10.6 Part 2

Monday, April 5, 2021 11:46 AM

Th If $\sum a_n$ conv. Abs. $\Rightarrow$ $\sum a_n$ conv.

عكس النظرية

If $\sum a_n$ conv by AST $\not\Rightarrow$ $\sum a_n$ conv. Abs.

$\sum |a_n|$ not necessarily conv.

مثال

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ Alternating harmonic series

conv. by AST since

① $u_n = \frac{1}{n} > 0$

② $u_n \downarrow$

③ $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\sum |a_n| = \sum \frac{1}{n}$ div harmonic series

Test (Alternating p-series Test) $\leftrightarrow$ conv (Abs./Cond.) / div.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p} = \begin{cases} \text{conv. Abs.} & \text{if } p > 1 \\ \text{conv. Cond.} & \text{if } 0 < p \le 1 \\ \text{div by } n^{th} \text{ term test} & \text{if } p < 0 \end{cases}$$

Exp (1) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3}$ conv. Abs. $\Rightarrow$ conv.

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since $\sum |a_n| = \sum \frac{1}{n^3}$ ← conv. p-series

② $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{2/3}}$ conv. cond. $0 < p = \frac{2}{3} \leq 1$

⇓

conv. but not Abs.

by AST $\Rightarrow u_n = \frac{1}{n^{2/3}}$

① $u_n > 0$ ② $u_n \downarrow$ ③ $\lim_{n\to\infty} u_n = 0$

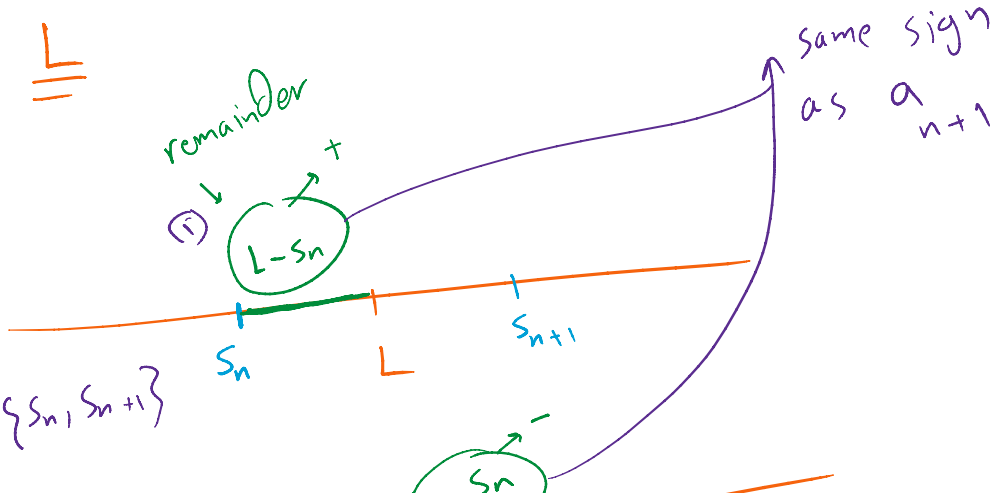
$\sum |a_n| = \sum \frac{1}{n^{2/3}}$ p-series div since $p = \frac{2}{3} < 1$

③ $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{-2}} = \sum (-1)^n n^2$ div by n^{th} term test

$u_n = n^2 \Rightarrow$ ③ $\lim_{n\to\infty} u_n \neq 0$

$\sum (-1)^{n+1} u_n = L$

S_n



③ $\min\{S_n, S_{n+1}\} < L < \max\{S_n, S_{n+1}\}$

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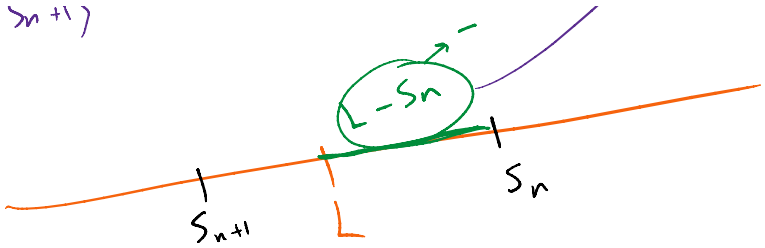
(3) $\min\{S_n, S_{n+1}\} < L < \max\{S_n, S_{n+1}\}$



(2) $E = |\text{Remainder}| = |L - S_n| < u_{n+1} = |a_{n+1}|$

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