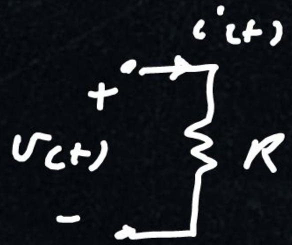
## 1.6) Energy and power of Contineous Time Signals

Ohm's Law




$v(t) = R\, i(t)$

$p(t) = v(t)\, i(t) = \dfrac{v^2(t)}{R} = i^2(t)\, R$

instantaneous power

in general

Let $R = 1\,\Omega \Rightarrow p(t) = v^2(t) = i^2(t) = x^2(t)$

$$E = \int_{-\infty}^{\infty} p(t)\,dt = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

Total Energy

$$P = \lim_{T\to\infty} \left[ \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt \right]$$

→ This is for Non-periodic signals

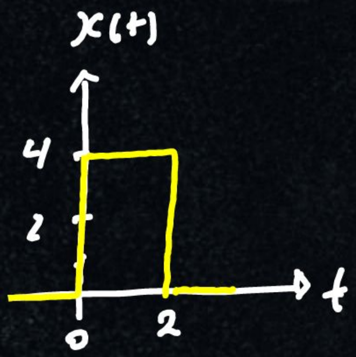
Average power

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2\,dt$$

→ This is for Periodic signals

## Energy signals

$E = \text{finite}$




$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{0}^{2} (4)^2 \, dt = 16(2) = 32 \text{ J}$$

$\downarrow$ finite

→ Average value of finite duration signal = 0

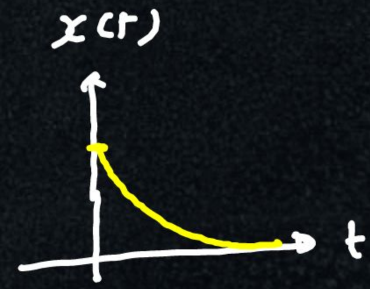
$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \lim_{T \to \infty} \frac{E}{T} = 0$$

when $E = \text{finite} \Rightarrow P = 0$

Ex :- Calculate the total energy

① $x(t) = e^{-at} u(t)\ ,\ a > 0$

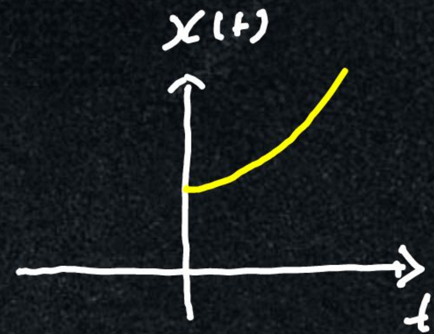
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{0}^{\infty} e^{-2at} dt = \frac{1}{-2a} e^{-2at} \Big|_{0}^{\infty}$$




$$E = \frac{1}{-2a} (0 - 1) = \frac{1}{2a}$$

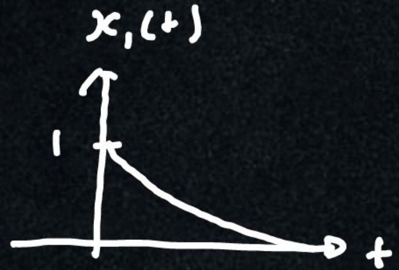
② $x(t) = e^{-at} u(t)\ ,\ a < 0$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{0}^{\infty} e^{-2at} dt = \frac{1}{-2a} e^{-2at} \Big|_{0}^{\infty} = \infty$$



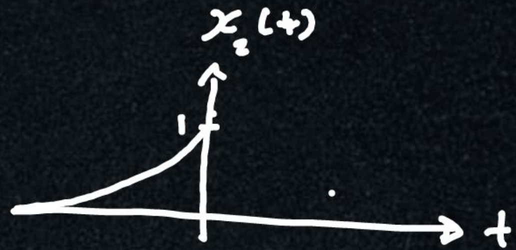
EX :- Calculate the total energy for the following signals

① $x_1(t) = e^{-at} u(t) \; ; \; a > 0$




$$E_1 = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_{0}^{\infty} e^{-2at} dt = \frac{1}{2a}$$

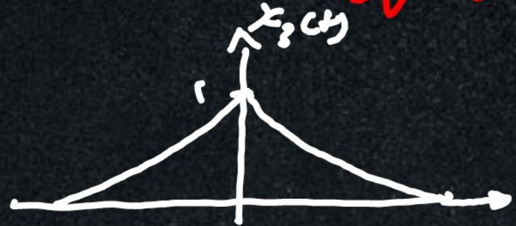
② $x_2(t) = x_1(-t)$




$$E_2 = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{0} e^{2at} dt = \frac{1}{2a}$$

⇒ No effect of T.R on total energy of signal

③ $x_3(t) = x_1(t) + x_2(t)$




$$E_3 = \int_{-\infty}^{\infty} |x_3(t)|^2 dt = \frac{1}{2a} + \frac{1}{2a} = \frac{1}{a} \rightarrow E_1 + E_2$$

## Power signals

condition of power signal $\rightarrow$ $P = \text{finite}$ $\Rightarrow$ $E = \underset{\substack{\uparrow \\ \text{finite}}}{P}\, \underset{\infty}{T} = \infty$

$(0 < P < \infty)$

properties : 1) Periodic signals are power signals

2) $P = (RSM)^2$

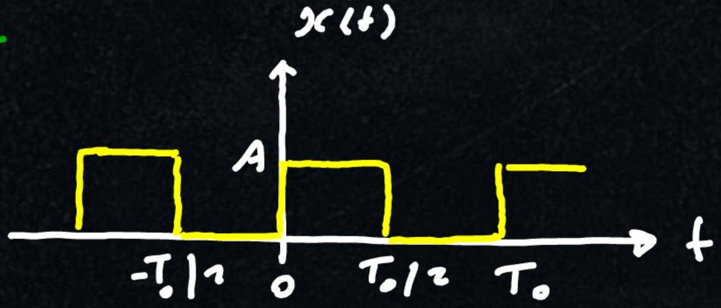
RMS $\rightarrow$ It is the square root of the mean value of the squared function

$$RMS = \sqrt{\frac{1}{T} \int_T x^2(t)\, dt}$$

$\rightarrow$ for $x(t) = A\cos(\omega t + \theta)$

$\Rightarrow RMS = \frac{A}{\sqrt{2}}$

**Ex:-**




Calculate P + RMS

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2\, dt = \frac{1}{T_0}\left[0 + A_0^2 \frac{T_0}{2}\right] = \frac{A_0^2}{2}$$

$$RMS = \sqrt{\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2\, dt} = \sqrt{P} = \frac{A_0}{\sqrt{2}}$$

**Ex:** Calculate P for $x(t) = A_0 \sin(\omega_0 t)$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A_0^2 \sin^2(\omega_0 t)\, dt = \frac{A_0^2}{2T_0}\int_{-T_0/2}^{T_0/2} \left(1 - \cos(2\omega_0 t)\right) dt = \frac{A_0^2}{2}$$

$\rightarrow \quad x(t) = A\sin(\omega t)$

Time scaling $\rightarrow$ $y(t) = x(2t)$ $\Big\}$ $P = \frac{A^2}{2}$ , $RMS = \frac{A}{\sqrt{2}}$

phase shift $\rightarrow$ $y(t) = x(t+t_0)$ $\Big\}$ $P = \frac{A^2}{2}$ , $RMS = \frac{A}{\sqrt{2}}$

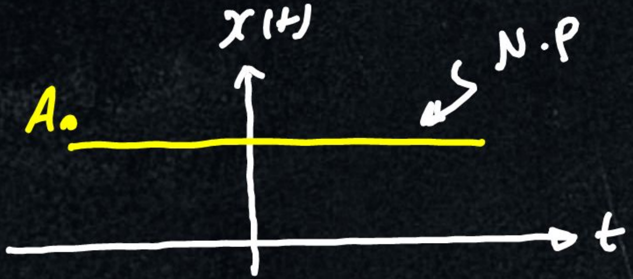
Amplitude Reversal $\rightarrow$ $y(t) = -x(t)$ $\Big\}$ $P = \frac{A^2}{2}$ , $RMS = \frac{A}{\sqrt{2}}$

Amplitude Scaling $\rightarrow$ $y(t) = k\,x(t)$ $\Big\}$ $P = \frac{(Ak)^2}{2}$ , $RMS = \frac{Ak}{\sqrt{2}}$

EX :- Calculate the average power of the following signals

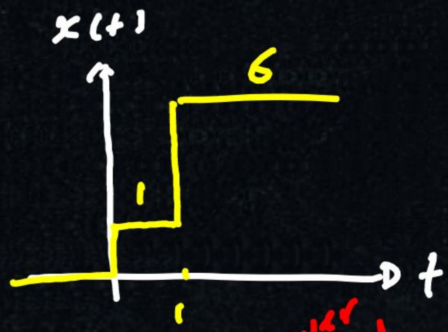
① 


$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} A_0^2\, dt = \frac{A_0^2}{2} \quad , \quad RMS = \frac{A_0}{\sqrt{2}} = \sqrt{P}$$

② 


$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = \lim_{T\to\infty} \int_{-T/2}^{T/2} A_0^2\, dt = A_0^2 \quad , \quad RMS = A_0 = \sqrt{P}$$

EX :- check if the following signals are energy or power signals:- [redacted]

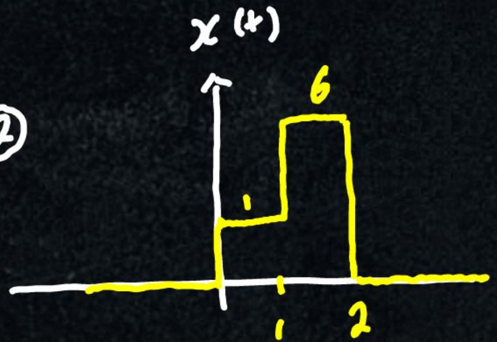
① 


power signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^1 (1)^2 dt + \int_1^{\infty} (6)^2 dt = \boxed{\infty}$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T\to\infty} \left[ \frac{1}{T}\int_0^1 (1)^2 dt + \frac{1}{T}\int_1^{T/2} (6)^2 dt \right]$$

$$P = \lim_{T\to\infty} \frac{1}{T} + \lim_{T\to\infty} \left[ \frac{36\,(T/2 - 1)}{T} \right] = \boxed{18}\ W$$

② 


Energy signal

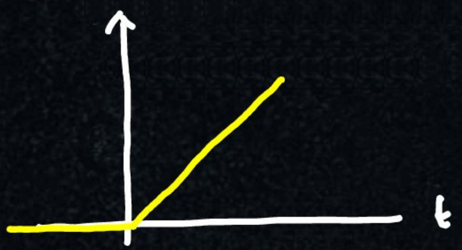
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^1 (1)^2 dt + \int_1^2 (6)^2 dt = \boxed{37}\ J$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T\to\infty} \left[ \frac{1}{T} \int_0^1 (1)^2 dt + \int_1^2 (6)^2 dt \right] = \boxed{0}$$

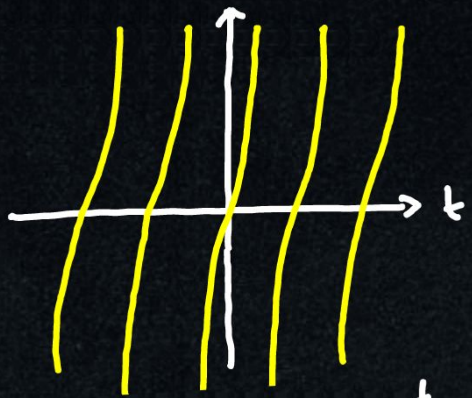
# Neither Energy Nor power Signals (NENP)

→ If $x(t_0) = \infty$ then $x(t)$ is NENP signal where $t_0$ is any instant of time

$x(t) = t\,u(t)$



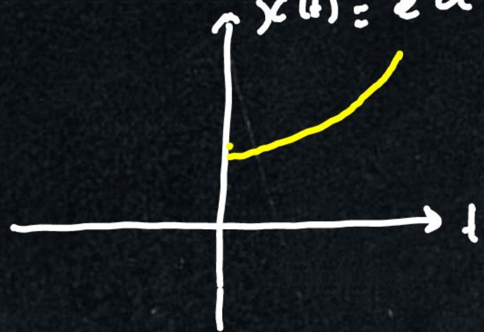

$x(t) = \tan(t)$



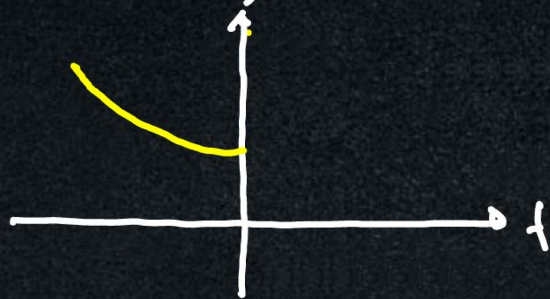

$x(t) = t^2$


$x(t) = \frac{1}{t}$


$x(t) = e^{t}u(t)$




$x(t) = e^{-t}u(-t)$



Ex :- $x(t) = t\,u(t)$ → check if it is E or P or NENP

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{0}^{\infty} t^2\,dt = \frac{t^3}{3}\Big|_{0}^{\infty} = \boxed{\infty}$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt = \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} t^2\,dt = \lim_{T\to\infty} \frac{1}{T}\,\frac{t^3}{3}\Big|_{0}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{3T}\left(\frac{T^3}{8}\right) = \lim_{T\to\infty} \frac{T^2}{24} = \boxed{\infty}$$

→ The signal is NENP


Note:- ① E → x(t) is decreasing in nature

② P → x(t) is neither increasing nor decreasing in nature

③ NENP → x(t) is increasing in nature