time for m_1 to reach the ground

$y = \cancel{V_{1y}}^{0}\, t - \frac{1}{2} g t^2$

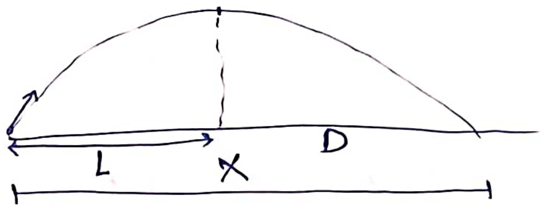
$-15.3 = -\frac{1}{2}(9.8)t^2$

$t = 1.77$ sec



$D = V_{1x}\, t$

$D = 20(1.77) = 35.3\ m$



To find X :

Find L ??

62| Elastic Collision

$m_1 = 250\,g$, m_2 ??

$\vec{V}_{1i} = V$, $\vec{V}_{2i} = -V$

$V_{1f} = 0$, V_{2f} ??

a) Find m_2

$$\vec{P}_i = \vec{P}_f$$

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f}$$

$$m_1 V + m_2(-V) = m_1(0) + m_2 V_{2f}$$

$$\boxed{(m_1 - m_2) V = m_2 V_{2f}}$$

$$K_i = K_f$$

$$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

$$\frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2 = \frac{1}{2} m_1 (0) + \frac{1}{2} m_2 V_{2f}^2$$

$$\cancel{\tfrac{1}{2}}(m_1 + m_2) V^2 = \cancel{\tfrac{1}{2}} m_2 V_{2f}^2$$

$$V_{2f} = \sqrt{\frac{m_1 + m_2}{m_2}}\, V$$

$$(m_1 - m_2)\cancel{V} = m_2 \sqrt{\frac{m_1 + m_2}{m_2}}\, \cancel{V}$$

$$\frac{(m_1 - m_2)^2}{m_2^2} = \frac{m_1 + m_2}{\cancel{m_2}}$$

$$m_1^2 - 2 m_1 m_2 + \cancel{m_2^2} = m_1 m_2 + \cancel{m_2^2}$$

$$m_1^2 = 3 \cancel{m_1} m_2$$

$$m_2 = \frac{m_1}{3} = \frac{250\,g}{3} = 83.8\,g$$

or from eq. (9.75)

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2 m_2}{m_1 + m_2} V_{2i}$$

b) what is the speed of the two sphere Center of mass if the initial speed of each sphere is 2 m/s ?

$$M \vec{V}_{com} = m_1 \vec{V_1} + m_2 \vec{V_2}$$

$$(m_1 + m_2) V_{com} = m_1 V + m_2 (-V)$$

$$V_{com} = \frac{250 (2) + 83.8 (-2)}{250 + 83.8} \simeq 1 \text{ m/s}$$

V_{com} after Collision ??

sinc $\vec{F}_{net(ext)} = 0 \Rightarrow \vec{a}_{com} = 0 \Rightarrow V_{com} = \text{Cunst.}$

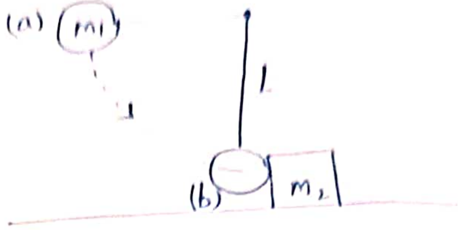
V_{com} after Collision $= 1$ m/s

$$(V_{com})_i = (V_{com})_f$$

" if $V_{com} = 0 \Rightarrow r_{com}$ will not change "

64) $m_1 = 0.6$ kg , $L = 70$ cm

$m_2 = 2.8$ kg



Find V_1, V_2 after the Collision??
"Elastic collision"

we need to find V_{1i} :

$E_a = E_b$ (for the ball)

$K_a + U_a = K_b + U_b$

$0 + \cancel{m_1} g L = \frac{1}{2} \cancel{m_1} V_1^2 + 0$

$V_1^2 = 2gL = 2(9.8)(0.7)$

$V_1 = 3.7$ m/s $= V_{1i}$ (befor Collision)

$\vec{P}_i = \vec{P}_f$

$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f}$

$m_1 V_{1i} + \cancel{m_2 (0)} = m_1 V_{1f} + m_2 V_{2f}$

$0.6(3.7) = 0.6 V_{1f} + 2.8 V_{2f}$

$\boxed{2.2 = 0.6 V_{1f} + 2.8 V_{2f}}$ — 1

$K_i = K_f$

$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

$\frac{1}{2}(0.6)(3.7)^2 + 0 = \frac{1}{2}(0.6) V_{1f}^2 + \frac{1}{2}(2.8) V_{2f}^2$

$\boxed{4.1 = 0.3 V_{1f}^2 + 1.4 V_{2f}^2}$ — 2

Instead of solving ① & ②, use eq. (9.75) & (9.76)

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2 m_2}{m_1 + m_2} V_{2i}$$

$$V_{1f} = \frac{0.6 - 2.8}{0.6 + 2.8}(3.7) + \cancel{\frac{2 m_2}{m_1 + m_2}(0)}$$

$$= -2.4 \ m/s$$

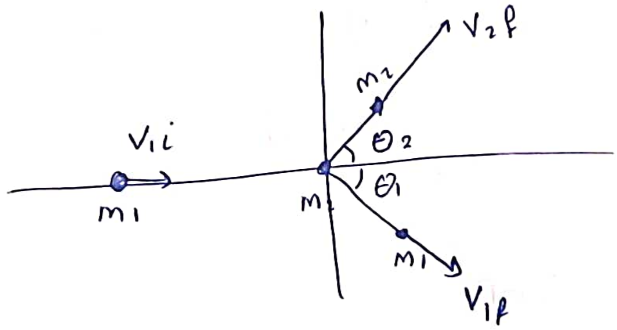
$$V_{2f} = \frac{2 m_1}{m_1 + m_2} V_{1i} + \cancel{\frac{m_2 - m_1}{m_1 + m_2} V_{2i}}$$

$$= \frac{2(0.6)}{0.6 + 2.8}(3.7) + 0$$

$$= 1.3 \ m/s$$

* Collisions in 2D :-

$$\vec{P_i} = \vec{P_f}$$



x- Comp.

$$(P_i)_x = (P_f)_x$$

$$m_1 V_{1i} + m_2(0) = m_1 V_{1f} \cos\theta_1 + m_2 V_{2f} \cos\theta_2$$

y- Comp.

$$\boxed{0 = m_1(-V_{1f} \sin\theta_1) + m_2 V_{2f} \sin\theta_2}$$

for elastic collision: $K_i = K_f$

$$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

Example :

$M_A = 2\,kg \quad , \quad \vec{V}_{Ai} = 15\hat{i} + 30\hat{j} \ m/s$

collides with

$M_B = 2\,kg \quad , \quad \vec{V}_{Bi} = -10\hat{i} + 5\hat{j} \ m/s$

after collision : $\vec{V}_{Af} = -5\hat{i} + 20\hat{j} \ m/s$

Find $\vec{V}_{Bf}$?

$$\vec{P}_i = \vec{P}_f$$

X - Comp

$$(P_i)_x = (P_f)_x$$

$$(m_A V_{Ai} + m_B V_{Bi} = m_A V_{Af} + m_B V_{Bf})_x$$

$$2(15) + 2(-10) = 2(-5) + 2(V_{Bf})_x$$

$$(V_{Bf})_x = 10 \ m/s$$

y - Comp

$$(P_i)_y = (P_f)_y$$

$$2(30) + 2(5) = 2(20) + 2(V_{Bf})_y$$

$$(V_{Bf})_y = 15 \ m/s$$

$$V_{Bf} = 10\hat{i} + 15\hat{j} \ m/s$$

Find ΔK ?

$K_f - K_i$:

$$K_i = K_{Ai} + K_{Bi}$$

$$= \frac{1}{2} m_A V_{Ai}^2 + \frac{1}{2} m_B V_{Bi}^2$$

$$= \frac{1}{2}(2)\left(\sqrt{15^2 + 30^2}\right)^2 + \frac{1}{2}(2)\left(\sqrt{10^2 + 5^2}\right)^2$$

$$= 1250 \text{ J}$$

$$K_f = \frac{1}{2} m_A V_{Af}^2 + \frac{1}{2} m_B V_{Bf}^2$$

$$= \frac{1}{2}(2)\left(\sqrt{5^2 + 20^2}\right)^2 + \frac{1}{2}(2)\left(\sqrt{10^2 + 15^2}\right)^2$$

$$= 750 \text{ J}$$

$$\Delta K = 750 - 1250 = -500 \text{ J}$$

(Not elastic Collision).