Chapter 13

Solutions

2 a. $\bar{\bar{x}} = (153 + 169 + 158)/3 = 160$

$$SSTR = \sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2 = 4(153 - 160)^2$$
$$+ 4(169 - 160)^2$$
$$+ 4(158 - 160)^2 = 536$$

$MSTR = SSTR/(k - 1) = 536/2 = 268$

b. $$SSE = \sum_{j=1}^{k}(n_j - 1)s_j^2 = 3(96.67) + 3(97.33)$$
$$+ 3(82.00) = 828.00$$

$MSE = SSE/(n_T - k) = 828.00/(12 - 3) = 92.00$

c. $F = MSTR/MSE = 268/92 = 2.91$

Using F table (2 degrees of freedom numerator and 9 denominator), p-value is greater than 0.10

Actual p-value = 0.1060

Because p-value $> \alpha = 0.05$, we cannot reject the null hypothesis.

d.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments	2	536	268	2.91
Error	9	828	92	
Total	11	1364		

4 a.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments	1200	3	400	80
Error	300	60	5	
Total	1500	63		

b. Using F table (3 degrees of freedom numerator and 60 denominator), p-value is less than 0.01

Because p-value $\le \alpha = 0.05$, we reject the null hypothesis that the means of the four populations are equal.

6

	Manufacturer 1	Manufacturer 2	Manufacturer 3
Sample Mean	23	28	21
Sample Variance	6.67	4.67	3.33

$\bar{\bar{x}} = (23 + 28 + 21)/3 = 24$

$$SSTR = \sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2 = 4(23 - 24)^2$$
$$+ 4(28 - 24)^2 + 4(21 - 24)^2 = 104$$

$MSTR = SSTR/(k - 1) = 104/2 = 52$

$$SSE = \sum_{j=1}^{k}(n_j - 1)s_j^2 = 5(0.8) + 5(0.03)$$
$$+ 5(0.4) = 7.50$$

$MSE = SSE/(n_T - k) = 44.01/(12 - 3) = 4.89$

$F = MSTR/MSE = 52/4.89 = 10.63$

Using F table (2 degrees of freedom numerator and 9 denominator), p-value is less than 0.01

Actual p-value = 0.0043

Because p-value $< \alpha = 0.05$, we reject the null hypothesis that the mean time needed to mix a batch of material is the same for each manufacturer.

8

	Marketing managers	Marketing research	Advertising
Sample Mean	5	4.5	6
Sample Variance	0.8	0.3	0.4

$\bar{\bar{x}} = (5 + 4.5 + 6)/3 = 5.17$

$SSTR = 7.00$

$MSTR = SSTR/(k - 1) = 7.00/2 = 3.5$

$SSE = 7.50$

$MSE = SSE/(n_T - k) = 7.50/(18 - 3) = 0.5$

$F = MSTR/MSE = 3.5/0.50 = 7.00$

Using F table (2 degrees of freedom numerator and 15 denominator), p-value is less than 0.01

Actual p-value = 0.0071

Because p-value $\le \alpha = 0.05$, we reject the null hypothesis that the mean perception score is the same for the three groups of specialists.

10 Because p-value $\le \alpha = 0.05$, we reject the null hypothesis that the mean service ratings are equal.

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One-way ANOVA: small, medium, large

Source    DF      SS      MS      F      P
Factor     3   226.1   113.0   3.70  0.042
Error     21   640.8    30.5
Total     23   866.9
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12 a. $\bar{\bar{x}} = 62$

$SSTR = 1448$

$MSTR = SSTR/(k - 1) = 1448/2 = 724$

$SSE = 828$

$MSE = SSE/(n_T - k) = 828/(12 - 3) = 92$

$F = MSTR/MSE = 724/92 = 7.87$

Using F table (2 degrees of freedom numerator and 9 denominator), p-value is between 0.01 and 0.025

Actual p-value = 0.0106

Because p-value $\le \alpha = 0.05$, we reject the null hypothesis that the means of the three populations are equal.

b.

$$LSD = t_{\alpha/2}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = t_{0.025}\sqrt{92\left(\frac{1}{4} + \frac{1}{4}\right)}$$
$$= 2.262\sqrt{46} = 15.34$$

$|\bar{x}_1 - \bar{x}_2| = |51 - 77| = 26 > LSD$; significant difference

$|\bar{x}_1 - \bar{x}_3| = |51 - 58| = 7 < LSD$; no significant difference

$|\bar{x}_2 - \bar{x}_3| = |77 - 58| = 19 > LSD$; significant difference

14 $\bar{x}_1 - \bar{x}_2 \pm LSD$

$23 - 28 \pm 3.54$

$-5 \pm 3.54 = -8.54$ to -1.46

16 a.

	Machine 1	Machine 2	Machine 3	Machine 4
Sample Mean	7.1	9.1	9.9	11.4
Sample Variance	1.21	0.93	0.70	1.02

$\bar{\bar{x}} = (7.1 + 9.1 + 9.9 + 11.4)/4 = 9.38$

$$SSTR = \sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2 = 6(7.1 - 9.38)^2$$
$$+ 6(9.1 - 9.38)^2 + 6(9.9 - 9.38)^2$$
$$+ 6(11.4 - 9.38)^2 = 57.77$$

$MSTR = SSTR/(k - 1) = 57.77/3 = 19.26$

$$SSE = \sum_{j=1}^{k}(n_j - 1)s_j^2 = 5(1.21) + 5(0.93)$$
$$+ 5(0.70) + 5(0.07) + 5(1.02) = 19.30$$

$MSE = SSE/(n_T - k) = 19.30/(24 - 4) = 0.97$

$F = MSTR/MSE = 19.26/0.97 = 19.86$

Using F table (3 degrees of freedom numerator and 20 denominator), p-value is less than 0.01

Actual p-value = 0.0000 (to 4 decimal places)

Because p-value $\le \alpha = 0.05$, we reject the null hypothesis that the mean time between breakdowns is the same for the four machines.

b. Note: $t_{\alpha/2}$ is based upon 20 degrees of freedom

$$LSD = t_{\alpha/2}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = t_{0.025}\sqrt{0.97\left(\frac{1}{6} + \frac{1}{6}\right)}$$
$$= 2.086\sqrt{0.3233} = 1.19$$

$|\bar{x}_2 - \bar{x}_4| = |9.1 - 11.4| = 2.3 > LSD$:

significant difference

18 $n_1 = 8, n_2 = 8, n_3 = 8$

$t_{\alpha/2}$ is based upon 21 degrees of freedom

$$LSD = t_{0.025}\sqrt{30.5\left(\frac{1}{8} + \frac{1}{8}\right)} = 2.080\sqrt{7.6250} = 5.74$$

Comparing Small and Medium

$92.20 - 89.65 = 2.55 < LSD$; no significant difference

Comparing Small and Large

$92.20 - 84.80 = 7.40 > LSD$; significant difference

Comparing Medium and Large

$89.65 - 84.80 = 4.85$ LSD; no significant difference

20 a.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments	2	1488	744	5.50
Error	15	2030	135.3	
Total	17	3518		

b.

$$LSD = t_{\alpha/2}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = 2.131\sqrt{135.3\left(\frac{1}{6} + \frac{1}{6}\right)}$$
$$= 14.31$$

$|156 - 142| = 14 < 14.31$; no significant difference

$|156 - 134| = 22 > 14.31$; significant difference

$|142 - 134| = 8 < 14.31$; no significant difference

22 a. H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

H_1: Not all the population means are equal

b. Using F table (4 degrees of freedom numerator and 30 denominator), p-value is less than 0.01

Actual p-value = 0.0000

Because p-value $\le \alpha = 0.05$, we reject H_0

24

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments	2	1200	600	43.99
Error	44	600	13.64	
Total	46	1800		