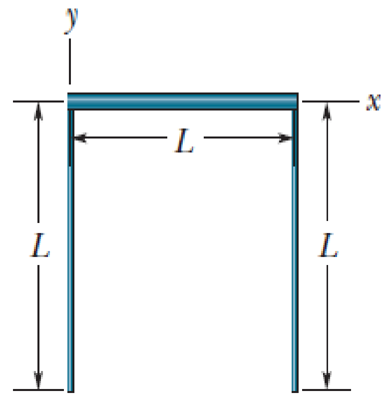
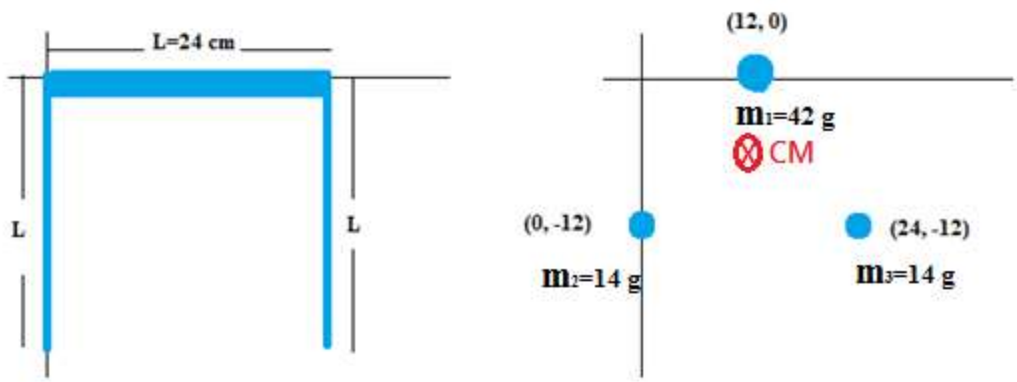
Chapter 9: Center of Mass and Linear Momentum

9-4) In the below figure, three uniform thin rods, each of length L = 24 cm, form an inverted U. The vertical rods each have a mass of 14 g; the horizontal rod has a mass of 42 g. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass?



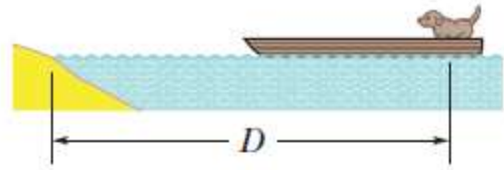


Note: the mass is uniformly distributed along each rod, so the CM of each rod is in its canter (see the figure above)

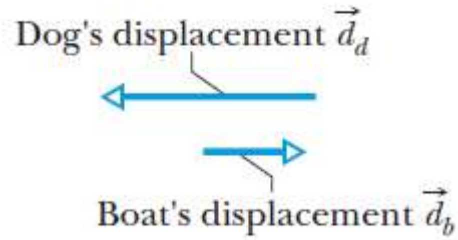
$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{42 * 12 + 14 * 0 + 14 * 24}{42 + 14 + 14} = 12\ cm$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{42 * 0 + 14 * (-12) + 14 * (-12)}{42 + 14 + 14} = -4.8\ cm$$

9-17) In the below figure (a), a 4.5 kg dog stands on an 18 kg flatboat at distance D = 6.1 m from the shore. It walks 2.4 m along the boat toward shore and then stops. Assuming no friction between the boat and the water, find how far the dog is then from the shore. (*Hint:* See Fig.b.)



(*a*)



(*b*)

No External Force: X_{com} is constant

(Center of mass for dog and boat system does not change)

$$\Delta X_{com} = 0 = m_D\, \Delta x_{DG} + m_B\, \Delta x_{BG} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$-2.4\ m = \Delta x_{DB} = \Delta x_{DG} + \Delta x_{GB} = \Delta x_{DG} - \Delta x_{BG} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$(\textit{ the negative sigh since the dog moves to the left})$

I need Δx_{DG}:

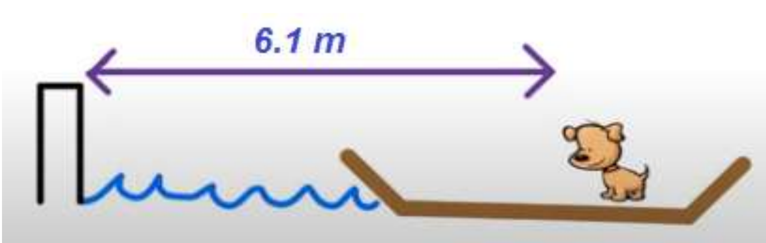
((Equation 1) / m_B) + (Equation 2): $-2.4\ m = \left(1 + \frac{m_D}{m_B}\right) \Delta x_{DG}$

$$\Delta x_{DG} = \frac{-2.4\ m}{\left(1 + \frac{m_D}{m_B}\right)} = \frac{2.4\ m}{\left(1 + \frac{4.5\ Kg}{18\ Kg}\right)} = -1.92\ m$$

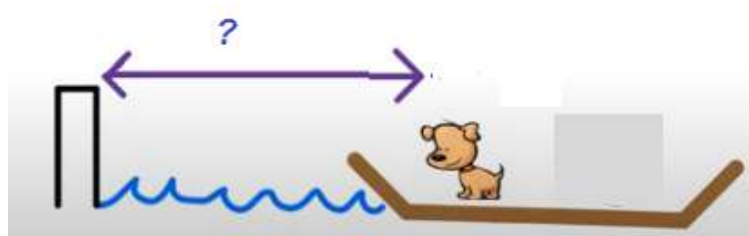
$$\Delta x_{DG} = x_{DG,f} - x_{DG,i} \rightarrow\rightarrow x_{DG,f} = x_{DG,i} + \Delta x_{DG}$$

$$x_{DG} = 6.1 - 1.92 = 4.18\ m$$

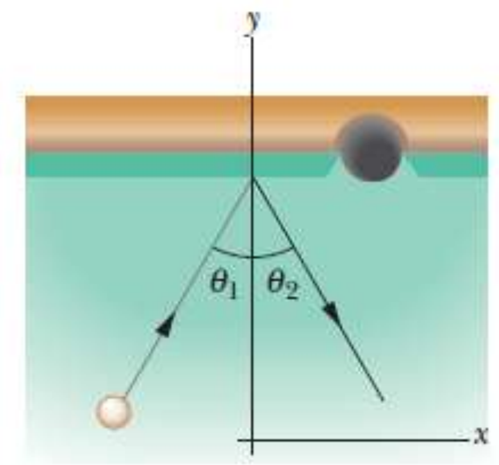
Before



After



9-22) The below figure gives an overhead view of the path taken by a 0.165 kg cue ball as it bounces from a rail of a pool table. The ball's initial speed is 2.00 m/s, and the angle θ_1 *is* 30.0°. The bounce reverses the *y* component of the ball's velocity but does not alter the *x* component. What are (a) angle θ_2 and (b) the change in the ball's linear momentum in unit-vector notation? (The fact that the ball rolls is irrelevant to the problem.)



(a) Since the x component of the ball's velocity does not change and the force of impact on the ball is in the y direction, the linear momentum in x direction is conserved.

P_x is conserved $\rightarrow\rightarrow P_{xi} = P_{xf}$

$$m\, v_i sin\, \theta_1 = m\, v_f \sin\theta_2$$

Use the fact $v_i = v_f$

Thus;

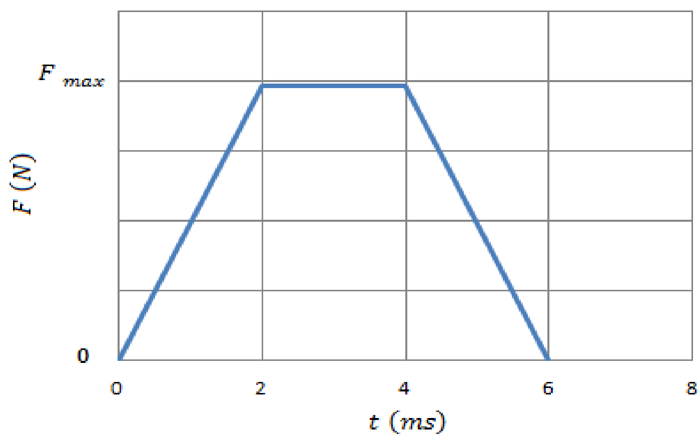
$$\theta_1 = \theta_2 = 30.0^\circ$$

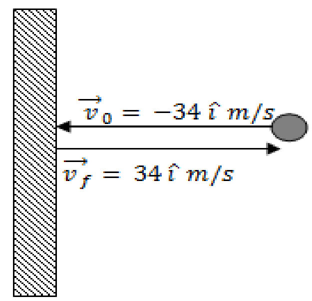
(b) $\Delta\vec{P} = \vec{P}_{yf} - \vec{P}_{yi} = mv_i \cos\theta_2\ (-\hat{j}) - mv_i \cos\theta_1\ (+\hat{j}) = -2\, mv_i \cos\theta_1\ \hat{j}$

$$\Delta\vec{P} = -2\,(0.165\ Kg)\left(2.00\frac{m}{s}\right)(\cos 30.0^\circ)\,\hat{j}$$

$\Delta\vec{P} = -0.572\ Kg\frac{m}{s}\,\hat{j}$

9-35) The below figure shows an approximate plot of force magnitude F versus time t during the collision versus time *t* during the collision of a 58 g Superball with a wall. The initial velocity of the ball is 34 m/s perpendicular to the wall; the ball rebounds directly back with approximately the same speed, also perpendicular to the wall. What is F_{max}, the maximum magnitude of the force on the ball from the wall during the collision?





Mass of the ball m = 58 g = 0.058 kg

The initial velocity of the ball before collision $v_0 = 34\, m/s$

The ball rebounds directly back with the same speed $|v_f| = |v_0|$

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = (0.058 * 34\hat{\imath}) - \left(0.058 * (-34\hat{\imath})\right) = 3.944\ kg.\frac{m}{s}\hat{\imath}$$

$$J = \int_{t_i}^{t_f} F(t)dt = Area\ under\ the\ curve$$

$$= Area\ \ of\ trapezoid(\text{شبه المنحرف})$$

$$= \frac{1}{2}\ (6+2) * 10^{-3} *\ F_{max}$$

$$= (4 * 10^{-3})\ F_{max}$$

From impulse – linear momentum theorem:

$$\Delta\vec{p} = \vec{J}$$

Thus, $|\Delta\vec{p}| = |\vec{J}| \rightarrow\rightarrow\rightarrow\ (4 * 10^{-3})\ F_{max} = 3.944$

$$F_{max} = 986\ N$$

47 A vessel at rest at the origin of an xy coordinate system explodes into three pieces. Just after the explosion, one piece, of mass m, moves with velocity $(-30 m/s)\hat{i}$ and a second piece, also of mass m, moves with velocity $(-30 m/s)\hat{j}$. The third piece has mass 3m. Just after the explosion, what are the (a) magnitude and (b) direction of the velocity of the third piece?

By conservation of linear momentum:

$$m_{vessel} \vec{v}_0 = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3$$

"Vessel at rest $\Rightarrow \vec{v}_0 = 0$"

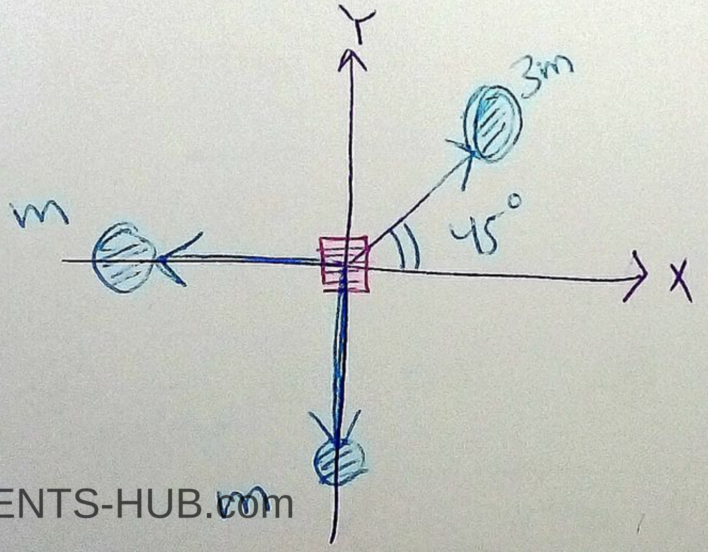
$$m_{vessel}\, 0 = m(-30\hat{i}) + m(-30\hat{j}) + 3m\, \vec{v}_3$$

$$\vec{v}_3 = \frac{1}{3}(+30\hat{i} + 30\hat{j})$$

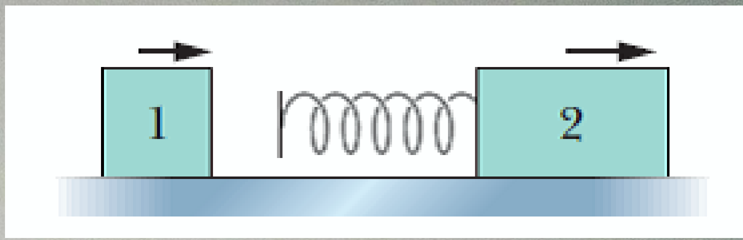
$$\boxed{\vec{v}_3 = (+10\hat{i} + 10\hat{j})\ m/s}$$

(a) $v_3 = 14.1\ m/s$

45° counter clock wise from +X



59 Block 1 (mass 2.0 kg) is moving rightward at 10 m/s and block 2 (mass 5.0 kg) is moving rightward at 3.0 m/s. The surface is frictionless, and a spring with a spring constant of 1120 N/m is fixed to block 2. When the blocks collide, the compression of the spring is maximum at the instant the blocks have the same velocity. Find the maximum compression?



⇒ Maximum compression ⇒

The Two blocks have the same velocity (v)

By conservation of linear momentum

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$$

$$2.0\,kg\,(10\,m/s) + 5.0\,kg\,(3.0\,m/s) = (2.0 + 5.0)\,kg\ v$$

$$v = 5\,m/s$$

By conservation of mechanical energy

$$\Delta E_{mec} = \Delta K + \Delta U = Zero \Rightarrow \Delta U = -\Delta K = U_f - U_i$$

$U_i = Zero$

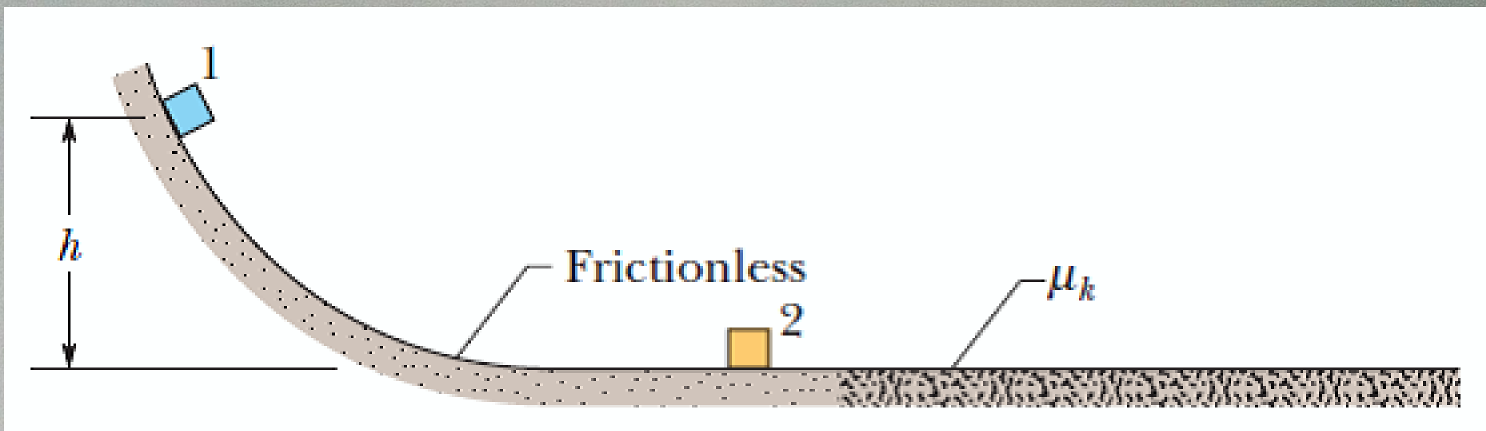
$$\star\ \Delta K = \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_2 v_{2i}^2$$

$$\Delta K = -35\,J$$

$$\frac{1}{2} k x_{Compression}^2 = -\Delta K \Rightarrow x_{Compression} = \sqrt{\frac{-2\Delta K}{k}}$$

$$x_{Compression} = 0.25\,m$$

68 Block 1 of mass m_1 slides from rest along a frictionless ramp from height $h = 2.50$ m and then collides with stationary block 2, which has mass $m_2 = 2.00 m_1$. After the collision, block 2 slides into a region where the coefficient of kinetic friction μ_k is 0.500 and comes to a stop in distance d within that region. What is the value of distance d if the collision is (a) elastic and (b) completely inelastic?



By conservation of mechanical energy: $v_{1i} = \sqrt{2gh}$

$mgh = \frac{1}{2} m v_i^2$ "block 1"

(a) Elastic collision

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \quad ; \quad m_2 = 2.00 m_1$$

$$v_{2f} = \frac{2m_1}{m_1 + 2m_1} \sqrt{2gh} = \frac{2}{3}\sqrt{2(9.8\,m/s^2)(2.50\,m)}$$

$$v_{2f} = 4.67\,m/s$$

⇒ Block 2: $\frac{1}{2} m_2 v_{2f}^2 = \Delta E_{th} = f_k d = \mu_k m_2 g d$

$$d = \frac{v_{2f}^2}{2\mu_k g} = 2.23\,m$$

(b) In elastic collision

$$v_{2f} = \frac{m_1}{m_1 + m_2} v_{1i} = \frac{m_1}{m_1 + 2m_1} v_{1i}$$

$$v_{2f} = \frac{\sqrt{2gh}}{3} = 2.33 \text{ m/s}$$

$$d = \frac{v_{2f}^2}{2\mu_k g} = 0.56 \text{ m}$$

76 A 6090 kg space probe moving nose-first toward Jupiter at 105 m/s relative to the Sun fires its rocket engin, ejecting 80.0 kg of exhaust at a speed of 253 m/s relative to the space probe. What is the final velocity of the probe?

$$M_f = 6090 - 80 = 6010 \text{ kg}$$

$$v_f = v_i + v_{rel} \ln\left(\frac{M_i}{M_f}\right)$$

$$v_f = 105 \frac{m}{s} + 253 \frac{m}{s} \ln\left[\frac{6090 \text{ kg}}{6010 \text{ kg}}\right]$$

$$v_f = 108.3 \frac{m}{s}$$

2 Figure 9-24 shows an overhead view of four particles of equal mass sliding over a frictionless surface at constant velocity. The directions of the velocities are indicated; their magnitudes are equal. Consider pairing the particles. Which pairs form a system with a center of mass that (a) is stationary, (b) is stationary and at the origin, and (c) passes through the origin?

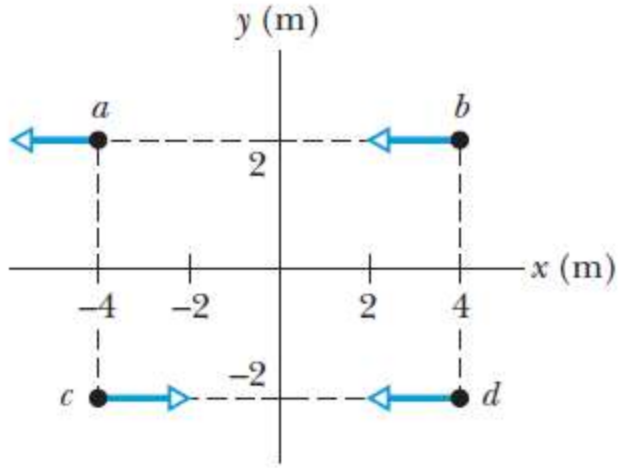


Figure 9-24 Question 2.

a) Choose pairs with v in opposite directions
ac, cd, bc

b) Choose pairs with v in opposite directions AND each particle in pair must be on opposite sides of the origin AND the same distance from origin.
bc

c) Since the center of mass (CM) must pass through the origin the CM must be moving, so v's cannot be in opposite directions.
bd, ad

3 Consider a box that explodes into two pieces while moving with a constant positive velocity along an x axis. If one piece, with mass m_1, ends up with positive velocity $\vec{v}_1$, then the second piece, with mass m_2, could end up with (a) a positive velocity $\vec{v}_2$ (Fig. 9-25*a*), (b) a negative velocity $\vec{v}_2$ (Fig. 9-25*b*), or (c) zero velocity (Fig. 9-25*c*). Rank those three possible results for the second piece according to the corresponding magnitude of $\vec{v}_1$, greatest first.

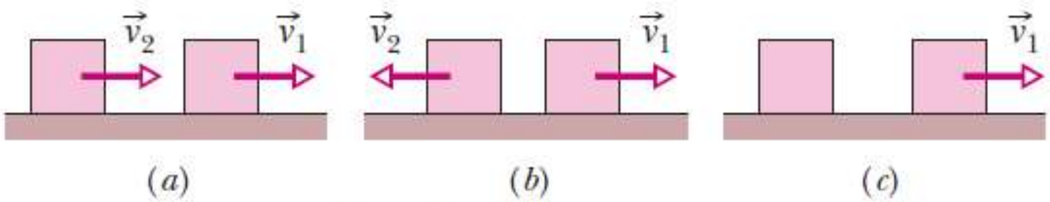


Figure 9-25 Question 3.

b > **c** > **a**. If m_2 is stopped, then $m_1 v_1$ has to carry all of the original momentum. If $v_2 > 0$, then it makes a positive contribution. If $v_2 < 0$, then it makes a *negative* contribution, so v_1 must be greatest.

4 Figure 9-26 shows graphs of force magnitude versus time for a body involved in a collision. Rank the graphs according to the magnitude of the impulse on the body, greatest first.

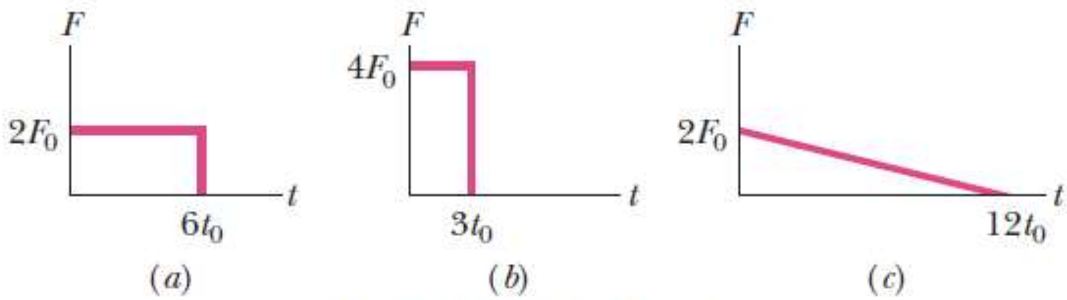


Figure 9-26 Question 4.

all are equal. The impulse is the area under the curve, which is $J = 12\,F_0\,t_0$ in each case.