

QUIZ 1

Student Name: Answer Key Student Number: _____

1. Prove that if $ab = 0$ and $a + b = 0$, then $a = 0$ and $b = 0$. Justify each step.

Suppose $ab = 0$ and $a + b = 0$

$ab = 0 \Rightarrow a = 0$ or $b = 0$, we proved this result in class.

Case 1. $a = 0$

$$\Rightarrow b = 0 + b, \text{ additive identity}$$

$$= a + b, \text{ since } a = 0$$

$$= 0, \text{ since } a + b = 0 \text{ by hypothesis}$$

Case 2. $b = 0$, repeat the argument in case 1 but replace b by a

Case 1 & Case 2 $\Rightarrow a = 0$ & $b = 0$.

2. Let $a, b, c \in \mathbb{R}$, prove that if $a \leq b$ and $c \geq 0$, then $ac \leq bc$. Justify each step.

Case 1. $a < b$ and $c = 0$

$$\Rightarrow a \cdot 0 = 0 \text{ \& } b \cdot 0 = 0 \Rightarrow ac = bc \Rightarrow ac \leq bc$$

Case 2. $a < b$ & $c > 0$

$$\Rightarrow ac < bc$$

$$\Rightarrow ac \leq bc$$

, multiplicative property

Case 3. $a = b$ & $c = 0$

$$\Rightarrow ac = bc \Rightarrow ac \leq bc, \text{ similar to case 1}$$

Case 4. $a = b$ & $c > 0$

$$\Rightarrow ac = bc, \text{ replace } a \text{ by } b \text{ since } a = b$$

$$\Rightarrow ac \leq bc$$