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		Factor B		Factor A
		Small	Large	Means
	A	$\bar{x}_{11} = 10$	$\bar{x}_{12} = 10$	$\bar{x}_{1} = 10$
Factor A	B_2	$\bar{x}_{21} = 18$	$\bar{x}_{22} = 28$	$\bar{x}_{2} = 23$
	C	$\bar{x}_{31} = 14$	$\bar{x}_{32} = 16$	$\bar{x}_{3} = 15$
Factor B	Means	$\bar{x}_{.1} = 14$	$\bar{x}_{.2} = 18$	$\bar{\bar{x}} = 16$

SST = 544
SSA =344
SSB = 48
SSAB = 56
SSE = 96

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Factor A	2	344	172	172/16 = 10.75
Factor B	1	48	48	48/16 = 3.00
Interaction	2	56	28	28/16 = 1.75
Error	6	96	16	
Total	11	544		

Using F table for Factor A (2 degrees of freedom numerator and 6 denominator), p-value is between 0.01 and 0.025

Actual p-value = 0.0104

Because p-value $\le \alpha = 0.05$, Factor A is significant; there is a difference due to the type of advertisement design

Using F table for Factor B (1 degree of freedom numerator and 6 denominator), p-value is greater than 0.01

Actual p-value = 0.1340

Because p-value $> \alpha = 0.05$, Factor B is not significant; there is not a significant difference due to size of advertisement

Using F table for Interaction (2 degrees of freedom numerator and 6 denominator), p-value is greater than 0.10

Actual p-value = 0.2519

Because p-value $> \alpha = 0.05$, Interaction is not significant.

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$\bar{x}_{1.} = (1.13 + 1.56 + 2.00)/3 = 1.563$

$\bar{x}_{2.} = (0.48 + 1.68 + 2.86)/3 = 1.673$

$\bar{x}_{.1} = (1.13 + 0.48)/2 = 0.805$

$\bar{x}_{.2} = (1.56 + 1.68)/2 = 1.620$

$\bar{x}_{.3} = (2.00 + 2.86)/2 = 2.43$

$\bar{\bar{x}} = 1.618$

SST = 327.50 (given in problem statement)
SSA = 0.4538
SSB = 66.0159
SSAB = 14.2525
SSE = 246.7778

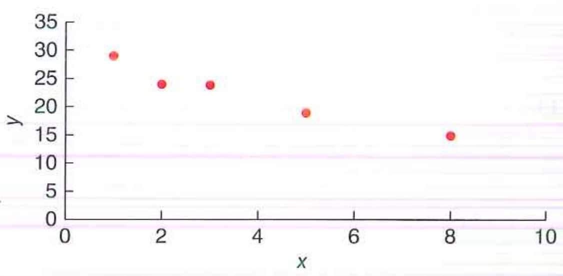
Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Factor A	1	0.4538	0.4538	0.2648
Factor B	2	66.1059	33.0080	19.2608
Interaction	2	14.2525	7.1263	4.1583
Error	144	246.7778	1.7137	
Total	149	327.5000		

Factor A: Actual p-value = 0.6076. Because p-value $> \alpha = 0.05$, Factor A is not significant. Factor B: Actual p-value = 0.0000. Because p-value $\le \alpha = 0.05$, Factor B is significant. Interaction: Actual p-value = 0.0176. Because p-value $\le \alpha = 0.05$. Interaction is significant.

Chapter 14

Solutions

2 a.



b. There appears to be a linear relationship between x and y.

c. Many different straight lines can be drawn to provide a linear approximation of the relationship between x *and* y; in part d we will determine the equation of a straight line that 'best' represents the relationship according to the least squares criterion.

d. Summations needed to compute the slope and y-intercept are:

$\sum x_i = 19 \quad \sum y_i = 116 \quad \sum(x_i - \bar{x})(y_i - \bar{y}) = -57.8$

$\sum(x_i - \bar{x})^2 = 30.8$

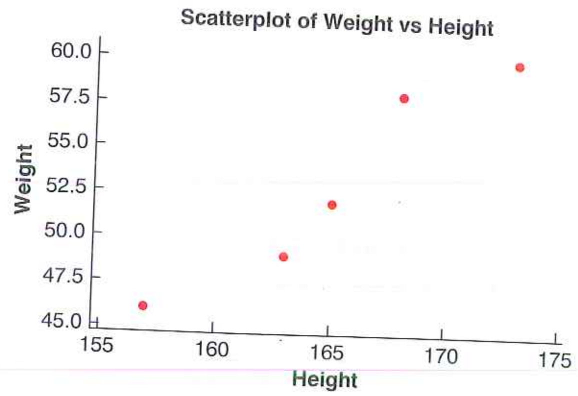
$b_1 \dfrac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{57.8}{30.8} = -1.8766$

$b_0 = \bar{y} - b_1\bar{x} = 23.2 - (-1.8766)(3.8) = 30.3311$

$\hat{y} = 30.33 - 1.88x$

e. $\hat{y} = 30.33 - 1.88(6) = 19.05$

4 a.



b. There appears to be a linear relationship between x and y.

c. Many different straight lines can be drawn to provide a linear approximation of the relationship between x *and* y; in

d. Summations needed to compute the slope and y-intercept are:

$\sum x_i = 826 \quad \sum y_i = 265 \quad \sum(x_i - \bar{x})(y_i - \bar{y})$
$= 135 \quad \sum(x_i - \bar{x})^2 = 140.8$

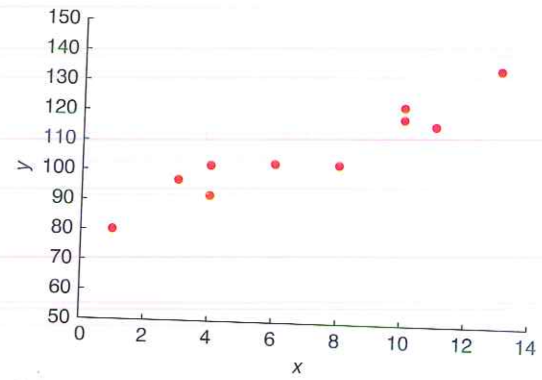
$b_1 = \dfrac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = \dfrac{135}{140.8} = 0.9588$

$b_0 = \bar{y} - b_1\bar{x} = 53 - (0.9588)(165.2) = -105.39$

$\hat{y} = -105.39 + 0.9588x$

e. $\hat{y} = -105.39 + 0.9588(160) = 48.02$ kg

6 a.



b. Summations needed to compute the slope and y-intercept are:

$\sum x_i = 70 \quad \sum y_i = 1080 \quad \sum(x_i - \bar{x})(y_i - \bar{y}) = 568$

$\sum(x_i - \bar{x})^2 = 142$

$b_1 = \dfrac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = \dfrac{568}{142} = 4$

$b_0 = \bar{y} - b_1\bar{x} = 108 - (4)(7) = 80$

$\hat{y} = 80 + 4x$

c. $\hat{y} = 80 + 4(9) = 116$

8 a. The estimated regression equation and the mean for the dependent variable are:

$\hat{y} = 30.33 - 1.88x \quad \bar{y} = 23.2$

The sum of squares due to error and the total sum of squares are

$SSE = \sum(y_i - \hat{y}_i)^2 = 6.33$

$SST = \sum(y_i - \bar{y})^2 = 114.80$

Thus, SSR = SST − SSE = 114.80 − 6.33 = 108.47

b. r^2 = SSR/SST = 108.47/114.80 = 0.945

The least squares line provided an excellent fit; 94.5 per cent of the variability in y has been explained by the estimated regression equation.

c. $r = \sqrt{0.945} = 0.9721$

Note: the sign for r is negative because the slope of the estimated regression equation is negative. ($b_1 = -1.88$)

10 a. The estimated regression equation and the mean for the dependent variable are:

$\hat{y} = -75.586 + 0.115x \quad \bar{y} = 784.215$

The sum of squares due to error and the total sum of squares are

SSE = 10083.87
SST = 1481257

Thus, SSR = 1471173.13

b. r^2 = SSR/SST = 1471173.13/1481257 = 0.99

We see that 99 per cent of the variability in y has been explained by the least squares line.

c. $r = \sqrt{0.99} = +0.99$

12 a. Let x = speed (ppm) and y = price (€)

The summations needed in this problem are:

$\Sigma x_i = 188 \quad \Sigma y_i = 953.97 \quad \Sigma(x_i - x)(y_i - y)$
$= 324.864 \quad \Sigma(x_i - \bar{x})^2 = 83.6$

$b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{324.864}{83.6} = 3.886$

$b_0 = \bar{y} - b_1\bar{x} = 95.397 - (3.886)(18.8) = 22.341$

$\hat{y} = 22.341 + 3.886x$

b. The sum of squares due to error and the total sum of squares are:

SSE = 3746.309
SST = 5008.708

Thus, SSR = 1262.399

r^2 = SSR/SST = 1262.399/5008.708 = 0.252

Approximately 25 per cent of the variability in price is explained by the speed.

c. $r = \sqrt{0.252} = +0.50$

It reflects a weak linear relationship.

14 a. s^2 = MSE = SSE/(n − 2) = 6.33/3 = 2.11

b. $s = \sqrt{\text{MSE}} = \sqrt{2.11} = 1.453$