Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Engineering Probability and Statistics ENEE 2307
Quiz # 1

Instructors: Dr. Wael Hashlamoun, Date: October 12, 2017

Problem:
A system is made up of two components A and B connected in parallel. The system works if at least one component works. The probability that A works is $P(A) = 1/4$ and the probability that B works is $P(B) = 1/3$. Given that $P(A \cap B) = 1/8$

a. Are A and B independent?
b. Find the probability that the system works.
c. Find the probability that either A or B but not both work.

a. $P(A \cap B) = \frac{1}{8} \neq P(A) \cdot P(B) = \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{12}$

$\Rightarrow$ A and B are dependent 3/10

b. $P(\text{system works}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{3} - \frac{1}{8}$

$= \frac{44}{96}$ $\frac{4}{10}$

c. $P(\text{A or B but not both}) = P(A \cup B) - P(A \cap B)$

$= \frac{44}{96} - \frac{1}{8} = \frac{256}{768} = \frac{1}{3}$

$\frac{3}{10}$

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The discrete probability function for a sample space S is:

$$P(x) = kx(x+1) \; ; \qquad 1 \le x \le 4.$$

Let $A = \{1, 2\}$, $B = \{2\}$ be two events defined over S.

1. Find k so that $P(x)$ is a valid discrete probability function.
2. Are A and B independent? Explain

1. $\sum_{x=1}^{4} P(x) = 1$

$k[2 + 6 + 12 + 20] = 1 \Rightarrow k = \frac{1}{40}$ $\frac{4}{10}$

2/10

2. $P(A) = P(1) + P(2) = \frac{1}{40}[2+6] = \frac{8}{40}$

$P(B) = P(2) = \frac{6}{40}$

2/10

$\{A \cap B\} = \{2\} \Rightarrow P(A \cap B) = P(2) = \frac{6}{40}$

Test $P(A \cap B) \overset{?}{=} P(A)\,P(B)$ $\frac{2}{10}$

$\left(\frac{6}{40}\right) \underset{NO}{\neq} \left(\frac{8}{40}\right)\left(\frac{6}{40}\right)$

$\Rightarrow$ A and B are dependent

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Quiz # 2

Instructors: Dr. Wael Hashlamoun, Date: November 2, 2017

Mugs produced by a certain pottery factory have an 8 % defective rate. A quality assurance inspector randomly selects five mugs and inspects them.

a. Find the probability that the inspector finds at least one defective mug.
b. What is the probability that all five mugs are detective?

Binomial Distribution

$$p(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

know that $\Rightarrow \frac{4}{10}$

a. $p = 0.08$, $n = 5$

$$p(X \geq 1) = 1 - p(X=0)$$
$$= 1 - \binom{5}{0}(0.08)^0 (0.92)^5$$
$$= 1 - 0.65908$$
$$= 0.3409$$

$\frac{4}{10}$

b. $$p(X=5) = \binom{5}{5}(0.08)^5 (1-0.08)^0$$
$$= (0.08)^5$$
$$= 3.2768 \times 10^{-6}$$

$\frac{2}{10}$

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Quiz # 2

Instructors: Dr. Wael Hashlamoun, Date: November 2, 2017

Let X be a random variable with the following probability density function:

$$f_X(x) = \begin{cases} kx, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$$

a. Find k so that $f_X(x)$ is a valid probability density function
b. Find the variance of X

a. $\int_0^2 f_x(x) = 1 = \int_0^2 k\, x\, dx = k\left(\frac{x^2}{2}\right|_0^2 = 2k$

$\Rightarrow k = 1/2$ 4/10

b. $\sigma_x^2 = E(x^2) - E^2(x)$ 2/10

$E(x) = \int_0^2 x\, f_x(x) dx = \int_0^2 k x^2 dx = \frac{4}{3}$ 2/10

same

$E(x^2) = \int_0^2 x^2 f_x(x) dx = \int_0^2 k x^3 dx = 2$ 2/10

$\sigma_x^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$

Also, $\sigma_x^2 = \int_0^2 \left(x - \frac{4}{3}\right)^2 k x\, dx = \frac{2}{9}$

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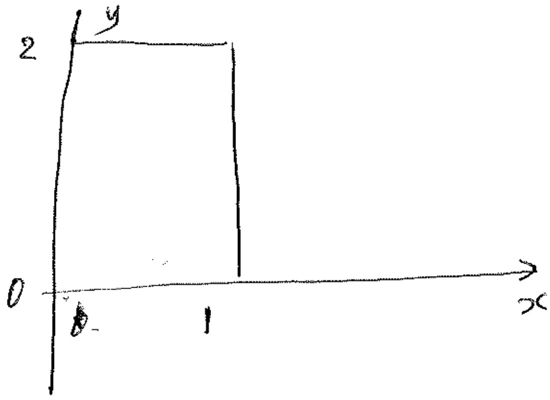
Quiz # 3

Instructors: Dr. Wael Hashlamoun, Date: December 7, 2017

The two random variables X and Y are related by the following joint probability density function (pdf)

$$f_{X,Y}(x,y) = \begin{cases} kx, & 0 \le x \le 1, \quad 0 \le y \le 2 \\ 0, & otherwise \end{cases}$$



a) Find k so that $f_{X,Y}(x,y)$ is a valid joint pdf.
b) Compute the marginal pdfs $f_X(x)$ and $f_Y(y)$

a. $K\int_0^2\int_0^1 kx \, dx \, dy = 1$

$K\int_0^2 \frac{x^2}{2}\Big|_0^1 dy = 1$

$\frac{K}{2}\int_0^2 dy = 1 \Rightarrow \frac{K}{2} y\Big|_0^2 = 1 \Rightarrow \boxed{k = 1}$ $\frac{4}{10}$

b. $f_X(x) = \int_0^2 x \, dy$ 3/10 $= \begin{cases} 2x & 0 \le x \le 1 \\ 0 & o.w \end{cases}$

$f_Y(y) = \int_0^1 x \, dx$ 3/10 $= \begin{cases} \frac{1}{2} & 0 \le y \le 2 \\ 0 & o.w \end{cases}$

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Quiz # 3

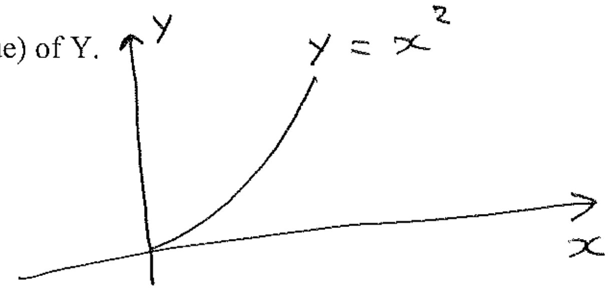
Instructors: Dr. Wael Hashlamoun, Date: November 7, 2017

Let X be a random variable with the following probability density function:

$$f_X(x) = \begin{cases} \frac{1}{2}x, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$$

If Y is another random variable, related to X by $Y = X^2$,

a. Find the probability density function of Y and the interval over which it is define.
b. Find the mean value (i.e., the expected value) of Y.



$$f_X(x) = \begin{cases} \frac{1}{2}x & 0 \leq x \leq 2 \\ 0 & o.w \end{cases}$$

a. $f_Y(y) = \dfrac{f_X(x)}{|dy/dx|}$

$= \dfrac{\frac{1}{2}x}{2x} = \dfrac{1}{4}$ 4/10 part a: $\frac{6}{10}$

when $x = 0$, $y = 0$
$x = 2$, $y = 4$ $0 \leq y \leq 4$ 2/10

$\Rightarrow f_Y(y) = \begin{cases} \frac{1}{4} \\ 0 & o.w \end{cases}$

b. can be solved in two ways (both are correct)

1. $E(y) = E(x^2) = \int_0^2 x^2 f_X(x)\,dx = \int_0^2 x^2 \cdot \frac{1}{2} x\,dx$ 4/10

$= \frac{1}{2} \left.\frac{x^4}{4}\right|_0^2 = \frac{16}{8} = \boxed{2}$

2. $E(y) = \int_0^4 y\, f_Y(y)\,dy = \int_0^4 y \cdot \frac{1}{4}\,dy = \frac{1}{4} \left.\frac{y^2}{2}\right|_0^4 = \frac{16}{8}$

$\boxed{= 2}$

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Quiz # 4

Instructors: Dr. Wael Hashlamoun, Date: January 9, 2018

The speed of a vehicle, measured at different times, is given in the table

Time(s) x	1	1.3	2.3	3	3.5	4
Speed (m/s) y	5.7	6.3	7.4	8.4	11.9	13.7

We suspect that the data follow a linear function of the form $y = \alpha x + \beta$.

a. Set up the necessary equations needed to determine α and β.
b. Solve the equations in Part a for α and β.

$$y = \alpha x + \beta$$

$$\begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix}$$

$$\begin{pmatrix} 6 & 15.1 \\ 15.1 & 45.23 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} 53.4 \\ 152.56 \end{pmatrix} \qquad \frac{7}{10}$$

$$\begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} 2.5738 \\ 2.5137 \end{pmatrix} \qquad \frac{3}{10}$$

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Quiz # 2

Instructors: Dr. Wael Hashlamoun, Date: October 25, 2018

The continuous probability function for a sample space S is:

$$f(x) = \frac{2}{15}x \; ; \qquad 1 \le x \le 4.$$

a. Find the mean (expected) value of X
b. Find the variance of X

$$E(X) = \mu_X = \int_1^4 x \, f_X(x) \, dx = \int_1^4 x \left(\frac{2}{15}\right) x \, dx$$

$$= \frac{2}{15} \int_1^4 x^2 \, dx = \frac{2}{15} \left(\frac{x^3}{3}\right|_1^4$$

$$= \frac{126}{45} = 2.8 \quad 4$$

$$\sigma_X^2 = E(X^2) - \mu_X^2 \quad \text{Formula} \quad 2$$

$$E(X^2) = \int_1^4 x^2 \, f_X(x) \, dx = \int_1^4 x^2 \cdot \left(\frac{2}{15} x\right) dx$$

$$= \frac{2}{15} \cdot \int_1^4 x^3 \, dx = \frac{2}{15} \left(\frac{x^4}{4}\right|_1^4$$

$$= \frac{2}{60} (256 - 1) = \frac{510}{60} = 8.5 \quad 3$$

$$\sigma_X^2 = 8.5 - (2.8)^2$$

$$= 0.66 \qquad 1 \quad \text{answer}$$

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Quiz # 4

Instructors: Dr. Wael Hashlamoun   Date: August 9, 2018

Let X and Y be two independent random variables with the marginal pdf's

$$f_X(x) = \begin{cases} 1/2, & 0 \le x \le 2, \\ 0, & otherwise \end{cases} \qquad f_Y(y) = \begin{cases} 1/3, & 0 \le y \le 3, \\ 0, & otherwise \end{cases}$$

Define Z=X + 2Y

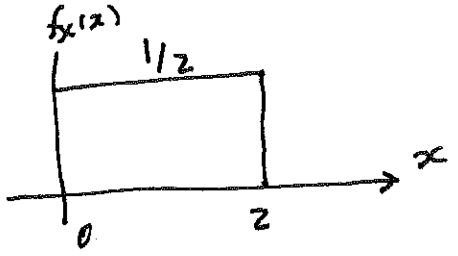
a) Find the joint pdf $f_{X,Y}(x,y)$ of the two variables and the region over which it is defined

b) Find the mean and variance of Z.

a. since X and Y are independent, then   4/10

$$f_{x,y}(x,y) = f_x(x)\, f_y(y)$$

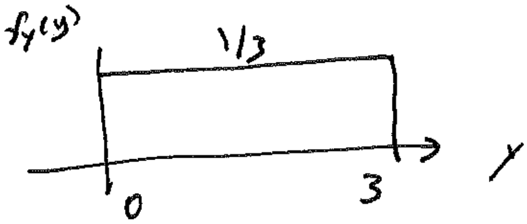
$$= \begin{cases} \frac{1}{6} & 0 \le x \le 2, \quad 0 \le y \le 3 \\ 0 & o.w \end{cases}$$



b. For X

$$\mu_x = \frac{a+b}{2} = 1$$ 1.5

$$\sigma_x^2 = \frac{(b-a)^2}{12} = \frac{4}{12} = \frac{1}{3}$$



For Y

$$\mu_y = \frac{a+b}{2} = \frac{3}{2}$$ 1.5

$$\sigma_y^2 = \frac{(b-a)^2}{12} = \frac{9}{12}$$

$$z = x + 2y$$

1.5 $$\mu_z = \mu_x + 2\mu_y = (1) + 2\left(\frac{3}{2}\right) = 4$$

1.5 $$\sigma_z^2 = \sigma_x^2 + 4\sigma_y^2$$

$$= \frac{1}{3} + 4 \cdot \frac{9}{12} = \frac{1+9}{3} = \frac{10}{3}$$