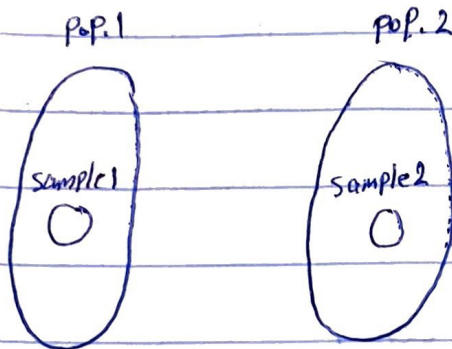


10.4: Inferences about the difference between two pop. proportions.



π_1 : proportion in population 1

π_2 : proportion in population 2.

p_1 : proportion in sample 1

p_2 : proportion in sample 2.

n_1 : sample 1 size

n_2 : sample 2 size.

★ point estimator for $\pi_1 - \pi_2 = p_1 - p_2$.

★ $(1-\alpha)$ CI for $\pi_1 - \pi_2 = (p_1 - p_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

★ margin of error (E) = $Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

★ standard error of $p_1 - p_2$: $\sigma_{p_1 - p_2} = \sqrt{\frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}}$

★ Assumptions:

1) samples 1 and 2 Random.

large enough.

2) samples 1 and 2 indep.

pop. $n_1 \pi_1 \geq 5$, $n_1(1-\pi_1) \geq 5$

3) samples 1 and 2 large enough.

$n_2 \pi_2 \geq 5$, $n_2(1-\pi_2) \geq 5$

sample. $n_1 p_1 \geq 5$, $n_1(1-p_1) \geq 5$

$n_2 p_2 \geq 5$, $n_2(1-p_2) \geq 5$

سؤال

★ Hypothesis Tests about $\pi_1 - \pi_2$:

$H_0 : \pi_1 - \pi_2 \geq 0$	$H_0 : \pi_1 - \pi_2 \leq 0$	$H_0 : \pi_1 - \pi_2 = 0$
$H_1 : \pi_1 - \pi_2 < 0$	$H_1 : \pi_1 - \pi_2 > 0$	$H_1 : \pi_1 - \pi_2 \neq 0$
Lower Tail test	upper tail test	two tail test

Remark: Hypothesized value for $\pi_1 - \pi_2$ is zero.

Remark: under the H_0 when H_0 is true an equality we get $\pi_1 = \pi_2 = \pi$

↳ standard error ($\pi_1 = \pi_2 = \pi$) :
$$\delta_{p_1 - p_2} = \sqrt{\frac{\pi(1-\pi)}{n_1} + \frac{\pi(1-\pi)}{n_2}}$$

$$= \sqrt{\pi(1-\pi)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

★ test statistic ($\pi_1 = \pi_2 = \pi$) :
$$Z = \frac{(p_1 - p_2)}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

★ pooled estimate of π when $\pi_1 = \pi_2 = \pi$:
$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

★ The assumption for using this test are the same as the assumption for the CI.

- ★ Reject H_0 if
- UTT : $Z \geq Z_\alpha$
- LTT : $Z \leq -Z_\alpha$
- TTT : $|Z| \geq Z_{\frac{\alpha}{2}}$

★ Reject H_0 if p-value $\leq \alpha$.