CH2: System Modeling in Time-Domain

2.1) Basic system properties

① Static and Dynamic systems

$x(t)$: input i/p
$y(t)$: output o/p

Static system :- $y(t)$ depends on present value of $x(t)$

ex:-
$$y(t) = 2x(t)$$
$$y(t) = e^{-(t+1)} x(t)$$
$$y(t) = (t-1)\,x(t)$$

[Note: Static system is known as memoryless or instantaneous system]

Dynamic system :- $y(t)$ depends on past or future values of $x(t)$

ex: $y(t) = x(t-1)$ ⇝ past i/p → dynamic

$y(t) = x(t+1)$ ⇝ future i/p → dynamic

$y(t) = \underbrace{x(t)}_{\text{present i/p}} + \underbrace{x(t-1)}_{\text{past i/p}}$ → dynamic

Ex:- $y(t) = x(2t)$

$y(0) = x(0)$ present i/p

$y(-1) = x(-2)$ past i/p

$y(1) = x(2)$ future i/p

$\Rightarrow$ Dynamic system

Ex:- $y(t) = x(-t)$

$y(0) = x(0)$ present i/p

$y(-1) = x(1)$ future i/p

$y(1) = x(-1)$ past i/p

$\Rightarrow$ Dynamic system

EX :- $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

Dynamic

EX :- $y(t) = x(\sin t)$

$y(\pi) = x(0) \Longrightarrow$ Dynamic

EX :- $y(t) = \sin(2t)\,x(t)$

Static

② Causal and Non-Causal Systems

Causal system :- $y(t)$ is independent of future value of $x(t)$

ex :- $y(t) = x(t)$

$y(t) = x(t) + x(t-1)$

Non-Causal system : $y(t)$ depends on future values of $x(t)$

ex :- $y(t) = x(t+2)$

$y(t) = x(t) + x(t-1) + x(t+1)$

Anti-Causal system : $y(t)$ depends only on future values of $x(t)$

ex :- $y(t) = x(t+2)$

EX :- $y(t) = x(3t)$

$y(0) = x(0)$
$y(1) = x(3)$
$\Big\} \rightarrow$ N.C

EX :- $y(t) = \begin{cases} x(3t) & t < 0 \\ x(t-1) & t \geq 0 \end{cases}$

$y(-1) = x(-3)$
$y(0) = x(-1)$
$y(1) = x(0)$
$\Big\} \rightarrow$ C

EX :- $y(t) = \sin(t+1)\, x(t-1)$

C

EX :- $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

Causal

EX :- $y(t) = \int_{-\infty}^{t+1} x(\tau)\, d\tau$

N.C

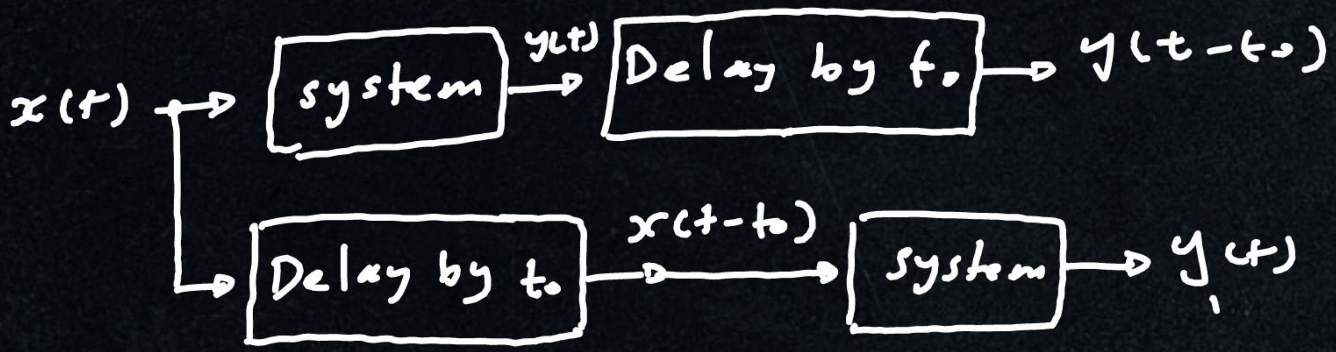
EX :- $y(t) = \int_{-\infty}^{t-1} x(\tau)\, d\tau$

Causal

EX :- $y(t) = \int_{-\infty}^{t} x(3\tau)\, d\tau$

N.C

③ Time-Invariant & Time-Variant Systems



$$y_1(t) = y(t-t_0) \Rightarrow \text{TIV system}$$

$$y_1(t) \neq y(t-t_0) \Rightarrow \text{TV system}$$

Ex :- $y(t) = x(2t)$

$$\left.\begin{aligned} & y(t) \xrightarrow{t_0} y(t-t_0) = x(2(t-t_0)) = x(2t-2t_0) \\ & x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{system} \rightarrow x(2t-t_0) \end{aligned}\right\} \Rightarrow \text{TV system}$$

Ex :- $y(t) = 2 + x(t)$

$$\left.\begin{array}{l} y(t) \xrightarrow{t_0} y(t-t_0) \\ x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{system} \rightarrow 2 + x(t-t_0) \end{array}\right\} \text{TIV system}$$

Ex :- $y(t) = x(\cos t)$

$$\left.\begin{array}{l} y(t) \xrightarrow{t_0} y(t-t_0) = x(\cos(t-t_0)) \\ x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{system} \rightarrow x((\cos t) - t_0) \end{array}\right\} \text{TV system}$$

Ex :- $y(t) = \cos t \; x(t)$

$$\left.\begin{array}{l} y(t) \xrightarrow{t_0} y(t-t_0) = \cos(t-t_0)\, x(t-t_0) \\ x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{system} \rightarrow \cos t \; x(t-t_0) \end{array}\right\} \text{TV system}$$

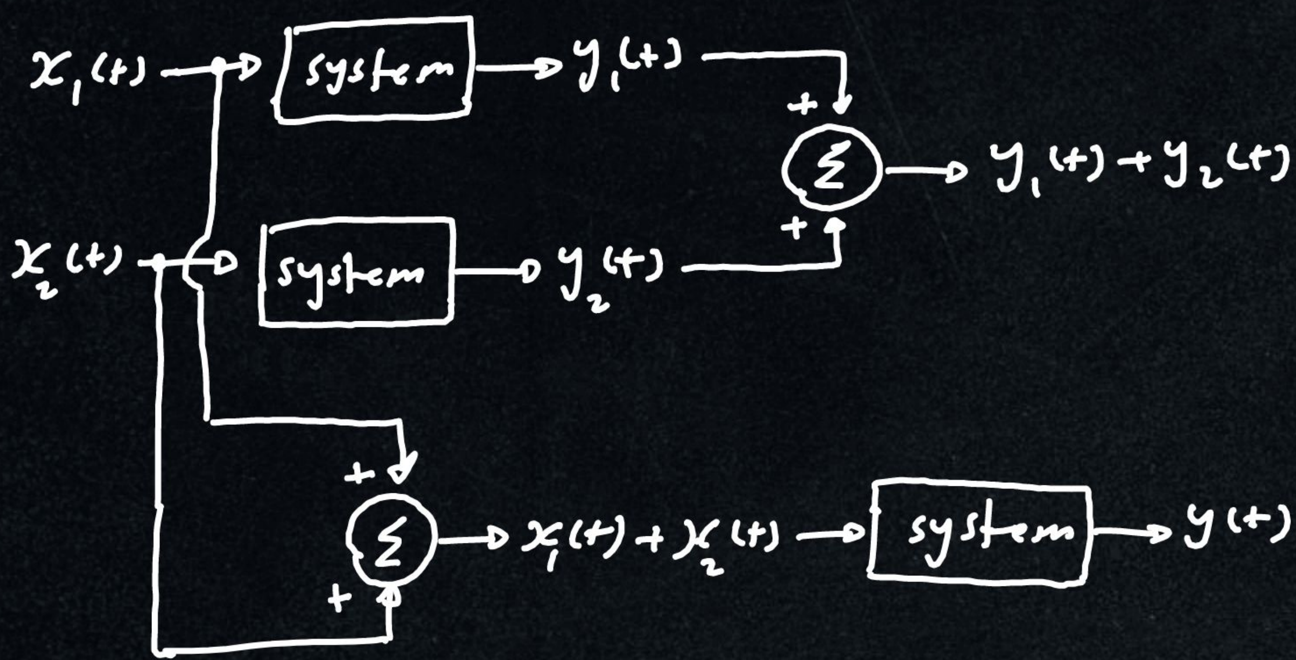
④ Linear and Non-Linear Systems

Linear system :- The system which follows the principle of superposition

The principle of superposition is the combination of two different laws :-

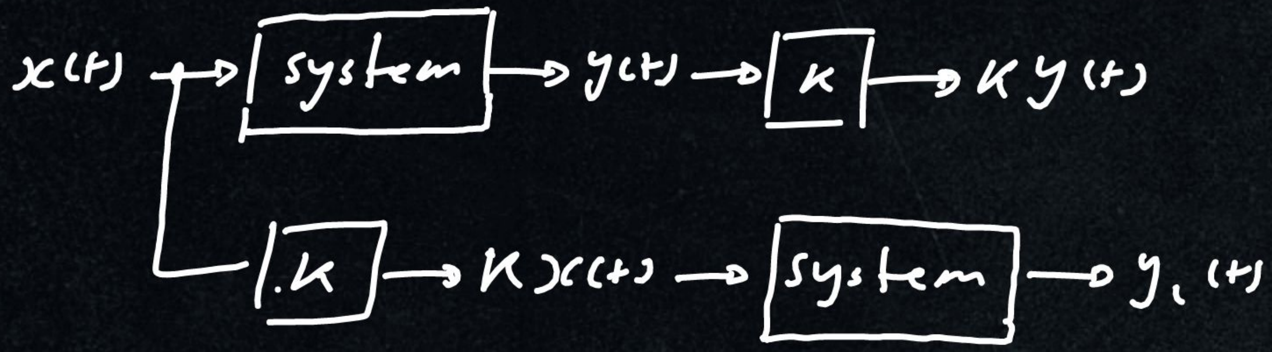
1) Law of additivity
2) Law of homogeneity

Law of Additivity



if $y(t) = y_1(t) + y_2(t) \Rightarrow$ The system is following the LOA

Law of Homogeneity



If $y_1(t) = k\,y(t) \Rightarrow$ The system is following the LOH

EX :- $y(t) = x(\sin t)$

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t) = x(\sin t)$$

$$\left.\begin{aligned} y_1(t) &= x_1(\sin t) \\ y_2(t) &= x_2(\sin t) \end{aligned}\right] \Rightarrow y_1(t) + y_2(t) = x_1(\sin t) + x_2(\sin t)$$

$$x_1(t) + x_2(t) \longrightarrow \text{system} \longrightarrow x_1(\sin t) + x_2(\sin t) = y_1(t) + y_2(t)$$

LOA ✓

$$k\,y(t) = k\,x(\sin t)$$

LOH ✓

$$k\,x(t) \longrightarrow \text{system} \longrightarrow k\,x(\sin t) = k\,y(t)$$

$\Rightarrow$ Linear System

Ex :- $y(t) = x^2(t)$

$y_1(t) + y_2(t) = x_1^2(t) + x_2^2(t)$ LOA ✗

$x_1(t) + x_2(t) \longrightarrow \text{system} \longrightarrow (x_1(t) + x_2(t))^2 \neq y_1(t) + y_2(t)$

$\Rightarrow$ Non-Linear

Ex :- $y(t) = (\sin t)\, x(t)$

$y_1(t) + y_2(t) = \sin t\, x_1(t) + \sin t\, x_2(t) = \sin t\,(x_1(t) + x_2(t))$

$x_1(t) + x_2(t) \longrightarrow \text{system} \longrightarrow \sin t\,(x_1(t) + x_2(t)) = y_1(t) + y_2(t)$

LOA ✓

$k\,y(t) = k \sin t\, x(t)$ LOH ✓

$k\,x(t) \longrightarrow \text{system} \longrightarrow \sin t\,(k\,x(t)) = k \sin t\, x(t) = k\,y(t)$

$\Rightarrow$ Linear system

Ex :- $y(t) = 2t + x(t)$

$y_1(t) + y_2(t) = 2t + x_1(t) + 2t + x_2(t) = 4t + x_1(t) + x_2(t)$

$x_1(t) + x_2(t) \longrightarrow \text{system} \longrightarrow 2t + (x_1(t) + x_2(t)) \neq y_1(t) + y_2(t)$

LOA ✗

$\Rightarrow$ Non-linear

Ex :- $y(t) = x(t+1) + x(t-1)$

$y_1(t) + y_2(t) = x_1(t+1) + x_1(t-1) + x_2(t+1) + x_2(t-1)$

LOA ✓

$x_1(t) + x_2(t) \longrightarrow \text{system} \longrightarrow x_1(t+1) + x_2(t+1) + x_1(t-1) + x_2(t-1) = y_1(t) + y_2(t)$

$k y(t) = k x(t+1) + k x(t-1)$

LOH ✓

$k x(t) \longrightarrow \text{system} \longrightarrow k x(t+1) + k x(t-1)$

$\Rightarrow$ Linear

EX :- $\frac{dy(t)}{dt} + 3y(t) = 2x(t)$

$x_1(t) \rightarrow \text{system} \rightarrow y_1(t) \Rightarrow y_1'(t) + 3y_1(t) = 2x_1(t)$ ... (1)

$x_2(t) \rightarrow \text{system} \rightarrow y_2(t) \Rightarrow y_2'(t) + 3y_2(t) = 2x_2(t)$ ... (2)

$(1) + (2) \Rightarrow (y_1(t) + y_2(t))' + 3(y_1(t) + y_2(t)) = 2(x_1(t) + x_2(t))$ .... (*)

$x_1(t) + x_2(t) \rightarrow \text{system} \rightarrow y'(t) + 3y(t) = 2(x_1(t) + x_2(t))$ ... (**)

$y'(t) + 3y(t) = (y_1(t) + y_2(t))' + 3(y_1(t) + y_2(t))$

$y(t) = y_1(t) + y_2(t)$ LDA ✓

multiply by k

$$k y'(t) + 3k y(t) = 2k x(t)$$

$$(k y(t))' + 3(k y(t)) = 2k x(t)$$

$$k x(t) \rightarrow \text{system} \rightarrow y_H(t)$$

$$y_H'(t) + 3 y_H(t) = 2(k x(t))$$

$$\Rightarrow \quad y_H'(t) + 3 y_H(t) = (k y(t))' + 3(k y(t))$$

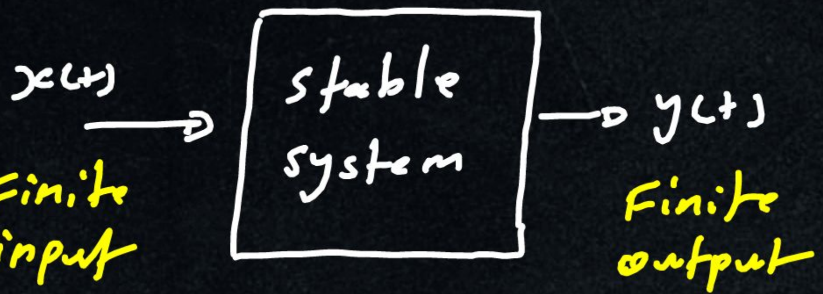
$$\Rightarrow \quad y_H(t) = k y(t)$$

LOH ✓

Linear

⑤ Stable and Unstable systems

Bounded Input - Bounded Output Criteria (BIBO)



Bounded signals : DC, sin t, cos t, u(t)

For a stable system, y(t) should be bounded for bounded x(t) at each and every instant of time.

Ex :- $y(t) = t\,x(t)$

let $x(t) = u(t)$

$\Rightarrow y(t) = t\,u(t) = r(t)$

unbounded

$\Rightarrow$ unstable

Ex :- $y(t) = \sin(x(t))$

$y(t)$ is bounded

$\Rightarrow$ stable

Ex :- $y(t) = x(t) + 2$

let $x(t) = 4$

$y(t) = 4 + 2 = 6$

$y(t)$ is Bounded

$\Rightarrow$ stable

Ex :- $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

let $x(t) = u(t)$

$\Rightarrow y(t) = r(t)$ unbounded

$\Rightarrow$ Unstable

Ex :- $y(t) = \frac{x(t)}{t}$

let $x(t) = 2$

$y(t) = \frac{2}{t} \longrightarrow$ unbounded

$\Rightarrow$ Unstable

Ex :- $y(t) = \frac{dx(t)}{dt}$

let $x(t) = u(t)$

$\Rightarrow y(t) = \delta(t)$

$\hookrightarrow$ Unbounded

$\Rightarrow$ Unstable