6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 **[a]** $i_g = 8e^{-300t} - 8e^{-1200t}\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.44766, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}$$

$$x = 11.0523, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu$s

[e] $p_{\max} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\text{ms}$.

[g] $i_{\max} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\max} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

AP 6.2 **[a]** $i = C\frac{dv}{dt} = 24 \times 10^{-6}\frac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 **[a]** $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv^2_{\text{max}} = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 **[a]** $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$

$i_1 + i_2 = i$

AP 6.5 $v_1 = 0.5 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$v_2 = 0.125 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$

AP 6.6 **[a]** Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

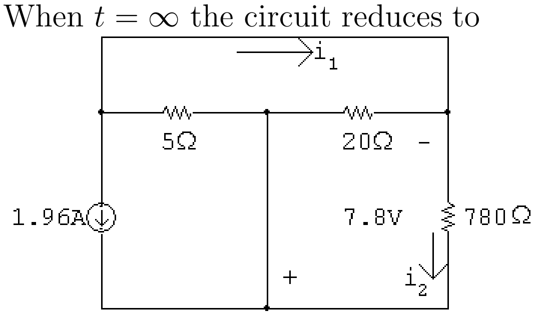
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\quad \dfrac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$

$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$

$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$

$$-8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$

$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

Problems

P 6.1 **[a]** $v = L\frac{di}{dt}$

$$= (150 \times 10^{-6})(25)[e^{-500t} - 500te^{-500t}] = 3.75e^{-500t}(1 - 500t)\,\text{mV}$$

[b] $i(5\,\text{ms}) = 25(0.005)(e^{-2.5}) = 10.26\,\text{mA}$

$$v(5\,\text{ms}) = 0.00375(e^{-2.5})(1 - 2.5) = -461.73\,\mu\text{V}$$

$$p(5\,\text{ms}) = vi = (10.26 \times 10^{-3})(-461.73 \times 10^{-6}) = -4.74\,\mu\text{W}$$

[c] delivering $4.74\,\mu$W

[d] $i(5\,\text{ms}) = 10.26\,\text{mA}$ (from part [b])

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(150 \times 10^{-6})(0.01026)^2 = 7.9\,\text{nJ}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 500t = 0 \quad \text{or} \quad t = 2\,\text{ms}$$

$$i_{\text{max}} = 25(0.002)e^{-1} = 18.39\,\text{mA}$$

$$w_{\text{max}} = \frac{1}{2}(150 \times 10^{-6})(0.01839)^2 = 25.38\,\text{nJ}$$

P 6.2 **[a]**

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 4t\,\text{A} && 0 \le t \le 25\,\text{ms} \\ i &= 0.2 - 4t\,\text{A} && 25 \le t \le 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\frac{di}{dt} = 500 \times 10^{-3}(4) = 2\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$$v = 500 \times 10^{-3}(-4) = -2\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 2\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -2\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$$p = vi$$

$$\begin{aligned}
p &= 0 && t < 0 \\
p &= (4t)(2) = 8t\,\text{W} && 0 < t < 25\,\text{ms} \\
p &= (0.2 - 4t)(-2) = 8t - 0.4\,\text{W} && 25 < t < 50\,\text{ms} \\
p &= 0 && 50\,\text{ms} < t
\end{aligned}$$

$$\begin{aligned}
w &= 0 && t < 0 \\
w &= \int_0^t (8x)\,dx = 8\frac{x^2}{2}\Big|_0^t = 4t^2\,\text{J} && 0 < t < 25\,\text{ms} \\
w &= \int_{0.025}^t (8x - 0.4)\,dx + 2.5 \times 10^{-3} \\
&= 4x^2 - 0.4x\Big|_{0.025}^t + 2.5 \times 10^{-3} \\
&= 4t^2 - 0.4t + 10 \times 10^{-3}\,\text{J} && 25 < t < 50\,\text{ms} \\
w &= 0 && 10\,\text{ms} < t
\end{aligned}$$

P 6.3 **[a]** $i(0) = A_1 + A_2 = 0.12$

$$\frac{di}{dt} = -500A_1e^{-500t} - 2000A_2e^{-2000t}$$

$$v = -25A_1e^{-500t} - 100A_2e^{-2000t}\,\text{V}$$

$$v(0) = -25A_1 - 100A_2 = 3$$

Solving, $A_1 = 0.2$ and $A_2 = -0.08$

Thus,

$$i = 200e^{-500t} - 80e^{-2000t}\,\text{mA} \qquad t \geq 0$$

$$v = -5e^{-500t} + 8e^{-2000t}\,\text{V}, \qquad t \geq 0$$

[b] $i = 0$ when $200e^{-500t} = 80e^{-2000t}$

Therefore

$e^{1500t} = 0.4$ so $t = -610.86\,\mu\text{s}$ which is not possible!

$v = 0$ when $5e^{-500t} = 8e^{-2000t}$

Therefore

$e^{1500t} = 1.6$ so $t = 313.34\,\mu\text{s}$

Thus the power is zero at $t = 313.34\,\mu\text{s}$.

P 6.4 [**a**] From Problem 6.3 we have

$$i = A_1e^{-500t} + A_2e^{-2000t}\,\text{A}$$

$$v = -25A_1e^{-500t} - 100A_2e^{-2000t}\,\text{V}$$

$$i(0) = A_1 + A_2 = 0.12$$

$$v(0) = -25A_1 - 100A_2 = -18$$

Solving, $A_1 = -0.08;$ $A_2 = 0.2$

Thus,

$$i = -80e^{-500t} + 200e^{-2000t}\,\text{mA} \quad t \geq 0$$

$$v = 2e^{-500t} - 20e^{-2000t}\,\text{V} \quad t \geq 0$$

[**b**] $i = 0 \quad \text{when} \quad 80e^{-500t} = 200e^{-2000t}$

$$\therefore \quad e^{1500t} = 2.5 \quad \text{so} \quad t = 610.86\,\mu\text{s}$$

Thus,

$$i > 0 \quad \text{for} \quad 0 \leq t < 610.86\,\mu\text{s} \qquad \text{and} \qquad i < 0 \quad \text{for} \quad 610.86\,\mu\text{s} < t < \infty$$

$$v = 0 \quad \text{when} \quad 2e^{-500t} = 20e^{-2000t}$$

$$\therefore \quad e^{1500t} = 10 \quad \text{so} \quad t = 1535.06\,\mu\text{s}$$

Thus,

$$v < 0 \quad \text{for} \quad 0 \leq t < 1535.06\,\mu\text{s} \qquad \text{and} \qquad v > 0 \quad \text{for} \quad 1535.06\,\mu\text{s} < t < \infty$$

Therefore,

$$p < 0 \quad \text{for} \quad 0 \leq t < 610.86\,\mu\text{s} \qquad \text{and} \qquad 1535.06\,\mu\text{s} < t < \infty$$

(inductor delivers energy)

$$p > 0 \quad \text{for} \quad 610.86\,\mu\text{s} < t < 1535.06\,\mu\text{s} \quad \text{(inductor stores energy)}$$

[**c**] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(0.05)(0.12)^2 = 360\,\mu\text{J}$$

$$p = vi = -0.16e^{-1000t} + 2e^{-2500t} - 4e^{-4000t}\,\text{W}$$

For $t > 0$:

$$w = \int_0^\infty -0.16e^{-1000t}\,dt + \int_0^\infty 2e^{-2500t}\,dt - \int_0^\infty 4e^{-4000t}\,dt$$

$$= \frac{-0.16e^{-1000t}}{-1000}\Big|_0^\infty + \frac{2e^{-2500t}}{-2500}\Big|_0^\infty - \frac{4e^{-4000t}}{-4000}\Big|_0^\infty$$

$$= (-1.6 + 8 - 10) \times 10^{-4}$$

$$= -360\,\mu\text{J}$$

Thus, the energy stored equals the energy extracted.

P 6.5 $i = (B_1 \cos 200t + B_2 \sin 200t)e^{-50t}$

$i(0) = B_1 = 75\,\text{mA}$

$$\frac{di}{dt} = (B_1 \cos 200t + B_2 \sin 200t)(-50e^{-50t}) + e^{-50t}(-200B_1 \sin 200t + 200B_2 \cos 200t)$$

$$= [(200B_2 - 50B_1)\cos 200t - (200B_1 + 50B_2)\sin 200t]e^{-50t}$$

$$v = 0.2\frac{di}{dt} = [(40B_2 - 10B_1)\cos 200t - (40B_1 + 10B_2)\sin 200t]e^{-50t}$$

$v(0) = 4.25 = 40B_2 - 10B_1 = 40B_2 - 0.75 \quad \therefore \quad B_2 = 125\,\text{mA}$

Thus,

$i = (75 \cos 200t + 125 \sin 200t)e^{-50t}\,\text{mA}, \qquad t \geq 0$

$v = (4.25 \cos 200t - 4.25 \sin 200t)e^{-50t}\,\text{V}, \qquad t \geq 0$

$i(0.025) = -28.25\,\text{mA}; \qquad v(0.025) = 1.513\,\text{V}$

$p(0.025) = (-28.25)(1.513) = -42.7\,\text{mW}$ delivering

P 6.6 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$\text{W} = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\text{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.7 **[a]** $0 \leq t \leq 50\,\text{ms}$:

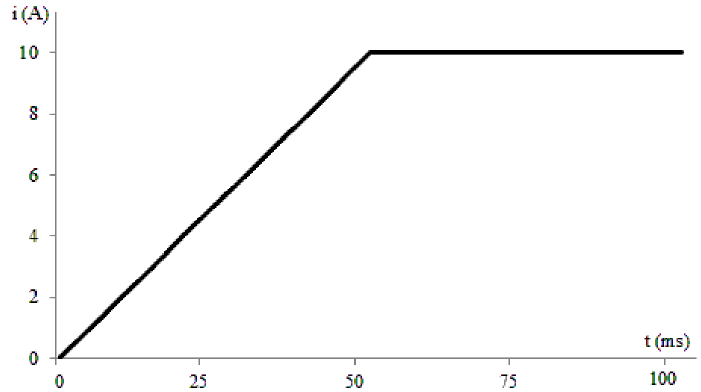
$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{750}\int_0^t 0.15\,dx + 0$$

$$= 200x\,\Big|_0^t = 200t\,\text{A}$$

$i(0.05) = 200(0.05) = 10\,\text{A}$

$t \geq 50\,\text{ms}$: $\qquad i = \dfrac{10^6}{750}\displaystyle\int_{50\times10^{-3}}^t (0)\,dx + 10 = 10\,\text{A}$

[b] $i = 200t$ A, $0 \le t \le 50$ ms; $i = 10$ A, $t \ge 50$ ms



P 6.8 $0 \le t \le 100\,\text{ms}:$

$$i_L = \frac{10^3}{50}\int_0^t 2e^{-100x}\,dx + 0.1 = 40\frac{e^{-100x}}{-100}\Big|_0^t + 0.1$$

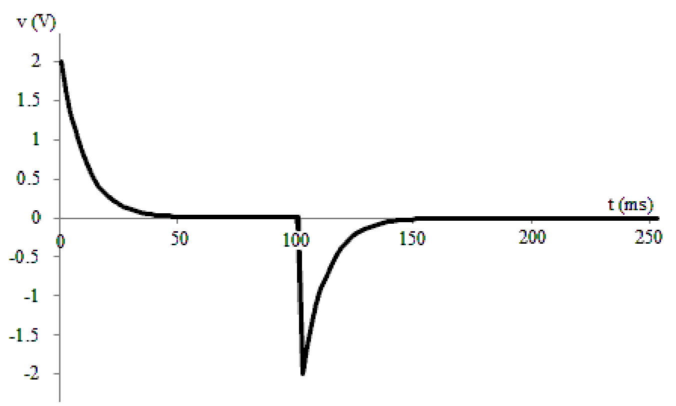
$$= -0.4e^{-100t} + 0.5\,\text{A}$$

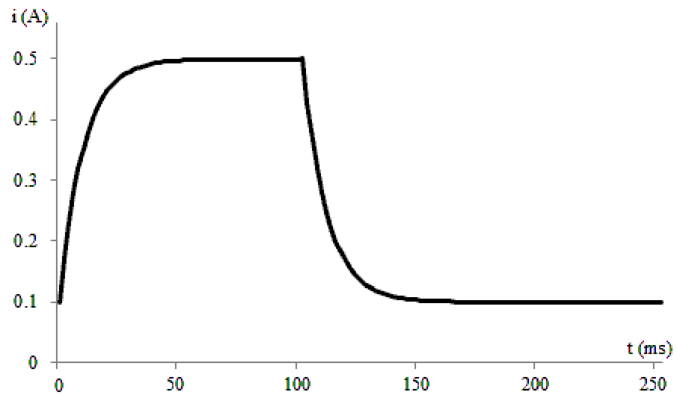
$$i_L(0.1) = -0.4e^{-10} + 0.5 = 0.5\,\text{A}$$

$t \ge 100\,\text{ms}:$

$$i_L = \frac{10^3}{50}\int_{0.1}^t -2e^{-100(x-0.1)}\,dx + 0.5 = -40\frac{e^{-100(x-0.1)}}{-100}\Big|_{0.1}^t + 0.5$$

$$= 0.4e^{-100(t-0.1)} + 0.1\,\text{A}$$





P 6.9 **[a]** $0 \le t \le 25\,\text{ms}:$

$$v = 800t$$

$$i = \frac{1}{10}\int_0^t 800x\,dx + 0 = 80\frac{x^2}{2}\Big|_0^t$$

$$i = 40t^2\ \text{A}$$

$25\,\text{ms} \le t \le 75\,\text{ms}:$

$$v = 20$$

$$i(0.025) = 25\,\text{mA}$$

$$\therefore\ i = \frac{1}{10}\int_{0.025}^t 20\,dx + 0.025$$

$$= 2x\Big|_{0.025}^t + 0.025$$

$$= 2t - 0.025\ \text{A}$$

$75\,\text{ms} \le t \le 125\,\text{ms}:$

$$v = 80 - 800t\ \text{V}$$

$$i(0.075) = 2(0.075) - 0.025 = 0.125\ \text{A}$$

$$i = \frac{1}{10}\int_{0.075}^t (80 - 800x)\,dx + 0.125$$

$$= \left(8x - \frac{80x^2}{2}\right)\Big|_{0.075}^t + 0.125$$

$$= 8t - 40t^2 - 0.25\ \text{A}$$

$125\,\text{ms} \le t \le 150\,\text{ms}:$

$$v = 800t - 120$$

$$i(0.125) = 8(0.125) - 40(0.125)^2 - 0.25 = 0.125\,\text{A}$$

$$i = \frac{1}{10}\int_{0.125}^{t} (800x - 120)\,dx + 0.125$$

$$= \left(\frac{80x^2}{2} - 12x\right)\Big|_{0.125}^{t} + 0.125$$

$$= 40t^2 - 12t + 1\,\text{A}$$

$t \geq 150\,\text{ms}:$

$v = 0$

$$i(0.150) = 40(0.15)^2 - 12(0.15) + 1 = 0.1\,\text{A}$$

$$i = \frac{1}{10}\int_{0.15}^{t} 0\,dx + 0.1$$

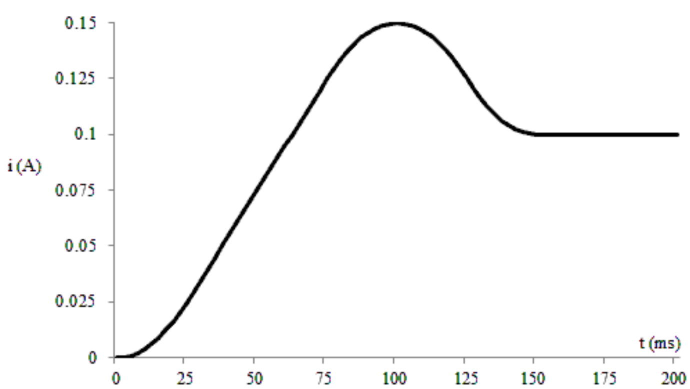
$$= 0.1\,\text{A}$$

[b] $v = 0$ at $t = 100\,\text{ms}$ and $t = 150\,\text{ms}$

$$i(0.1) = 8(0.1) - 40(0.1)^2 - 0.25 = 0.15\,\text{A}$$

$$i(0.15) = 0.1\,\text{A}$$

[c]



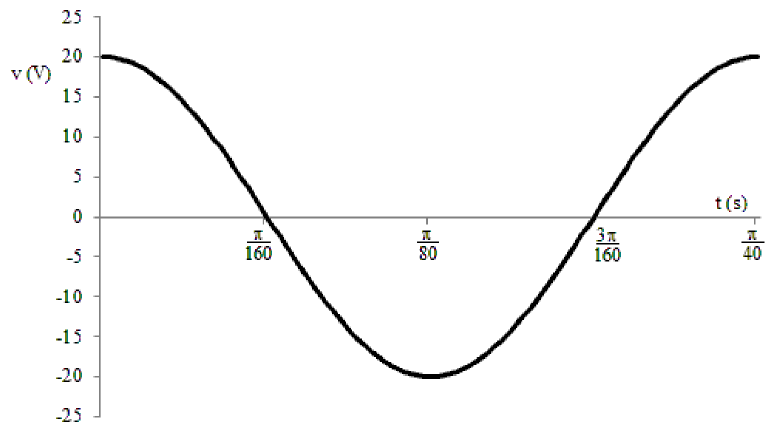
P 6.10 **[a]** $$i = \frac{1}{0.1}\int_{0}^{t} 20\cos 80x\,dx$$

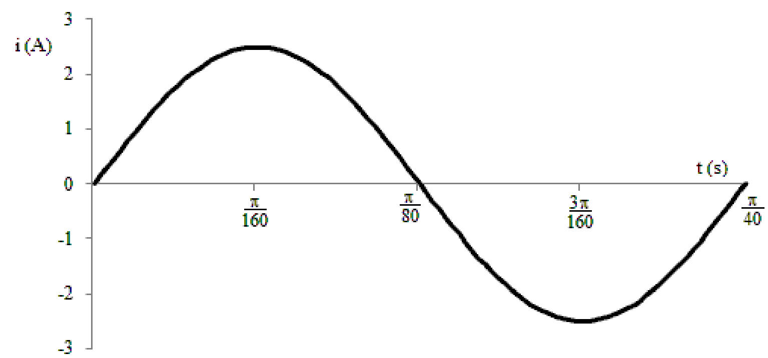
$$= 200\frac{\sin 80x}{80}\Big|_{0}^{t}$$

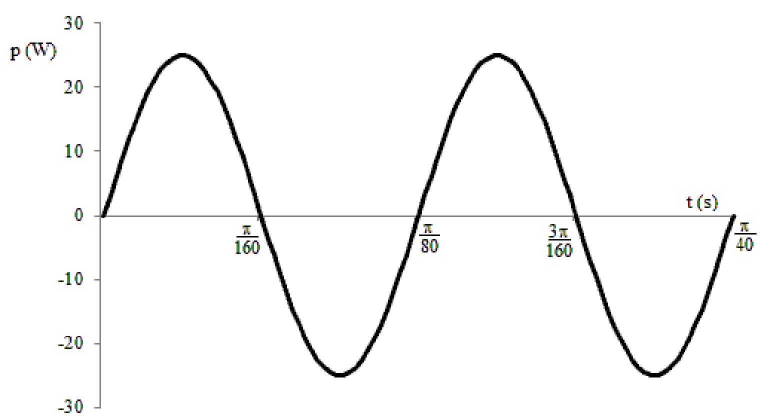
$$= 2.5\sin 80t\,\text{A}$$

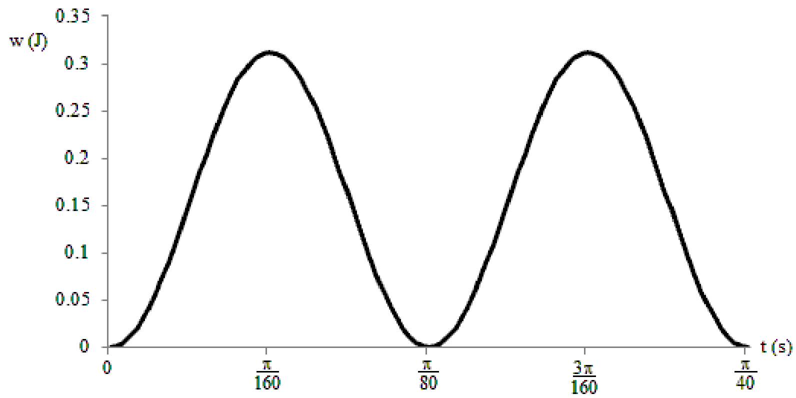
[b]

$$\begin{aligned}
p &= vi = (20\cos 80t)(2.5\sin 80t) \\
&= 50\cos 80t\sin 80t \\
p &= 25\sin 160t\,\text{W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(0.1)(2.5\sin 80t)^2 \\
&= 312.5\sin^2 80t\,\text{mJ} \\
w &= (156.25 - 156.25\cos 160t)\,\text{mJ}
\end{aligned}$$









[c] Absorbing power: Delivering power:

Absorbing power:	Delivering power:
$0 \le t \le \pi/160\,\mathrm{s}$	$\pi/160 \le t \le \pi/80\,\mathrm{s}$
$\pi/80 \le t \le 3\pi/160\,\mathrm{s}$	$3\pi/160 \le t \le \pi/40\,\mathrm{s}$

P 6.11 **[a]** $v = L\dfrac{di}{dt}$

$$v = -25 \times 10^{-3}\frac{d}{dt}[10\cos 400t + 5\sin 400t]e^{-200t}$$

$$= -25 \times 10^{-3}(-200e^{-200t}[10\cos 400t + 5\sin 400t]$$

$$+e^{-200t}[-4000\sin 400t + 2000\cos 400t])$$

$$v = -25 \times 10^{-3}e^{-200t}(-1000\sin 400t - 4000\sin 400t)$$

$$= -25 \times 10^{-3}e^{-200t}(-5000\sin 400t)$$

$$= 125e^{-200t}\sin 400t\,\mathrm{V}$$

$$\frac{dv}{dt} = 125(e^{-200t}(400)\cos 400t - 200e^{-200t}\sin 400t)$$

$$= 25{,}000e^{-200t}(2\cos 400t - \sin 400t)\ \mathrm{V/s}$$

$$\frac{dv}{dt} = 0 \qquad \text{when} \qquad 2\cos 400t = \sin 400t$$

$$\therefore\quad \tan 400t = 2, \qquad 400t = 1.11; \qquad t = 2.77\,\mathrm{ms}$$

[b] $v(2.77\text{ ms}) = 125e^{-0.55}\sin 1.11 = 64.27\,\mathrm{V}$

P 6.12 For $0 \le t \le 1.6\,\mathrm{s}$:

$$i_L = \frac{1}{5}\int_0^t 3 \times 10^{-3}\,dx + 0 = 0.6 \times 10^{-3}t$$

$$i_L(1.6\,\mathrm{s}) = (0.6 \times 10^{-3})(1.6) = 0.96\,\mathrm{mA}$$

$R_m = (20)(1000) = 20\,\text{k}\Omega$

$v_m(1.6\,\text{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\text{V}$

P 6.13 **[a]** $i = C\dfrac{dv}{dt} = (5 \times 10^{-6})[500t(-2500)e^{-2500t} + 500e^{-2500t}]$

$$= 2.5 \times 10^{-3}e^{-2500t}(1 - 2500t)\,\text{A}$$

[b] $v(100\,\mu) = 500(100 \times 10^{-6})e^{-0.25} = 38.94\,\text{mV}$

$i(100\,\mu) = (2.5 \times 10^{-3})e^{-0.25}(1 - 0.25) = 1.46\,\text{mA}$

$p(100\,\mu) = vi = (38.94 \times 10^{-3})(1.46 \times 10^{-3}) = 56.86\,\mu\text{W}$

[c] $p > 0$, so the capacitor is absorbing power.

[d] $v(100\,\mu) = 38.94\,\text{mV}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(5 \times 10^{-6})(38.94 \times 10^{-3})^2 = 3.79\,\text{nJ}$$

[e] The energy is maximum when the voltage is maximum:

$\dfrac{dv}{dt} = 0$ when $(1 - 2500t) = 0$ or $t = 0.4\,\text{ms}$

$v_{\text{max}} = 500(0.4 \times 10^{-3})^2e^{-1} = 73.58\,\text{mV}$

$p_{\text{max}} = \dfrac{1}{2}Cv_{\text{max}}^2 = 13.53\,\text{nJ}$

P 6.14 **[a]**

$$\begin{aligned}
v &= 0 && t < 0 \\
v &= 10t\,\text{A} && 0 \le t \le 2\,\text{s} \\
v &= 40 - 10t\,\text{A} && 2 \le t \le 6\,\text{s} \\
v &= 10t - 80\,\text{A} && 6 \le t \le 8\,\text{s} \\
v &= 0 && 8\,\text{s} < t
\end{aligned}$$

[b] $i = C\dfrac{dv}{dt}$

$$\begin{aligned}
i &= 0 && t < 0 \\
i &= 2\,\text{mA} && 0 < t < 2\,\text{s} \\
i &= -2\,\text{mA} && 2 < t < 6\,\text{s} \\
i &= 2\,\text{mA} && 6 < t < 8\,\text{s} \\
i &= 0 && 8\,\text{s} < t
\end{aligned}$$

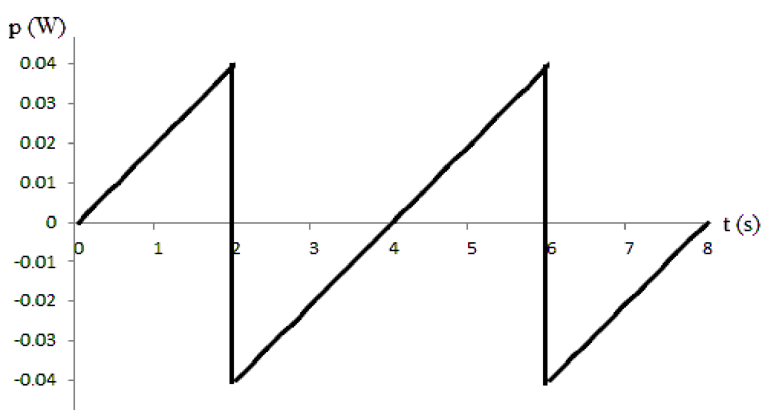
$p = vi$

$$\begin{aligned}
p &= 0 & t < 0\\
p &= (10t)(0.002) = 0.02t\,\text{W} & 0 < t < 2\,\text{s}\\
p &= (40 - 10t)(-0.002) = 0.02t - 0.08\,\text{W} & 2 < t < 6\,\text{s}\\
p &= (10t - 80)(0.002) = 0.02t - 0.16\,\text{W} & 6 < t < 8\,\text{s}\\
p &= 0 & 8\,\text{s} < t
\end{aligned}$$

$w = \int p\,dx$

$$\begin{aligned}
w &= 0 & t < 0\\
w &= \int_0^t (0.02x)\,dx = 0.01x^2 \Big|_0^t = 0.01t^2\,\text{J} & 0 < t < 2\,\text{s}\\
w &= \int_2^t (0.02x - 0.08)\,dx + 0.04\\
&= (0.01x^2 - 0.08x)\Big|_2^t + 0.04\\
&= 0.01t^2 - 0.08t + 0.16\,\text{J} & 2 < t < 6\,\text{s}\\
w &= \int_6^t (0.02x - 0.16)\,dx + 0.04\\
&= (0.01x^2 - 0.16x)\Big|_6^t + 0.04\\
&= 0.01t^2 - 0.16t + 0.64\,\text{J} & 6 < t < 8\,\text{s}\\
w &= 0 & 8\,\text{s} < t
\end{aligned}$$

[c]



From the plot of power above, it is clear that power is being absorbed for $0 < t < 2$ s and for $4\text{ s} < t < 6$ s, because $p > 0$. Likewise, power is being delivered for $2\text{ s} < t < 4$ s and $6\text{ s} < t < 8$ s, because $p < 0$.

P 6.15 **[a]** $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(5 \times 10^{-6})(60)^2 = 9\,\text{mJ}$

[b] $v = (A_1 + A_2 t)e^{-1500t}$

$v(0) = A_1 = 60\,\text{V}$

$$\frac{dv}{dt} = -1500e^{-1500t}(A_1 + A_2 t) + e^{-1500t}(A_2)$$

$$= (-1500A_2 t - 1500A_1 + A_2)e^{-1500t}$$

$$\frac{dv(0)}{dt} = A_2 - 1500A_1$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore \quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{100 \times 10^{-3}}{5 \times 10^{-6}} = 20 \times 10^3$$

$$\therefore \quad 20 \times 10^3 = A_2 - 1500(60)$$

Thus, $A_2 = 20 \times 10^3 + 90 \times 10^3 = 110 \times 10^3 \,\frac{\text{V}}{\text{s}}$

[c] $v = (60 + 110 \times 10^3 t)e^{-1500t}$

$$i = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(60 + 110 \times 10^3 t)e^{-1500t}$$

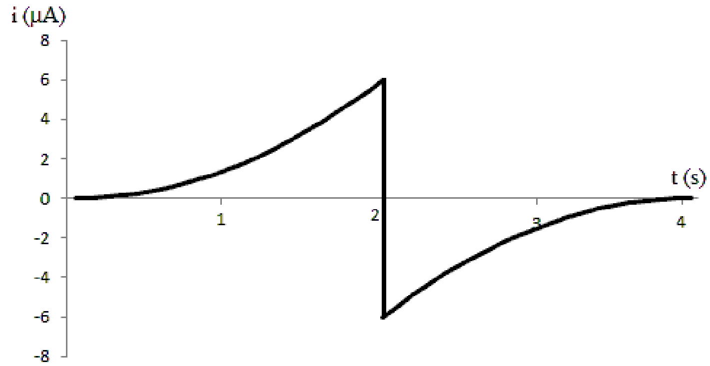
$$i = (5 \times 10^{-6})[110{,}000e^{-1500t} - 1500(60 + 110,000t)e^{-1500t}]$$

$$= (0.1 - 825t)e^{-1500t}\,\text{A}, \qquad t \geq 0$$

P 6.16 $i_C = C(dv/dt)$

$0 < t < 2\,\text{s}:\qquad i_C = 100 \times 10^{-9}(15)t^2 = 1.5 \times 10^{-6}t^2\,\text{A}$

$2 < t < 4\,\text{s}:\qquad i_C = 100 \times 10^{-9}(-15)(t-4)^2 = -1.5 \times 10^{-6}(t-4)^2\,\text{A}$



P 6.17 [**a**] $i = C\dfrac{dv}{dt} = 0, \quad t < 0$

[**b**] $i = C\dfrac{dv}{dt} = 120\times 10^{-6}\dfrac{d}{dt}[30 + 5e^{-500t}(6\cos 2000t + \sin 2000t)]$

$$= 120\times 10^{-6}[5(-500)e^{-500t}(6\cos 2000t + \sin 2000t)$$

$$+5(2000)e^{-500t}(-6\sin 2000t + \cos 2000t)]$$

$$= -0.6e^{-500t}[\cos 2000t + 12.5\sin 2000t]\,\text{A}, \quad t \geq 0$$

[**c**] no, $v(0^-) = 60$ V
$v(0^+) = 30 + 5(6) = 60$ V

[**d**] yes, $i(0^-) = 0$ A
$i(0^+) = -0.6$ A

[**e**] $v(\infty) = 30\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(120\times 10^{-6})(30)^2 = 54\,\text{mJ}$$

P 6.18 [**a**]

$$v(20\,\mu\text{s}) = 12.5\times 10^9(20\times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)}$$

$$v(20\,\mu\text{s}) = 10^6(20\times 10^{-6}) - (12.5)(400)\times 10^{-3} - 10$$

$$= 5\,\text{V (start of second interval)}$$

$$v(40\,\mu\text{s}) = 10^6(40\times 10^{-6}) - (12.5)(1600)\times 10^{-3} - 10$$

$$= 10\,\text{V (end of second interval)}$$

[**b**] $p(10\mu\text{s}) = 62.5\times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = (1.25)(50\,\text{m}) = 62.5\,\text{mW}$ (checks)

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

$p(30\,\mu\text{s}) = vi = (8.75)(0.05) = 62.5\,\text{mW}$ (checks)

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$$

$$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$$

$$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$$

P 6.19 **[a]**

$$\begin{aligned} v &= \frac{1}{0.5 \times 10^{-6}} \int_0^{500\times10^{-6}} 50 \times 10^{-3} e^{-2000t}\,dt - 20 \\ &= 100 \times 10^3 \frac{e^{-2000t}}{-2000}\Bigg|_0^{500\times10^{-6}} - 20 \\ &= 50(1 - e^{-1}) - 20 = 11.61\,\text{V} \\ w &= \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\,\mu\text{J} \end{aligned}$$

[b] $v(\infty) = 50 - 20 = 30\text{V}$

$$w(\infty) = \frac{1}{2}(0.5 \times 10^{-6})(30)^2 = 225\,\mu\text{J}$$

P 6.20 **[a]** $i = \dfrac{5}{2 \times 10^{-3}}t = 2500t \qquad 0 \le t \le 2\,\text{ms}$

$$i = \frac{-10}{4 \times 10^{-3}}t + 10 = 10 - 2500t \qquad 2 \le t \le 6\,\text{ms}$$

$$i = \frac{10}{4 \times 10^{-3}}t - 20 = 2500t - 20 \qquad 6 \le t \le 10\,\text{ms}$$

$$i = \frac{-5}{2 \times 10^{-3}}t + 30 = 30 - 2500t \qquad 10 \le t \le 12\,\text{ms}$$

$$\begin{aligned} q &= \int_0^{0.002} 2500t\,dt + \int_{0.002}^{0.006} (10 - 2500t)\,dt \\ &= \frac{2500t^2}{2}\Bigg|_0^{0.002} + \left(10t - \frac{2500t^2}{2}\right)\Bigg|_{0.002}^{0.006} \\ &= 0.005 - 0 + (0.06 - 0.045) - (0.02 - 0.005) \\ &= 5\,\text{mC} \end{aligned}$$

[b] $v = 0.5 \times 10^6 \displaystyle\int_0^{0.002} 2500x\,dx + 0.5 \times 10^6 \int_{0.002}^{0.006} (10 - 2500x)\,dx$

$$+\,0.5 \times 10^6 \int_{0.006}^{0.01} (2500x - 20)\,dx$$

$$= 0.5 \times 10^6 \left[\frac{2500x^2}{2}\Bigg|_0^{0.002} + 10x\Bigg|_{0.002}^{0.006} - \frac{2500x^2}{2}\Bigg|_{0.002}^{0.006} + \frac{2500x^2}{2}\Bigg|_{0.006}^{0.01} - 20x\Bigg|_{0.006}^{0.01}\right]$$

$$= 0.5 \times 10^6[(0.005 - 0) + (0.06 - 0.02) - (0.045 - 0.005)$$

$$+ (0.125 - 0.045) - (0.2 - 0.12)]$$

$$= 2500\,\text{V}$$

$$v(10\,\text{ms}) = 2500\,\text{V}$$

[c] $v(12\,\text{ms}) = v(10\,\text{ms}) + 0.5 \times 10^6 \int_{0.01}^{0.012} (30 - 2500x)\,dx$

$$= 2500 + 0.5 \times 10^6 \left(30x - \frac{2500x^2}{2}\right)\Bigg|_{0.01}^{0.012}$$

$$= 2500 + 0.5 \times 10^6(0.36 - 0.18 - 0.3 + 0.125)$$

$$= 2500 + 2500 = 5000\,\text{V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(2 \times 10^{-6})(5000)^2 = 25\,\text{J}$$

P 6.21 **[a]** $0 \le t \le 10\,\mu\text{s}$

$$C = 0.1\,\mu\text{F} \qquad \frac{1}{C} = 10 \times 10^6$$

$$v = 10 \times 10^6 \int_0^t -0.05\,dx + 15$$

$$v = -50 \times 10^4 t + 15\,\text{V} \qquad 0 \le t \le 10\,\mu\text{s}$$

$$v(10\,\mu\text{s}) = -5 + 15 = 10\,\text{V}$$

[b] $10\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$$v = 10 \times 10^6 \int_{10\times10^{-6}}^t 0.1\,dx + 10 = 10^6 t - 10 + 10$$

$$v = 10^6 t\,\text{V} \qquad 10 \le t \le 20\,\mu\text{s}$$

$$v(20\,\mu\text{s}) = 10^6(20 \times 10^{-6}) = 20\,\text{V}$$

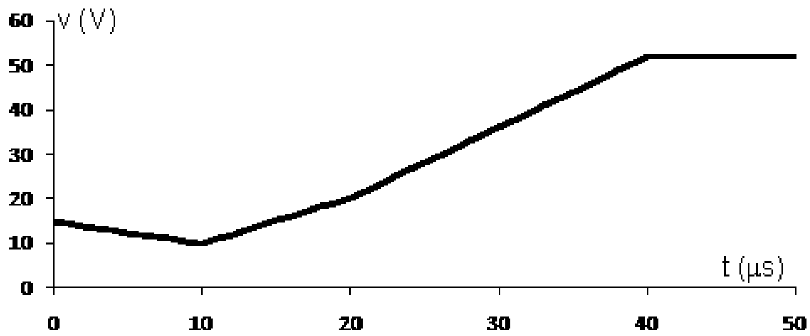
[c] $20\,\mu\text{s} \le t \le 40\,\mu\text{s}$

$$v = 10 \times 10^6 \int_{20\times10^{-6}}^t 1.6\,dx + 20 = 1.6 \times 10^6 t - 32 + 20$$

$$v = 1.6 \times 10^6 t - 12\,\text{V}, \qquad 20\,\mu\text{s} \le t \le 40\,\mu\text{s}$$

[d] $40\,\mu\text{s} \le t < \infty$

$$v(40\,\mu\text{s}) = 64 - 12 = 52\,\text{V} \qquad 40\,\mu\text{s} \le t < \infty$$



P 6.22 [a] $15\|30 = 10\,\text{mH}$

$10 + 10 = 20\,\text{mH}$

$20\|20 = 10\,\text{mH}$

$12\|24 = 8\,\text{mH}$

$10 + 8 = 18\,\text{mH}$

$18\|9 = 6\,\text{mH}$

$L_{\text{ab}} = 6 + 8 = 14\,\text{mH}$

[b] $12 + 18 = 30\,\mu\text{H}$

$30\|20 = 12\,\mu\text{H}$

$12 + 38 = 50\,\mu\text{H}$

$30\|75\|50 = 15\,\mu\text{H}$

$15 + 15 = 30\,\mu\text{H}$

$30\|60 = 20\,\mu\text{H}$

$L_{\text{ab}} = 20 + 25 = 45\,\mu\text{H}$

P 6.23 [a] Combine two 10 mH inductors in parallel to get a 5 mH equivalent inductor. Then combine this parallel pair in series with three 1 mH inductors:

$10\,\text{m}\|10\,\text{m} + 1\,\text{m} + 1\,\text{m} + 1\,\text{m} = 8\,\text{mH}$

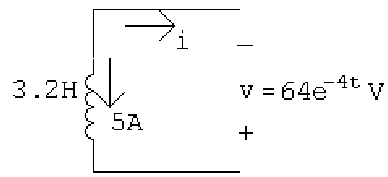
[b] Combine two $10\,\mu$H inductors in parallel to get a $5\,\mu$H inductor. Then combine this parallel pair in series with four more $10\,\mu$H inductors:

$10\,\mu\|10\,\mu + 10\,\mu + 10\,\mu + 10\,\mu + 10\,\mu = 45\,\mu\text{H}$

[c] Combine two 100 μH inductors in parallel to get a 50 μH inductor. Then combine this parallel pair with a 100 μH inductor and three 10 μH inductors in series:

$$100\,\mu\|100\,\mu + 100\,\mu + 10\,\mu + 10\,\mu + 10\,\mu = 180\,\mu\text{H}$$

P 6.24 **[a]**



$$3.2\frac{di}{dt} = 64e^{-4t} \qquad \text{so} \qquad \frac{di}{dt} = 20e^{-4t}$$

$$i(t) = 20\int_0^t e^{-4x}\,dx - 5$$

$$= 20\frac{e^{-4x}}{-4}\Big|_0^t - 5$$

$$i(t) = -5e^{-4t}\,\text{A}$$

[b] $4\dfrac{di_1}{dt} = 64e^{-4t}$

$$i_1(t) = 16\int_0^t e^{-4x}\,dx - 10$$

$$= 16\frac{e^{-4x}}{-4}\Big|_0^t - 10$$

$$i_1(t) = -4e^{-4t} - 6\,\text{A}$$

[c] $16\dfrac{di_2}{dt} = 64e^{-4t} \qquad \text{so} \qquad \dfrac{di_2}{dt} = 4e^{-4t}$

$$i_2(t) = 4\int_0^t e^{-4x}\,dx + 5$$

$$= 4\frac{e^{-4x}}{-4}\Big|_0^t + 5$$

$$i_2(t) = -e^{-4t} + 6\,\text{A}$$

[d] $p = -vi = (-64e^{-4t})(-5e^{-4t}) = 320e^{-8t}\,\text{W}$

$$w = \int_0^\infty p\,dt = \int_0^\infty 320e^{-8t}\,dt$$
$$= 320\frac{e^{-8t}}{-8}\bigg|_0^\infty$$
$$= 40\,\text{J}$$

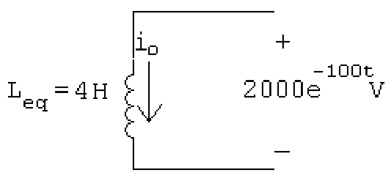
[e] $w = \frac{1}{2}(4)(-10)^2 + \frac{1}{2}(16)(5)^2 = 400\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 400 - 40 = 360\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(4)(-6)^2 + \frac{1}{2}(16)(6)^2 = 360\,\text{J}$ checks

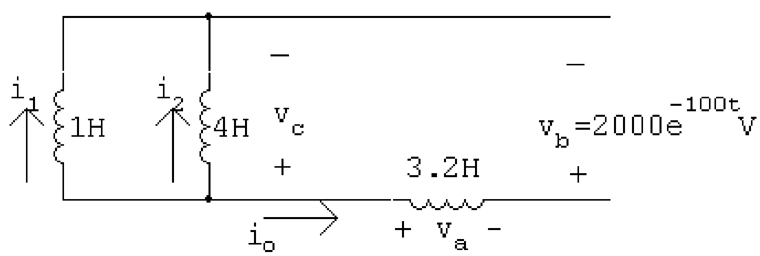
P 6.25 **[a]** $i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5\,\text{A}$

[b]



$$i_o = -\frac{1}{4}\int_0^t 2000e^{-100x}\,dx + 5 = -500\,\frac{e^{-100x}}{-100}\bigg|_0^t + 5$$
$$= 5(e^{-100t} - 1) + 5 = 5e^{-100t}\,\text{A}, \qquad t \geq 0$$

[c]



$$v_a = 3.2(-500e^{-100t}) = -1600e^{-100t}\,\text{V}$$

$$v_c = v_a + v_b = -1600e^{-100t} + 2000e^{-100t}$$
$$= 400e^{-100t}\,\text{V}$$
$$i_1 = \frac{1}{1}\int_0^t 400e^{-100x}\,dx - 6$$

$$= -4e^{-100t} + 4 - 6$$
$$i_1 = -4e^{-100t} - 2\,\text{A} \qquad t \geq 0$$

[d] $$i_2 = \frac{1}{4}\int_0^t 400e^{-100x}\,dx + 1$$

$$= -e^{-100t} + 2\,\text{A}, \qquad t \geq 0$$

[e] $w(0) = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(4)(5)^2 = 50\,\text{J}$

[g] $w_{\text{trapped}} = 60 - 50 = 10\,\text{J}$

or $\quad w_{\text{trapped}} = \frac{1}{2}(1)(2)^2 + \frac{1}{2}(4)(2)^2 + 10\,\text{J}$ (check)

P 6.26 $v_{\text{b}} = 2000e^{-100t}\,\text{V}$

$i_o = 5e^{-100t}\,\text{A}$

$p = 10{,}000e^{-200t}\,\text{W}$

$$w = \int_0^t 10^4 e^{-200x}\,dx = 10{,}000\frac{e^{-200x}}{-200}\Big|_0^t = 50(1 - e^{-200t})\,\text{W}$$

$w_{\text{total}} = 50\,\text{J}$

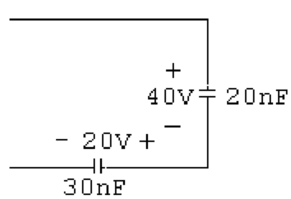
$80\% w_{\text{total}} = 40\,\text{J}$

Thus,

$50 - 50e^{-200t} = 40; \qquad e^{200t} = 5; \qquad \therefore \quad t = 8.05\,\text{ms}$

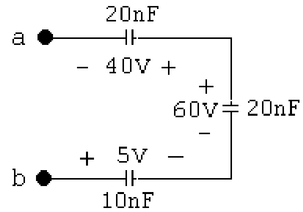
P 6.27 **[a]** $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16}; \qquad C_1 = 16\,\text{nF}$

$C_2 = 4 + 16 = 20\,\text{nF}$



$\frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12}; \qquad C_3 = 12\,\text{nF}$

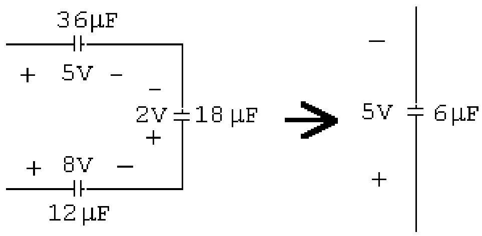
$C_4 = 12 + 8 = 20\,\text{nF}$

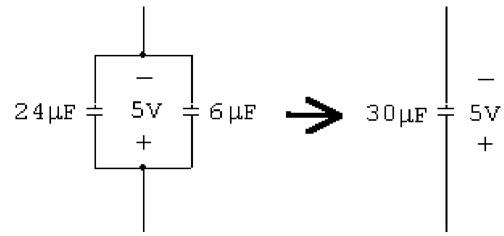


$$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5}; \qquad C_5 = 5\,\text{nF}$$

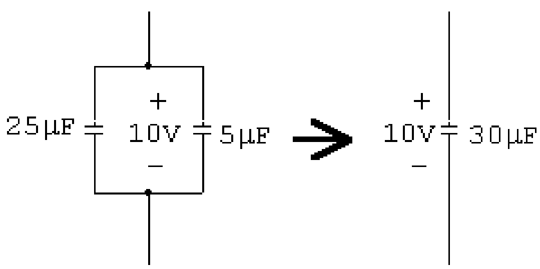
Equivalent capacitance is $5\,\text{nF}$ with an initial voltage drop of +15 V.

[b] $\frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1}{6} \quad \therefore \quad C_{\text{eq}} = 6\,\mu\text{F} \qquad 24 + 6 = 30\,\mu\text{F}$

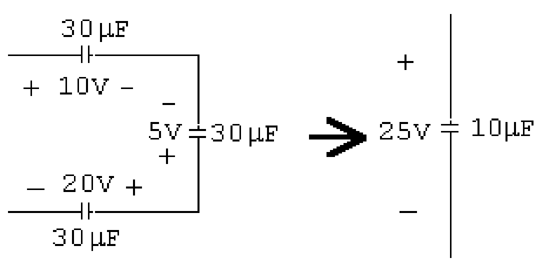




$25 + 5 = 30\,\mu\text{F}$



$$\frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{3}{30} \quad \therefore \quad C_{\text{eq}} = 10\,\mu\text{F}$$



Equivalent capacitance is $10\,\mu\text{F}$ with an initial voltage drop of +25 V.

P 6.28 **[a]** Combine a 470 pF capacitor and a 10 pF capacitor in parallel to get a 480 pF capacitor:

(470 p) in parallel with (10 p) = 470 p + 10 p = 480 pF

[b] Create a 1200 nF capacitor as follows:

$(1\,\mu)$ in parallel with $(0.1\,\mu)$ in parallel with $(0.1\,\mu)$

$$= 1000\,\text{n} + 100\,\text{n} + 100\,\text{n} = 1200\,\text{nF}$$

Create a second 1200 nF capacitor using the same three resistors. Place these two 1200 nF in series:

$$(1200\,\text{n}) \text{ in series with } (1200\,\text{n}) = \frac{(1200\,\text{n})(1200\,\text{n})}{1200\,\text{n} + 1200\,\text{n}} = 600\,\text{nF}$$

[a] Combine two $220\,\mu$F capacitors in series to get a $110\,\mu$F capacitor. Then combine the series pair in parallel with a $10\,\mu$F capacitor to get $120\,\mu$F:

$[(220\,\mu)$ in series with $(220\,\mu)]$ in parallel with $(10\,\mu)$

$$= \frac{(220\,\mu)(220\,\mu)}{220\,\mu + 220\,\mu} + 10\,\mu = 120\,\mu\text{F}$$

P 6.29 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i\,dx + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

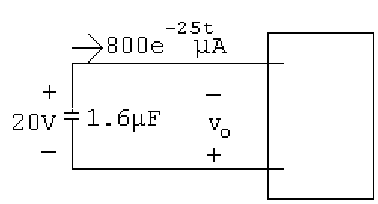
Therefore $\dfrac{1}{C_{\text{eq}}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right], \qquad v_{\text{eq}}(0) = v_1(0) + v_2(0) + \cdots$

P 6.30 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{\text{eq}} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 6.31 **[a]**



$$\begin{aligned} v_o &= \frac{10^6}{1.6}\int_0^t 800\times10^{-6}e^{-25x}\,dx - 20 \\ &= 500\frac{e^{-25x}}{-25}\Big|_0^t - 20 \\ &= -20e^{-25t}\,\text{V}, \qquad t\ge 0 \end{aligned}$$

[b]
$$\begin{aligned} v_1 &= \frac{10^6}{2}(800\times10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t + 5 \\ &= -16e^{-25t} + 21\,\text{V}, \qquad t\ge 0 \end{aligned}$$

[c]
$$\begin{aligned} v_2 &= \frac{10^6}{8}(800\times10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t - 25 \\ &= -4e^{-25t} - 21\,\text{V}, \qquad t\ge 0 \end{aligned}$$

[d]
$$\begin{aligned} p &= -vi = -(-20e^{-25t})(800\times10^{-6})e^{-25t} \\ &= 16\times10^{-3}e^{-50t} \\ w &= \int_0^\infty 16\times10^{-3}e^{-50t}\,dt \\ &= 16\times10^{-3}\frac{e^{-50t}}{-50}\Big|_0^\infty \\ &= -0.32\times10^{-3}(0-1) = 320\,\mu\text{J} \end{aligned}$$

[e]
$$\begin{aligned} w &= \tfrac{1}{2}(2\times10^{-6})(5)^2 + \tfrac{1}{2}(8\times10^{-6})(25)^2 \\ &= 2525\,\mu\text{J} \end{aligned}$$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 2525 - 320 = 2205\,\mu\text{J}$

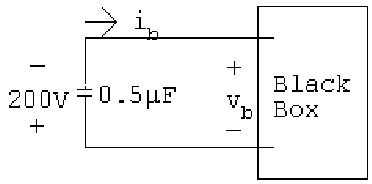
[g]
$$\begin{aligned} w_{\text{trapped}} &= \tfrac{1}{2}(2\times10^{-6})(21)^2 + \tfrac{1}{2}(8\times10^{-6})(-21)^2 \\ &= 2205\,\mu\text{J} \end{aligned}$$

P 6.32 $\dfrac{1}{C_e} = \dfrac{1}{1} + \dfrac{1}{5} + \dfrac{1}{1.25} = \dfrac{10}{5} = 2$

$\therefore\ \ C_2 = 0.5\,\mu\text{F}$

$v_{\text{b}} = 20 - 250 + 30 = -200\,\text{V}$

[a]



$$v_b = -\frac{10^6}{0.5}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 200$$

$$= 10{,}000\frac{e^{-50x}}{-50}\Big|_0^t - 200$$

$$= -200e^{-50t}\,\text{V}$$

[b] $$v_a = -\frac{10^6}{0.5}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 20$$

$$= 20(e^{-50t}-1) - 20$$

$$= 20e^{-50t} - 40\,\text{V}$$

[c] $$v_c = \frac{10^6}{1.25}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 30$$

$$= 80(e^{-50t}-1) - 30$$

$$= 80e^{-50t} - 110\,\text{V}$$

[d] $$v_d = 10^6\int_0^t -5\times10^{-3}e^{-50x}\,dx + 250$$

$$= 100(e^{-50t}-1) + 250$$

$$= 100e^{-50t} + 150\,\text{V}$$

CHECK: $v_b = -v_c - v_d - v_a$

$= -200e^{-50t}\,\text{V}$ (checks)

[e] $$i_1 = 0.2\times10^{-6}\frac{d}{dt}[100e^{-50t}+150]$$

$$= 0.2\times10^{-6}(-5000e^{-50t}) = -e^{-50t}\,\text{mA}$$

[f] $$i_2 = 0.8\times10^{-6}\frac{d}{dt}[100e^{-50t}+150] = -4e^{-50t}\,\text{mA}$$

CHECK: $i_b = i_1 + i_2 = -5e^{-50t}\,\text{mA}$ (OK)

P 6.33 **[a]**
$$w(0) = \tfrac{1}{2}(0.2\times10^{-6})(250)^2 + \tfrac{1}{2}(0.8\times10^{-6})(250)^2 + \tfrac{1}{2}(5\times10^{-6})(20)^2 + \tfrac{1}{2}(1.25\times10^{-6})(30)^2$$
$$= 32{,}812.5\,\mu\text{J}$$

[b]
$$w(\infty) = \tfrac{1}{2}(5\times10^{-6})(40)^2 + \tfrac{1}{2}(1.25\times10^{-6})(110)^2 + \tfrac{1}{2}(0.2\times10^{-6})(150)^2 + \tfrac{1}{2}(0.8\times10^{-6})(150)^2$$
$$= 22{,}812.5\,\mu\text{J}$$

[c] $w = \dfrac{1}{2}(0.5\times10^{-6})(200)^2 = 10{,}000\,\mu\text{J}$

CHECK: $32{,}812.5 - 22{,}812.5 = 10{,}000\,\mu\text{J}$

[d] $\%\text{ delivered} = \dfrac{10{,}000}{32{,}812.5}\times 100 = 30.48\%$

[e]
$$w = \int_0^t (-0.005e^{-50x})(-200e^{-50x})\,dx = \int_0^t e^{-100x}\,dx$$
$$= 10(1 - e^{-100t})\,\text{mJ}$$
$$\therefore\ 10^{-2}(1 - e^{-100t}) = 7.5\times10^{-3}; \qquad e^{-100t} = 0.25$$

Thus, $t = \dfrac{\ln 4}{100} = 13.86\,\text{ms}.$

P 6.34
$$v_c = -\frac{1}{10\times10^{-6}}\left(\int_0^t 0.2e^{-800x}\,dx - \int_0^t 0.04e^{-200x}\,dx\right) + 5$$
$$= 25(e^{-800t} - 1) - 20(e^{-200t} - 1) + 5$$
$$= 25e^{-800t} - 20e^{-200t}\ \text{V}$$
$$v_L = 150\times10^{-3}\frac{di_o}{dt}$$
$$= 150\times10^{-3}(-160e^{-800t} + 8e^{-200t})$$
$$= -24e^{-800t} + 1.2e^{-200t}\ \text{V}$$
$$v_o = v_c - v_L$$
$$= (25e^{-800t} - 20e^{-200t}) - (-24e^{-800t} + 1.2e^{-200t})$$
$$= 49e^{-800t} - 21.2e^{-200t}\ \text{V},\ t > 0$$

P 6.35
$$\frac{di_o}{dt} = (2)\{e^{-5000t}[-1000\sin 1000t + 5000\cos 1000t] + (-5000e^{-5000t})[\cos 1000t + 5\sin 1000t]\}$$
$$= e^{-5000t}\{-52{,}000\sin 1000t\}\ \text{V}$$

$$\frac{di_o}{dt}(0^+) = (1)[\sin(0)] = 0$$

$$\therefore \qquad 50 \times 10^{-3}\frac{di_o}{dt}(0^+) = 0 \qquad \text{so} \qquad v_2(0^+) = 0$$

$$v_1(0^+) = 25i_o(0^+) + v_2(0^+) = 25(2) + 0 = 50\,\text{V}$$

P 6.36 **[a]** Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$

$$+80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$

$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.37 **[a]** Yes, using KVL around the lower right loop

$$v_o = v_{20\Omega} + v_{60\Omega} = 20(i_2 - i_1) + 60i_2$$

[b]
$$\begin{aligned} v_o &= 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 20(-3 - 116e^{-5t} + 119e^{-4t}) + 60 - 3120e^{-5t} + 3060e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\ \text{V} \end{aligned}$$

[c]
$$\begin{aligned} v_o &= L_2\frac{d}{dt}(i_g - i_2) + M\frac{di_1}{dt} \\ &= 16\frac{d}{dt}(15 + 36e^{-5t} - 51e^{-4t}) + 8\frac{d}{dt}(4 + 64e^{-5t} - 68e^{-4t}) \\ &= -2880e^{-5t} + 3264e^{-4t} - 2560e^{-5t} + 2176e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\ \text{V} \end{aligned}$$

P 6.38 **[a]**
$$\begin{aligned} v_g &= 5(i_g - i_1) + 20(i_2 - i_1) + 60i_2 \\ &= 5(16 - 16e^{-5t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 60 + 5780e^{-4t} - 5840e^{-5t}\ \text{V} \end{aligned}$$

[b] $v_g(0) = 60 + 5780 - 5840 = 0\,\text{V}$

[c] $$\begin{aligned} p_{\text{dev}} &= v_g i_g \\ &= 960 + 92{,}480e^{-4t} - 94{,}400e^{-5t} - 92{,}480e^{-9t} + \\ &\quad 93{,}440e^{-10t}\,\text{W} \end{aligned}$$

[d] $p_{\text{dev}}(\infty) = 960\,\text{W}$

[e] $i_1(\infty) = 4\,\text{A}; \quad i_2(\infty) = 1\,\text{A}; \quad i_g(\infty) = 16\,\text{A};$

$$p_{5\Omega} = (16-4)^2(5) = 720\,\text{W}$$

$$p_{20\Omega} = 3^2(20) = 180\,\text{W}$$

$$p_{60\Omega} = 1^2(60) = 60\,\text{W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960\,\text{W}$$

P 6.39 [a] $0.5\dfrac{di_g}{dt} + 0.2\dfrac{di_2}{dt} + 10i_2 = 0$

$$0.2\frac{di_2}{dt} + 10i_2 = -0.5\frac{di_g}{dt}$$

[b] $i_2 = 625e^{-10t} - 250e^{-50t}\,\text{mA}$

$$\frac{di_2}{dt} = -6.25e^{-10t} + 12.5e^{-50t}\,\text{A/s}$$

$$i_g = e^{-10t} - 10\,\text{A}$$

$$\frac{di_g}{dt} = -10e^{-10t}\,\text{A/s}$$

$$0.2\frac{di_2}{dt} + 10i_2 = 5e^{-10t} \qquad \text{and} \qquad -0.5\frac{di_g}{dt} = 5e^{-10t}$$

[c] $$\begin{aligned} v_1 &= 5\frac{di_g}{dt} + 0.5\frac{di_2}{dt} \\ &= 5(-10e^{-10t}) + 0.5(-6.25e^{-10t} + 12.5e^{-50t}) \\ &= -53.125e^{-10t} + 6.25e^{-50t}\,\text{V}, \qquad t > 0 \end{aligned}$$

[d] $v_1(0) = -53.125 + 6.25 = -46.875\,\text{V}; \qquad$ Also

$$\begin{aligned} v_1(0) &= 5\frac{di_g}{dt}(0) + 0.5\frac{di_2}{dt}(0) \\ &= 5(-10) + 0.5(-6.25 + 12.5) = -46.875\,\text{V} \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

P 6.40 **[a]** $v_{ab} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$

It follows that $L_{ab} = (L_1 + L_2 + 2M)$

[b] $v_{ab} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$

Therefore $L_{ab} = (L_1 + L_2 - 2M)$

P 6.41 **[a]** $v_{ab} = L_1\frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{ab} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{ab}$$

from which we have

$$v_{ab} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.42 When the switch is opened the induced voltage is negative at the dotted terminal. Since the voltmeter kicks downscale, the induced voltage across the voltmeter must be negative at its positive terminal. Therefore, the voltage is negative at the positive terminal of the voltmeter.

Thus, the upper terminal of the unmarked coil has the same instantaneous polarity as the dotted terminal. Therefore, place a dot on the upper terminal of the unmarked coil.

P 6.43 **[a]** Dot terminal 1; the flux is up in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

[b] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[c] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[d] Dot terminal 1; the flux is down in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

P 6.44 **[a]** $\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1i_1 + \mathcal{P}_{21}N_1i_1 = N_1i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2i_2)(\phi_{21}/N_1i_1)}{(\phi_1/N_1i_1)(\phi_2/N_2i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.45 **[a]** $k = \frac{M}{\sqrt{L_1L_2}} = \frac{22.8}{\sqrt{576}} = 0.95$

[b] $M_{\text{max}} = \sqrt{576} = 24\,\text{mH}$

[c] $\dfrac{L_1}{L_2} = \dfrac{N_1^2\mathcal{P}_1}{N_2^2\mathcal{P}_2} = \left(\dfrac{N_1}{N_2}\right)^2$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{60}{9.6} = 6.25$$

$$\frac{N_1}{N_2} = \sqrt{6.25} = 2.5$$

P 6.46 **[a]** $L_2 = \left(\dfrac{M^2}{k^2L_1}\right) = \dfrac{(0.09)^2}{(0.75)^2(0.288)} = 50\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$$

[b] $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = \dfrac{0.288}{(1200)^2} = 0.2 \times 10^{-6}\ \text{Wb/A}$

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.05}{(500)^2} = 0.2 \times 10^{-6}\ \text{Wb/A}$$

P 6.47 **[a]** $W = (0.5)L_1i_1^2 + (0.5)L_2i_2^2 + Mi_1i_2$

$$M = 0.85\sqrt{(18)(32)} = 20.4\,\text{mH}$$

$$W = [9(36) + 16(81) + 20.4(54)] = 2721.6\,\text{mJ}$$

[b] $W = [324 + 1296 + 1101.6] = 2721.6\,\text{mJ}$

[c] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

[d] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

P 6.48 **[a]** $M = 1.0\sqrt{(18)(32)} = 24\,\text{mH}, \qquad i_1 = 6\,\text{A}$

Therefore $\quad 16i_2^2 + 144i_2 + 324 = 0, \qquad i_2^2 + 9i_2 + 20.25 = 0$

Therefore $\quad i_2 = -\left(\dfrac{9}{2}\right) \pm \sqrt{\left(\dfrac{9}{2}\right)^2 - 20.25} = -4.5 \pm \sqrt{0}$

Therefore $\quad i_2 = -4.5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.49 **[a]** $L_1 = N_1^2\mathcal{P}_1; \qquad \mathcal{P}_1 = \dfrac{72\times 10^{-3}}{6.25\times 10^4} = 1152 \text{ nWb/A}$

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.2; \qquad \mathcal{P}_{21} = 2\mathcal{P}_{11}$$

$$\therefore\quad 1152\times 10^{-9} = \mathcal{P}_{11} + \mathcal{P}_{21} = 3\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = 192 \text{ nWb/A}; \qquad \mathcal{P}_{21} = 960 \text{ nWb/A}$$

$$M = k\sqrt{L_1L_2} = (2/3)\sqrt{(0.072)(0.0405)} = 36\,\text{mH}$$

$$N_2 = \frac{M}{N_1\mathcal{P}_{21}} = \frac{36\times 10^{-3}}{(250)(960\times 10^{-9})} = 150 \text{ turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{40.5\times 10^{-3}}{(150)^2} = 1800 \text{ nWb/A}$

[c] $\mathcal{P}_{11} = 192$ nWb/A [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2 - \mathcal{P}_{12}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2}{\mathcal{P}_{12}} - 1$

$$\mathcal{P}_{21} = \mathcal{P}_{21} = 960 \text{ nWb/A}; \qquad \mathcal{P}_2 = 1800 \text{ nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{1800}{960} - 1 = 0.875$$

P 6.50 $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = 2 \text{ nWb/A}; \quad \mathcal{P}_2 = \dfrac{L_2}{N_2^2} = 2 \text{ nWb/A}; \quad M = k\sqrt{L_1L_2} = 180\,\mu\text{H}$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1N_2} = 1.2 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 0.8 \text{ nWb/A}$$

P 6.51 When the touchscreen in the mutual-capacitance design is touched at the point x, y, the touch capacitance C_t is present in series with the mutual capacitance at the touch point, C_{mxy}. Remember that capacitances combine in series the way that resistances combine in parallel. The resulting mutual capacitance is

$$C'_{mxy} = \frac{C_{mxy}C_t}{C_{mxy} + C_t}$$

P 6.52 **[a]** The self-capacitance and the touch capacitance are effectively connected in parallel. Therefore, the capacitance at the x-grid electrode closest to the touch point with respect to ground is

$$C_x = C_p + C_t = 30\,\text{pF} + 15\,\text{pF} = 45\,\text{pF}.$$

The same capacitance exists at the y-grid electrode closest to the touch point with respect to ground.

[b] The mutual-capacitance and the touch capacitance are effectively connected in series. Therefore, the mutual capacitance between the x-grid and y-grid electrodes closest to the touch point is

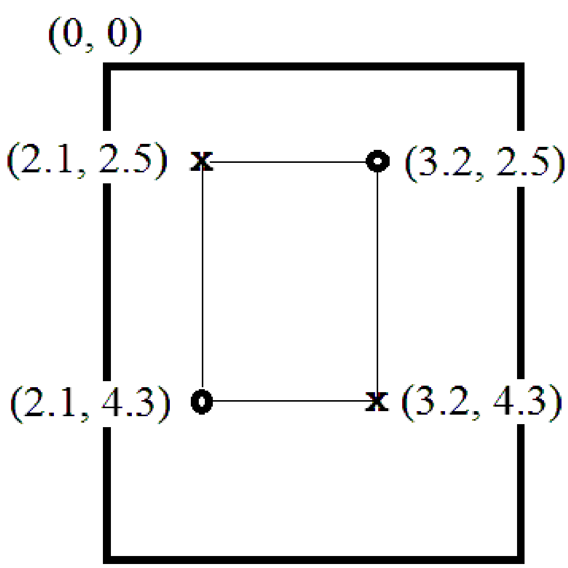
$$C'_{mxy} = \frac{C_{mxy}C_t}{C_{mxy} + C_t} = \frac{(30)(15)}{30 + 15} = 10\,\text{pF}.$$

[c] In the self-capacitance design, touching the screen increases the capacitance being measured at the point of touch. For example, in part (a) the measured capacitance before the touch is 30 pF and after the touch is 45 pF. In the mutual-capacitance design, touching the screen decreases the capacitance being measured at the point of touch. For example, in part (b) the measured capacitance before the touch is 30 pF and after the touch is 10 pF.

P 6.53 **[a]** The four touch points identified are the two actual touch points and two ghost touch points. Their coordinates, in inches from the upper left corner of the screen, are

$$(2.1, 4.3); \quad (3.2, 2.5); \quad (2.1, 2.5); \quad \text{and} \quad (3.2, 4.3)$$

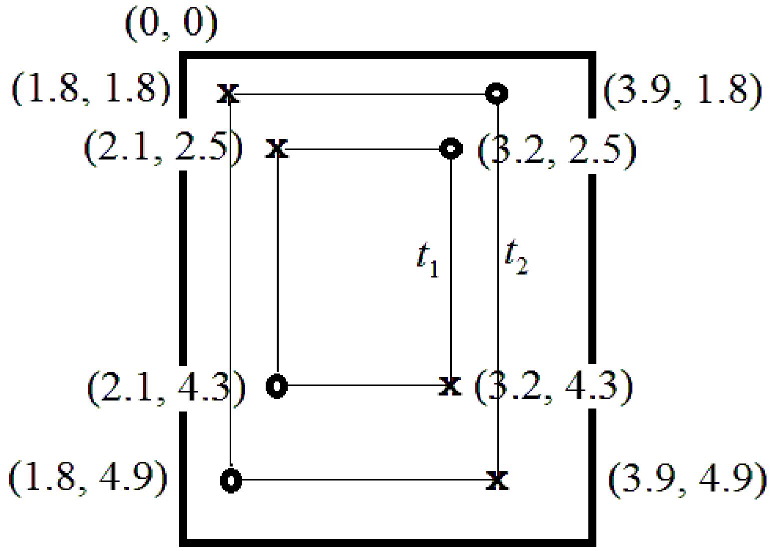
These four coordinates identify a rectangle within the screen, shown below.



[b] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$$(1.8, 4.9); \quad (3.9, 1.8); \quad (1.8, 1.8); \quad \text{and} \quad (3.9, 4.9)$$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2, as shown here:

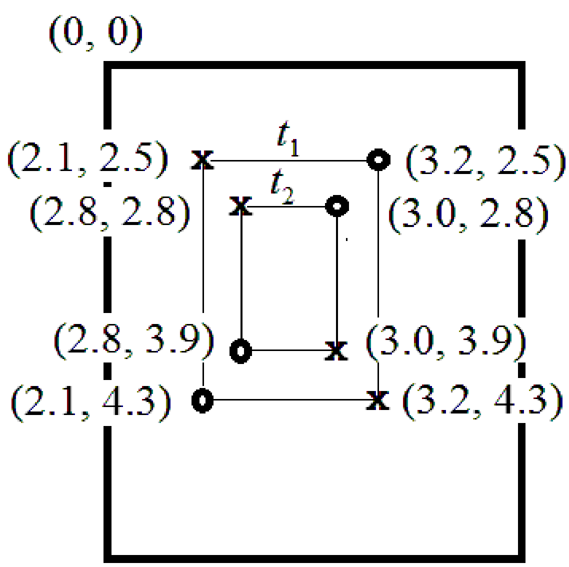


Note that the rectangle at time t_2 is larger than the rectangle at time t_1, so the software would recognize the two fingers are moving toward the edges of the screen. This pinch gesture thus specifies a zoom-in for the screen.

[**c**] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$$(2.8, 3.9); \quad (3.0, 2.8); \quad (2.8, 2.8); \quad \text{and} \quad (3.0, 3.9)$$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2, as shown here:



Here, the rectangle at time t_2 is smaller than the rectangle at time t_1, so the software would recognize the two fingers are moving toward the middle of the screen. This pinch gesture thus specifies a zoom-out for the screen.