BIRZEIT UNIVERSITY

**Faculty of Engineering and Technology**
**Electrical and Computer Engineering Department**
***Probability and Statistical Engineering, ENEE2307***
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*Quiz #1*

Date: Time: 25 minutes
Name: Student #:

**Problem 1** (10 pts):

In a game, a person rolls a die and then flips a fair coin several times equal to the number observed on the dice. The probability of observing any number on the dice is directly proportional to the number itself. Let A represent observing at least four heads when flipping the coin. Compute P(A).

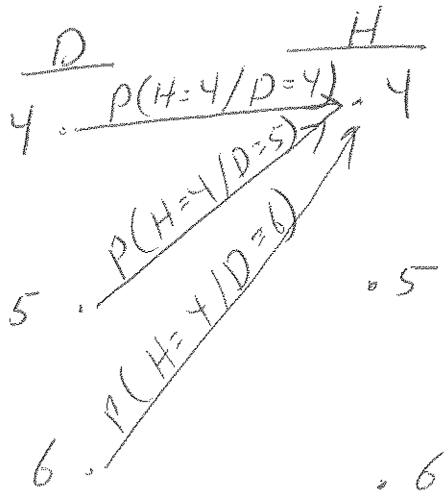
On the dice → $p(\{1,2,3,4,5,6\}) = p(S) = 1$

$p(\{1\}) + p(\{2\}) + p(\{3\}) + p(\{4\}) + p(\{5\}) + p(\{6\}) = 1$

$K + 2K + 3K + 4K + 5K + 6K = 21K = 1$

$\boxed{K = \frac{1}{21}}$

D = number observed on dice
H = number of heads observed




$p(H=4/D=4) = p(\{HHHH\}) = (p(H))^4 = \left(\frac{1}{2}\right)^4$

$p(H=4/D=5) = p(\{HHHHT, HHHTH, HHTHH, HTHHH, THHHH\})$

$= 5 * \{p(H)\}^4 p(T) = 5 * \left(\frac{1}{2}\right)^5$

$p(H=4/D=6) = \binom{6}{4} p(H)^4 p(T)^2 = \frac{6!}{(6-4)!4!} = 15 * \left(\frac{1}{2}\right)^6$

$p(H=4) = p(D=4)\,p(H=4/D=4) + p(D=5)\,p(H=4/D=5) + p(D=6)\,p(H=4/D=5)$

$= \frac{4}{21} * \left(\frac{1}{2}\right)^4 + \frac{5}{21} * 5 \left(\frac{1}{2}\right)^5 + 15 * \left(\frac{1}{2}\right)^6 \cong 0.283782143$

$p(H=5) = p(D=5)\,p(H=5/D=5) + p(D=6)\,p(H=5/D=6)$

$= \frac{5}{21} * \left(\frac{1}{2}\right)^5 + \frac{6}{21} * 6 * \left(\frac{1}{2}\right)^6 = 0.0342261 9$

$p(H=6) = p(D=6)\,p(H=6/D=6) = \frac{6}{21} * \left(\frac{1}{2}\right)^6 = 0.0053146 26$

$p(A) = p(H=4) + p(H=5) + p(H=6) = 0.32302295 9$

**Problem 2** (5+5 pts):

A factory has two production lines; A and B. The probability that production line A fails is 20%. The probability that production line B fails is 10%, and the probability that both production lines fail is 4%. What is the probability that:

a) at least one of the production lines will stay working?

$p(A) = 1 - p(\bar{A}) = 0.8$

$p(B) = 1 - p(\bar{B}) = 0.9$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$ ← same as part (b)

Given

$p(\bar{A}) = 0.2$

$p(\bar{B}) = 0.1$

$p(\bar{A} \cap \bar{B}) = 0.04$

$= 0.8 + 0.9 - 0.74$

$= 0.96$

b) both production lines will stay working?

$p(A \cap B) = ?$

$p(\bar{A} \cup \bar{B}) = p(\bar{A}) + p(\bar{B}) - p(\bar{A} \cap \bar{B})$

$p(\bar{A} \cup \bar{B}) = 0.2 + 0.1 - 0.04 = 0.26$

but using Demorgan's Law

$p(\bar{A} \cup \bar{B}) = p(\overline{A \cap B}) = 1 - p(A \cap B)$

so $p(A \cap B) = 1 - p(\bar{A} \cup \bar{B}) = 1 - 0.26 = \boxed{0.74}$