Exp
$$f(x) = \begin{cases} x^{\frac{2}{3}}, & x \geq 0 \\ x^{\frac{1}{3}}, & x < 0 \end{cases}$$

① Is f cont. at $x=0$

$f(0) = \lim_{x\to 0} f(x)$

$0 = 0$

$f(0) = 0^{\frac{2}{3}} = \sqrt[3]{0^2} = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sqrt[3]{x^2} = \sqrt[3]{0^2} = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt[3]{x} = \sqrt[3]{0} = 0$

Yes f is cont. at $x=0$

② Is f diff at $x=0$

$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$

check $f'_+(0) \stackrel{?}{=} f'_-(0)$

we will use the definition of derivative

$f(h) = \sqrt[3]{h^2}$

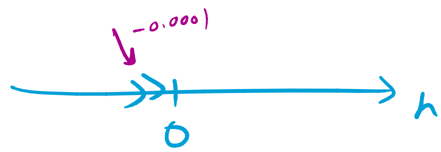
$f'_+(0) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0^+} \frac{f(0+h)}{h} = \lim_{h\to 0^+} \frac{f(h)}{h}$

$= \lim \quad h^{\frac{2}{3}} \quad \lim \quad h^{\frac{2}{3}-1} = \lim \quad h^{-\frac{1}{3}}$

$$= \lim_{h\to 0^+} \frac{h^{\frac{2}{3}}}{h^1} = \lim_{h\to 0^+} h^{3} = \lim_{h\to 0^+} h^{-3}$$

$$= \lim_{h\to 0^+} \frac{1}{\sqrt[3]{h}} = \frac{1}{\text{small}+} = +\infty$$

$$f'_-(0) = \lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0^-} \frac{f(h) - 0}{h}$$



$f(x) = \sqrt[3]{x}$

$$= \lim_{h\to 0^-} \frac{h^{\frac{1}{3}}}{h^1} = \lim_{h\to 0^-} h^{\frac{1}{3}-1} = \lim_{h\to 0^-} h^{-\frac{2}{3}}$$

$$= \lim_{h\to 0^-} \frac{1}{\sqrt[3]{h^2}} = \frac{1}{\text{small}+} = +\infty$$

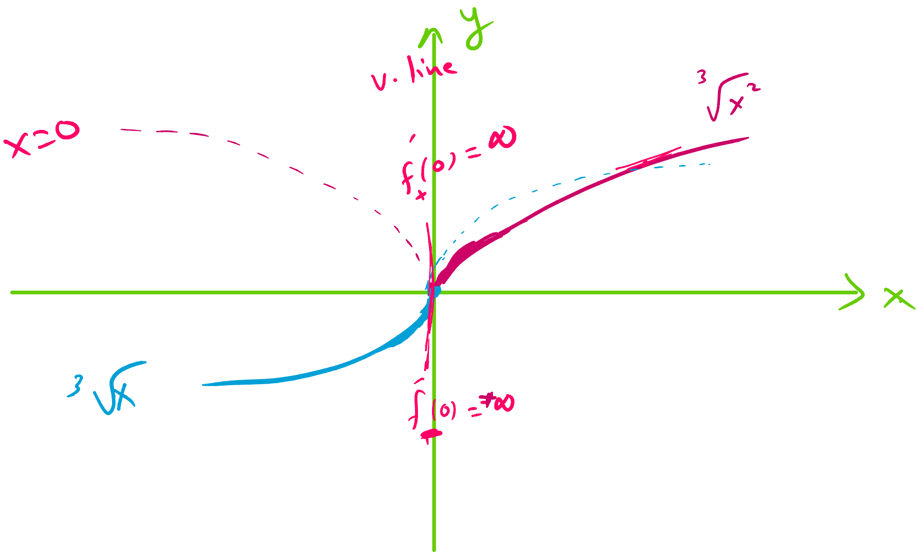
not real #

f is not diff at $x=0$

$$f(x) = \begin{cases} \sqrt[3]{x^2} & \text{if } x \geq 0 \\ \sqrt[3]{} & \text{if } x < 0 \end{cases}$$

$$f(x) = \begin{cases} \quad \\ \sqrt[3]{x} & \text{if } x < 0 \end{cases}$$

f cont. at $x=0$
but f is not diff $x=0$



In general if f is diff at $x=a$
then f is cont. at $x=a$

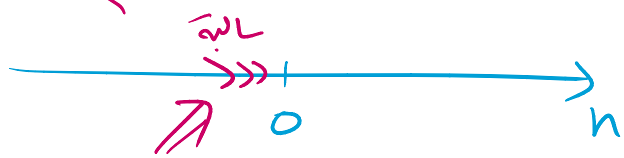
$$f(x) = \begin{cases} x^{\frac{2}{3}} & \text{if } x \geq 0 \\ x^{\frac{1}{3}} & \text{if } x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{2}{3} x^{-\frac{1}{3}} & \text{if } x > 0 \\ \frac{1}{3} x^{-\frac{2}{3}} & \text{if } x < 0 \end{cases}$$

$$f'_+(0) = \lim_{h \to 0^+} \frac{2}{3} \frac{1}{\sqrt[3]{0}} = \frac{1}{\text{small} +} = +\infty$$

$\frac{1}{0^+}$ $= \lim_{h \to 0^+} 3\sqrt[3]{0}$ small +

$f'_-(0) = \lim_{h \to 0^-} \frac{1}{3} \frac{1}{\sqrt[3]{x^2}} = \frac{1}{\text{small}+} = +\infty$



$$\frac{d}{dx}\left(a x^n\right) = a\, n\, x^{n-1}$$

Exp Find $y'(1)$ if $y(x) = (x^3 + 2x)^4$

$y'(x) = 4(x^3 + 2x)^3(3x^2 + 2)$

$y'(1) = 4(1+2)^3(3+2)$

$= 4(3)^3(5)$

$= 4(27)(5)$

$= (20)(27)$

$= 540$

Exp show that $\frac{d}{dx}(\sin x) = \cos x$

$f(x) = \sin x$

$\frac{d}{dx}(\sin x) = \lim \frac{f(x+h) - f(x)}{\ldots}$

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = \sin x$

$\sin(\overset{x}{A} + \overset{h}{B}) = \sin A \cos B + \sin B \cos A$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x (\cos h - 1) + \sin h \cos x}{h}$$

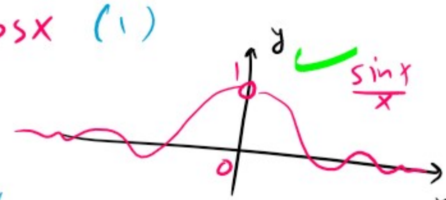
$$= \lim_{h\to 0} \frac{\sin x (\cos h - 1)}{h} + \lim_{h\to 0} \frac{\sin h \cos x}{h}$$

$$= \sin x \left(\lim_{h\to 0} \frac{\cos h - 1}{h} \right) + \cos x \left(\lim_{h\to 0} \frac{\sin h}{h} \right) \to 1$$

$\frac{0}{0}$??

$$+ \cos x \, (1)$$

$$= \sin x \lim_{h\to 0} \frac{-2\sin^2 \frac{h}{2}}{h}$$



$$= -\sin x \lim_{h\to 0} \frac{\sin^2 \frac{h}{2}}{\frac{h}{2}} + \cos x$$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$= \cos^2 x - \sin^2 x$

$$= -\sin x \left(\lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \right) \sin\frac{h}{2} + \cos x$$

$= \cos^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 2h = 1 - 2\sin^2 h$

$\cosh = 1 - 2\sin^2 \frac{h}{2}$

$\cosh - 1 = -2\sin^2\frac{h}{2}$

$$= -\sin x \quad (1) \boxed{\lim_{h\to 0} \sin\frac{h}{2}} +$$

$\sin 0 = 0$

$$= \cancel{-\sin(x)\,(1)\,(0)} + \cos x$$

$$= \cos x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

Exp Given the curve $xy + 2x - y = 0$

① write this curve in the form $y = f(x)$

$$y(x-1) + 2x = 0$$

$$y(x-1) = -2x \Rightarrow y = \frac{-2x}{x-1} = \frac{2x}{1-x}$$

$$\boxed{f(x) = \frac{2x}{1-x}}$$

② Find $D(f)$

$D(f) = \mathbb{R} \setminus \{1\} = (-\infty, 1) \cup (1, \infty)$

$1 - x \neq 0$

$x \neq 1$

③ Find Asy.

(3) Find Asy.

$f(x) = \frac{2x}{1-x}$ no O. Asy. $\Rightarrow \exists$ H. Asy.

H. Asy $\Rightarrow \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{2x}{1-x} = \lim_{x \to \infty} \frac{2}{\frac{1}{x} - 1} = \frac{2}{0-1}$

$= -2$

$\boxed{y = -2}$ is H. Asy.

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2x}{1-x} = \lim_{x \to -\infty} \frac{2}{\frac{1}{x} - 1} = \frac{2}{0-1} = -2$

V. Asy $\Rightarrow$ check zeros of denominator.

$\Rightarrow$ check $x = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{2x}{1-x} = \frac{2}{\text{small} -} = -\infty$

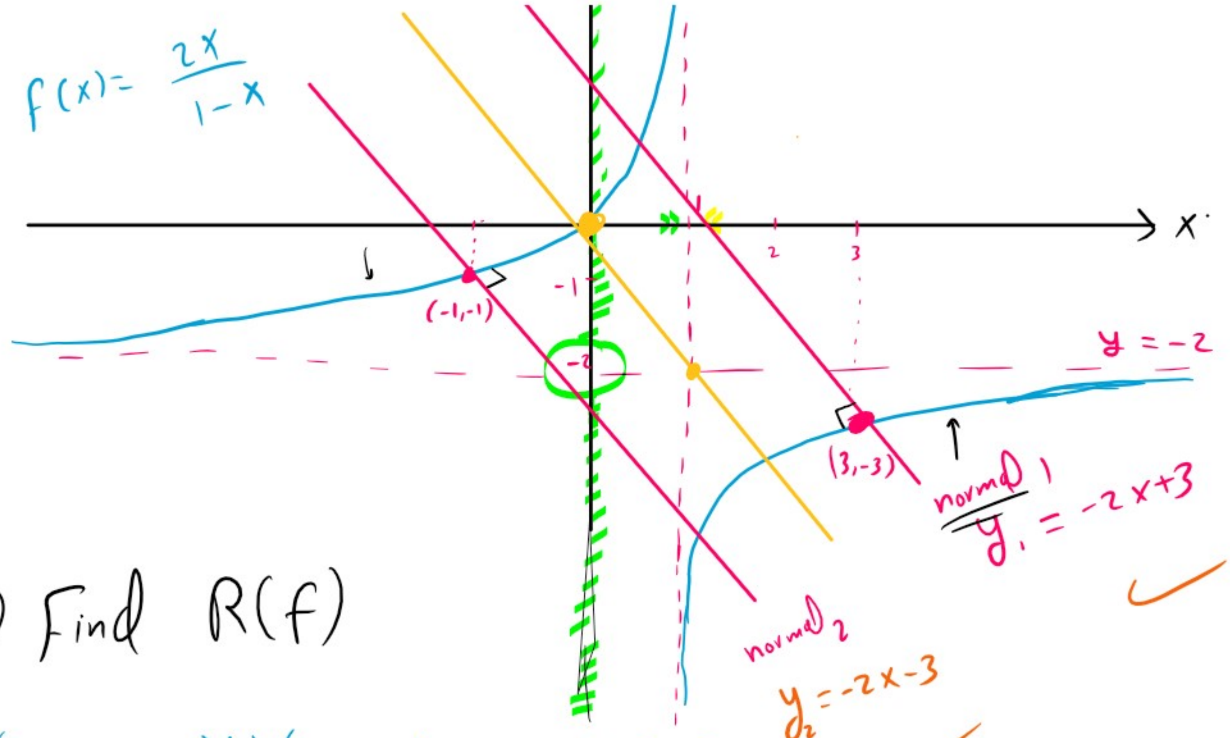
1.1

0.9 1 x

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{2x}{1-x} = \frac{2}{\text{small} +} = +\infty$

$\boxed{x = 1}$ is V. Asy.

(4) sketch $f(x) = \frac{2x}{1-x}$ keypoint (0,0)



④ Find R(f)

$(-\infty, -2) \cup (-2, \infty) = \mathbb{R} \setminus \{-2\}$

⑤ Draw the line $2x + y = 0$ on the graph

$\boxed{y = -2x}$ (0, 0) (1, -2)

⑥ Find normal lines to the curve f(x) that are parallel to the line $y = -2x$

- $y = -2x$ has slope $m_1 = -2$
- Normal line has slope $m_1 = -2$

L

$L \perp f(x) = \frac{2x}{1-x}$

ميل

m_2

-1 = (ميل العمودي)(ميل المماس)

$-1 = (-2)\ (m_2)$

$m_2 = \frac{1}{2}$

m_2

ميل المماس
Tangent

$y' = f'(x) = \dfrac{(1-x)(2) - (2x)(-1)}{(1-x)^2}$

$\frac{1}{2} = \dfrac{2 - \cancel{2x} + \cancel{2x}}{(1-x)^2}$

$\frac{1}{2} = \dfrac{2}{(1-x)^2}$

$(1-x)^2 = 4$

$1 - 2x + x^2 = 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad , \quad x = -1$

$f(3) = \dfrac{2(3)}{1-(3)} = \dfrac{6}{-2} = -3$

$f(-1) = \dfrac{2(-1)}{1-(-1)} = \dfrac{-2}{2} = -1$

$(3,-3) \qquad (-1,-1)$

Normal 1 $\quad (x_0, y_0) = (3,-3)\ ,\ m_1 = -2$

$y - y_0 = m_1(x - x_0)$

$y - -3 = -2(x-3)$

$y + 3 = -2x + 6$

$\boxed{y_1 = -2x + 3}$

Normal 2 $\quad (x_0, y_0) = (-1,-1)\ ,\ m_1 = -2$

$y - y_0 = m_1(x - x_0)$

$y - -1 = -2(x - -1)$

$y + 1 = -2x - 2$

$\boxed{y_2 = -2x - 3}$

⑧ Find tangent at $x = -1$ $\quad ((-1,-1))$

Tangent $\quad y - y_0 = m_2(x - x_0)$

$y - -1 = \frac{1}{2}(x - -1)$

$y - -1 = \frac{1}{2}(x - -1)$

$y + 1 = \frac{1}{2}x + \frac{1}{2}$

$\boxed{y = \frac{1}{2}x - \frac{1}{2}}$ $\Rightarrow$ slope $\frac{1}{2}$ $\times$ $= -1$

Tangent

$\Rightarrow$ slope -2

Tangent $\perp$ $y = -2x$

But $y = -2x$ not normal on the curve $f(x)$