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MATH 234
QUIZ 1

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Question 1 (1 point each) Answer by true or false:

1. F If a system of linear equations is undetermined, then it must have infinitely many solutions.
2. F If $A\mathbf{x} = \mathbf{b}$ is an overdetermined and consistent linear system, then it must have infinitely many solutions.
3. T A homogeneous system can have a nontrivial solution.
4. F If a matrix is in row echelon form, then it is also in reduced row echelon form.
5. F The method of solving a linear system by reducing its augmented matrix to reduced row echelon form is called Gaussian elimination.

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Question 2 (2 points each) Circle the most correct answer:

1. If $(A|\mathbf{b}) = \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & \alpha & \beta \end{array}\right]$ is the augmented matrix of the system $A\mathbf{x} = \mathbf{b}$. Answer the following questions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2+\alpha & 1+\beta \end{array}\right]$$

0 0 0

$2+\alpha=0$

$\alpha=-2$

$\beta \neq -1$

(a) The system has no solution if
   i. $\alpha = -2$ and $\beta \neq -1$
   ii. $\alpha = -2$ and $\beta = -1$
   iii. $\alpha \neq -2$ and $\beta \neq -1$
   iv. $\alpha \neq -2$ and $\beta = -1$

$\alpha \neq -2$,

(b) The system has exactly one solution if
   i. $\alpha = -2$ and $\beta = -1$
   ii. $\alpha \neq -2$
   iii. $\alpha = -2$
   iv. $\alpha \neq -2$ and $\beta \neq -1$

(c) The system has infinitely many solutions if
   i. $\alpha \neq -2$ and $\beta \neq -1$
   ii. $\alpha = -2$ and $\beta \neq -1$
   iii. $\alpha = -2$ and $\beta = -1$
   iv. $\alpha \neq -2$ and $\beta = -1$

$\alpha=-2$, $\beta=-1$
and

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RREF

2. One of the following matrices is in $\boxed{\text{reduced row echelon form}}$

(a) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ X

(b) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ ✓

(c) $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ X

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ x

Augemented → RREF

Question 2 (9 points) Use $\boxed{\text{Guauss-Jordan reduction}}$ to solve the system

dor determine no sol.

3X4 → infinit

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & - & 3x_3 & + & x_4 & = & 1 \\ -x_1 & - & x_2 & + & 4x_3 & - & x_4 & = & 6 \\ -2x_1 & - & 4x_2 & + & 7x_3 & - & x_4 & = & 1 \end{array}$$

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$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{array}\right] \xrightarrow[2R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_1} \left[\begin{array}{cccc|c} 1 & 0 & -5 & 0 & -13 \\ 0 & 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 0 & -5 & 0 & -13 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_2 - R_3} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right] \xrightarrow{-2R_2+R_1} \left[\begin{array}{cccc|c} 1 & 0 & -3 & 3 & -7 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right]$$

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$$\xrightarrow{+3R_3+R_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 2 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & 4 & 7 & -1 & 7 \end{array}\right] \xrightarrow[R_1+R_2 \to R_2]{R_1+R_3 \to R_3} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 7 \\ -1 & 6 & 4 & 0 & 8 \end{array}\right]$$

$$\xrightarrow{R_3+R_1} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 7 \\ 0 & 8 & 1 & 1 & 9 \end{array}\right] \xrightarrow{-4R_1+R_3} \Big[$$

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$x_4 = t$, because it's free variable

so infinite number of sol.

$x_3 + x_4 = 3$

$\boxed{x_3 = 3 - t}$

$x_2 - x_4 = 4$

$\boxed{x_2 = 4 + t}$

$x_1 + 6x_4 = 2$

$\boxed{x_1 = 2 - 6t}$

~~the set of sol =~~

$\cancel{\{(x_1, x_2, x_3, x_4) : (2-6t, 4+t}$

$\{(x_1, x_2, x_3, x_4) : (2-6t, 4+t, 3-t, t)$

$t \in \mathbb{R}$

the set of solution.