

7.3 Exponential Function:

[7] $e^{x^2} e^{2x+1} = e^t$ Solve for t

$$e^{x^2+2x+1} = e^t$$

$$x^2+2x+1 = t$$

$$\boxed{(x+1)^2 = t}$$

Find the derivative with respect to x , t , or θ

[14] $y = \ln(3\theta e^{-\theta})$

$$y = \ln 3 + \ln \theta + \ln e^{-\theta}$$

$$y = \ln 3 + \ln \theta - \theta$$

$$y' = \frac{1}{\theta} - 1 = \frac{1-\theta}{\theta}$$

[18] $y = \ln(2e^{-\theta} \sin \theta)$

$$y = \ln 2 + \ln e^{-\theta} + \ln \sin \theta$$

$$y = \ln 2 - \theta + \ln \sin \theta$$

$$y' = -1 + \frac{1}{\sin \theta} \cdot \cos \theta$$

$$y' = -1 + \cot \theta = \cot \theta - 1$$

10 Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{10}{x}\right)$

$$y = \ln 10 - \ln x$$

$$y' = -\frac{1}{x}$$

21 $y = e^{\cos t + \ln t}$

$$y = e^{\cos t} \cdot e^{\ln t}$$

$$y = t \cdot e^{\cos t}$$

$$y' = t \cdot e^{\cos t} (-\sin t) + e^{\cos t}$$

$$y' = e^{\cos t} [1 - t \sin t]$$

23 $y = \int_0^{\ln x} \sin(e^t) dt$ By Using F.T.C

$$y' = \sin(e^{\ln x}) \cdot \frac{1}{x} - \cancel{\sin(e^0) \cdot (0)}$$

$$y' = \frac{\sin x}{x}$$

$$\boxed{28} \quad \tan y = e^x + \ln x$$

$$y' \sec^2(y) = e^x + \frac{1}{x}$$

$$y' = \left[e^x + \frac{1}{x} \right] \sec^2(y)$$

$$y' = \left[e^x + \frac{1}{x} \right] \sec^2 \left[\tan^{-1}(e^x + \ln x) \right]$$

Evaluate the integral 29-50:-

$$\boxed{38} \quad \int \frac{e^{-\sqrt{r}}}{\sqrt{r}} dr$$

$$u = -\sqrt{r}$$

$$du = \frac{-1}{2\sqrt{r}} dr$$

$$\int \frac{e^u}{\sqrt{r}} \cdot -2\sqrt{r} du = -2 \int e^u du$$

$$= -2 e^u + C$$

$$= -2 e^{-\sqrt{r}} + C$$

$$\boxed{44} \quad \int_{\pi/4}^{\pi/2} (1 + e^{\cot \theta}) \csc^2 \theta d\theta$$

$$u = \cot \theta$$

$$du = -\csc^2 \theta d\theta$$

$$\int_1^0 (1 + e^u) \frac{du}{-\csc^2 \theta}$$

$$x = \frac{\pi}{4} \rightarrow u = 1$$

$$x = \frac{\pi}{2} \rightarrow u = 0$$

$$\int_1^0 (1 + e^u) du = \left[u + e^u \right]_1^0 = 1 + e - 1 = \boxed{e}$$

$$\boxed{48} \quad I = \int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2}) dx$$

$$\int \frac{2x e^{x^2} \cos(u) du}{2x e^{x^2}}$$

$$\int_1^{\pi} \cos(u) du = \sin(u) \Big|_1^{\pi}$$

$$= 0 - \sin(1) = -\sin(1)$$

$$u = e^{x^2}$$

$$du = e^{x^2} \cdot 2x dx$$

$$x=0 \longrightarrow u=1$$

$$x=\sqrt{\ln \pi} \longrightarrow u=\pi$$

$$\boxed{50} \quad I = \int \frac{1}{1+e^x} dx$$

$$= \int \frac{1}{e^x(e^x+1)} dx$$

$$\text{let } u = e^{-x} + 1$$

$$du = -e^{-x} dx$$

$$\int \frac{1}{e^x(u)} \frac{du}{-e^{-x}} = - \int \frac{1}{u} du$$

$$= - \ln |u| + C$$

$$= - \ln |e^{-x} + 1| + C$$

I.V.P
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$$\frac{dy}{dx} = e^{-t} \sec^2(\pi e^{-t})$$

$$y(\ln 4) = \frac{2}{\pi}$$

Initial condition given

Find $y = ??$

$$y = \int e^{-t} \sec^2(\pi e^{-t}) dt$$

$$u = \pi e^{-t}$$
$$du = -\pi e^{-t} dt$$

$$\int e^{-t} \sec^2(u) \frac{du}{-\pi e^{-t}}$$

$$y = \frac{-1}{\pi} [\tan(u)] + C = \frac{-1}{\pi} \tan(\pi e^{-t}) + C$$

To find $C = ??$

$$y(\ln 4) = \frac{2}{\pi}$$

$$\frac{2}{\pi} = y(\ln 4) = \frac{-1}{\pi} \tan(\pi e^{-\ln 4}) + C$$

$$\frac{2}{\pi} = \frac{-1}{\pi} \tan\left(\frac{\pi}{4}\right) + C$$

$$\frac{2}{\pi} = \frac{-1}{\pi} + C \longrightarrow C = \frac{3}{\pi}$$

$$e^{-\ln 4} = e^{\ln 4^{-1}} = 4^{-1} = \frac{1}{4}$$

$$y = \frac{-1}{\pi} \tan(\pi e^{-t}) + \frac{3}{\pi}$$

58 Find the derivative of y with respect to the given independent variable

$$y = 2^{(s^2)}$$

$$y' = 2^{s^2} \cdot (2s) (\ln 2)$$

63 $y = 7^{\sec \theta} \ln 7$

$$y' = (\ln 7) (7)^{\sec \theta} \cdot (\ln 7) (\sec \theta \tan \theta)$$

$$y' = (\ln 7)^2 7^{\sec \theta} (\sec \theta \tan \theta)$$

65 $y = 2^{\sin(3t)}$

$$y' = 2^{\sin(3t)} \cdot (3 \cos(3t)) (\ln 2)$$

$$= 3 \ln 2 \cos(3t) 2^{\sin(3t)}$$

$$= (\ln 8) \cos(3t) 2^{\sin(3t)}$$

73 $y = \log_3 \left[\frac{x+1}{x-1} \right]^{\ln 3}$

$$y = \ln 3 \left[\log_3 (x+1) - \log_3 (x-1) \right]$$

$$= \ln 3 \left[\frac{\ln (x+1)}{\ln 3} - \frac{\ln (x-1)}{\ln 3} \right]$$

$$y = \ln (x+1) - \ln (x-1)$$

$$y = \ln(x+1) - \ln(x-1)$$

$$y' = \frac{1}{x+1} - \frac{1}{x-1}$$

$$y' = \frac{(x-1) - (x+1)}{x^2-1} = \frac{-2}{x^2-1}$$

$$\boxed{75} \quad y = \theta \sin(\log_7 \theta)$$

$$y' = \cancel{\theta} \cos(\log_7 \theta) \cdot \frac{1}{\cancel{\theta} \ln 7} + \sin(\log_7 \theta) \cdot (1)$$

$$y' = \frac{\cos(\log_7 \theta)}{\ln 7} + \sin(\log_7 \theta)$$

$$\boxed{91} \quad \int_2^4 x^{2x} (1 + \ln x) dx$$

$$\int (u)^2 \frac{\cancel{1 + \ln x} du}{\cancel{1 + \ln x}}$$

$$\int_4^{256} u du = \left[\frac{u^2}{2} \right]_4^{256}$$

$$= \frac{1}{2} [(256)^2 - 16]$$

$$U = x^x$$

$$\ln U = x \ln x$$

$$\frac{1}{U} \cdot \frac{dU}{dx} = x \frac{1}{x} + \ln x$$

$$\frac{1}{U} \frac{dU}{dx} = (1 + \ln x)$$

$$x=2 \longrightarrow U=2^2=4$$

$$x=4 \longrightarrow U=4^4=256$$

$$\boxed{98} \int_1^4 \frac{\log_2 x}{x} dx$$

$$U = \log_2 x$$

$$U = \frac{\ln x}{\ln 2}$$

$$du = \frac{1}{\ln 2} \cdot \frac{1}{x} dx$$

$$\Rightarrow x=1 \rightarrow U = \log_2 1 = 0$$

$$x=4 \rightarrow U = \log_2 4 = 2$$

$$\int_0^2 \frac{U \cdot x \cdot \ln(2)}{x} du$$

$$= \ln 2 \int_0^2 U du$$

$$= \ln 2 \left[\frac{U^2}{2} \right]_0^2$$

$$= \ln 2 \left[\frac{4}{2} - 0 \right] = 2 \ln 2 = \ln 4$$

$$\boxed{166} \int \frac{dx}{x (\log_8 x)^2}$$

$$U = \log_8 x = \frac{\ln x}{\ln 8}$$

$$du = \frac{1}{\ln 8} \cdot \frac{1}{x} dx$$

$$\int \frac{1}{x} \cdot \frac{x \cdot \ln(8)}{U^2} du = \ln 8 \int \frac{1}{U^2} du$$

$$= \ln(8) \left[-\frac{1}{U} \right] + C$$

$$= -\frac{\ln(8)}{\log_8 x} + C$$

$$= -\frac{[\ln(8)]^2}{\ln x} + C$$

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$$\int_1^{e^x} \frac{1}{t} dt$$

$$= \ln |t| \Big|_1^{e^x} = \ln |e^x| - \ln(1)$$

$$= \ln e^x - 0 = x$$

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$$\ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\frac{y'}{y} = \frac{\sin x}{x} + \ln(x) \cos(x)$$

$$y' = \left[\frac{\sin x}{x} + (\ln x)(\cos x) \right] x^{\sin x}$$

Note:

$$y = (\text{Variable})^{\text{variable}}$$

Use logarithmic differentiation to find the derivative of y

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$$y = \int_1^{e^x} \frac{1}{t} dt$$

$$y = (\ln x)^{\ln x}$$

$$\ln y = \ln (\ln x)^{\ln x}$$

$$\ln y = (\ln x) (\ln \ln x)$$

$$\frac{y'}{y} = \cancel{(\ln x)} \cdot \frac{1}{\cancel{\ln x}} \cdot \frac{1}{x} + \ln(\ln x) \cdot \frac{1}{x}$$

$$y' = \left[\frac{1 + \ln(\ln x)}{x} \right] (\ln x)^{\ln x}$$