

## Frequency Response = Laplace Transform



$$y(t) = h(t) * x(t)$$

$$\mathcal{L} \rightarrow Y(s) = H(s) X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} \rightsquigarrow \text{Transfer function}$$

$\Rightarrow$  The frequency response is determined as:

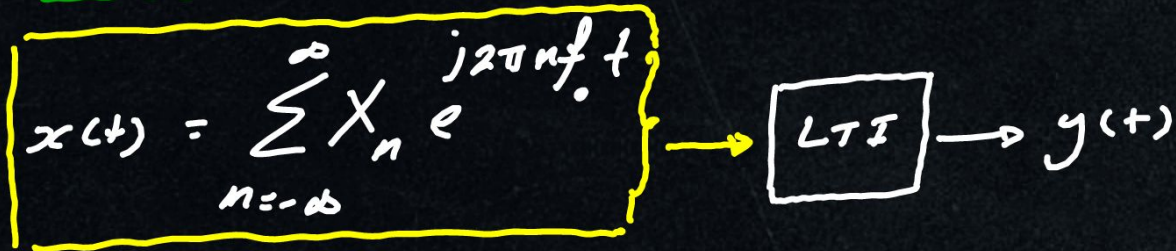
$$H(\omega) = H(s) \Big|_{s=j\omega}$$

EX :- Determine the frequency response of the system with transfer function :-

$$T(s) = \frac{3s+1}{s^2+5}$$

$$H(\omega) = T(s) \Big|_{s=j\omega} = \frac{1+j3\omega}{5-\omega^2}$$

4.8] Steady state response of LTI system using FT  
 "periodic signal."



$$Y(f) = H(f) X(f)$$

$$X(f) = \sum_{n=-\infty}^{\infty} X_n \delta(f - n f_0)$$

$$Y(f) = H(f) \sum_{n=-\infty}^{\infty} X_n \delta(f - n f_0) = \sum_{n=-\infty}^{\infty} H(n f_0) X_n \delta(f - n f_0)$$

$$y(t) = \sum_{n=-\infty}^{\infty} H(n f_0) X_n e^{j2\pi n f_0 t} = \sum_{n=-\infty}^{\infty} |H(n f_0)| |X_n| e^{j[2\pi n f_0 t + \theta_H(n f_0) + \theta_{X_n}]}$$

SS response

OR

$$y(t) = \sum_{n=-\infty}^{\infty} f_n X(nf_0) H(nf_0) e^{j 2\pi n f_0 t}$$

$$y(t) = \sum_{n=-\infty}^{\infty} f_n |X(nf_0)| |H(nf_0)| e^{j [2\pi n f_0 t + \theta_x(nf_0) + \theta_H(nf_0)]}$$

Ex :- Find the ss response of the system with the frequency response  $H(f) = \frac{10}{3 + j2\pi f}$  to the signal  $x(t) = \sum_{n=-\infty}^{\infty} \text{sinc}(t - nT_0)$

$$y(t) = \sum_{n=-\infty}^{\infty} f_0 X(nf_0) H(nf_0) e^{j2\pi n f_0 t}$$

$$X(f) = F[\text{sinc}(t)] = \pi(f)$$

$$H(f) = \frac{10}{\sqrt{9 + 4\pi^2 f^2}} \angle -\tan^{-1}\left(\frac{2\pi f}{3}\right)$$

$$y(t) = \sum_{n=-\infty}^{\infty} \left\{ \frac{10 f_0 \pi(nf_0)}{\sqrt{9 + 4\pi^2 n^2 f_0^2}} e^{j\left[2\pi n f_0 t - \tan^{-1}\left(\frac{2\pi n f_0}{3}\right)\right]} \right\}$$

## 4.9] Hilbert Transform

→ It aims to generate orthogonal signals

The Hilbert transform of  $x(t)$  is given by :-

$$x^H(t) = x(t) * \frac{1}{\pi t}$$

Orthogonal  
signals

$$X^H(f) = X(f) F\left[\frac{1}{\pi t}\right]$$

$$X^H(f) = X(f) F\left[\frac{1}{j\pi t}\right] j = -j \operatorname{sgn}(f) X(f)$$

$$X^H(f) = -j \operatorname{sgn}(f) X(f)$$

$$\left\{ \begin{array}{l} F[\operatorname{sgn}(t)] = \frac{1}{j\pi f} \\ F\left[\frac{1}{j\pi t}\right] = \operatorname{sgn}(-f) \\ \quad = -\operatorname{sgn}(f) \end{array} \right.$$

Ex :- Find the Hilbert transform of  $x(t) = A \cos(\omega t)$

$$X(f) = \frac{A}{2} \delta(f - f_0) + \frac{A}{2} \delta(f + f_0)$$

$$X_H(f) = -j \left[ \frac{A}{2} \delta(f - f_0) + \frac{A}{2} \delta(f + f_0) \right] \text{sgn}(f)$$

$$= A \left[ \frac{\delta(f - f_0) - \delta(f + f_0)}{2j} \right]$$

$$x_H(t) = A \sin(\omega t)$$